Appendix A. General Results for the Laplacian Operator in Bounded Domains

In Chapter 2, Section 2.9, we used the result that a function $u(x)$ with $u_x = 0$ on $x = 0, 1$ satisfies

$$\int_0^1 u_{xx}^2 \, dx \geq \pi^2 \int_0^1 u_x^2 \, dx \quad (A4.1)$$

and the more general result

$$\int_B |\nabla^2 u|^2 \, dr \geq \mu \int_{\partial B} \|\nabla u\|^2 \, dr, \quad (A4.2)$$

where $B$ is a finite domain enclosed by the simply connected surface $\partial B$ on which zero-flux (Neumann) conditions hold; namely, $n \cdot \nabla u = 0$ where $n$ is the unit outward normal to $\partial B$. In (A4.2), $\mu$ is the least positive eigenvalue of $\nabla^2 + \mu$ for $B$ with Neumann conditions on $\partial B$ and where $\| \cdot \|$ denotes a Euclidean norm. By the Euclidean norm here we mean, for example,

$$\| \nabla u \| = \max_{r \in B} \left[ \sum_{i,j} \left( \frac{\partial u_i}{\partial x_j} \right)^2 \right]^{1/2} \quad (A4.3)$$

$$r = (x_j), \quad j = 1, 2, 3; \quad u = (u_i), \quad i = 1, 2, \ldots, n.$$ 

We prove these standard results in this section: (A4.1) is a special case of (A4.2) in which $u$ is a single scalar and $r$ a single space variable.

By way of illustration we first derive the one-dimensional result (A4.1) in detail and then prove the general result (A4.2).

Consider the equation for the scalar function $w(x)$, a function of the single space variable $x$, given by

$$w_{xx} + \mu w = 0, \quad (A4.4)$$

where $\mu$ represents the general eigenvalue for solutions of this equation satisfying Neumann conditions on the boundaries; namely,

$$w_x(x) = 0 \quad \text{on} \quad x = 0, 1. \quad (A4.5)$$
The orthonormal eigenfunctions \( \{ \phi_k(x) \} \) and eigenvalues \( \{ \mu_k \} \), where \( k = 0, 1, 2, \ldots \), for (A4.4) and (A4.5) are

\[
\phi_k(x) = \cos \frac{\mu_k}{2} x, \quad \mu_k = k^2 \pi^2, \quad k = 0, 1, \ldots
\]  

(A4.6)

Any function \( w(x) \), such as we are interested in, satisfying the zero-flux conditions (A4.5) can be written in terms of a series (Fourier) expansion of eigenfunctions \( \phi_k(x) \) and so also can derivatives of \( w(x) \), which we assume exist. Let

\[
w_{xx}(x) = \sum_{k=0}^{\infty} a_k \phi_k(x) = \sum_{k=0}^{\infty} a_k \cos(k\pi x),
\]

(A4.7)

where, in the usual way,

\[
a_k = 2 \int_0^1 w_{xx}(x) \cos(k\pi x) \, dx, \quad k > 0
\]

\[
a_0 = \int_0^1 w_{xx}(x) \, dx = [w_x(x)]_0^1 = 0.
\]

Then, integrating (A4.7) twice and using conditions (A4.5) gives

\[
w(x) = \sum_{k=1}^{\infty} -\frac{a_k}{\mu_k} \phi_k(x) + b_0 \phi_0,
\]

where \( b_0 \) and \( \phi_0 \) are constants. Thus, since \( a_0 = 0 \),

\[
\int_0^1 w_x^2(x) \, dx = [ww_x]_0^1 - \int_0^1 ww_{xx} \, dx
\]

\[
= -\int_0^1 w_{xx} \, dx
\]

\[
= \int_0^1 \left[ \sum_{k=1}^{\infty} \frac{a_k}{\mu_k} \cos(k\pi x) \right] \left[ \sum_{k=1}^{\infty} a_k \cos(k\pi x) \right] \, dx
\]

\[
+ b_0 \phi_0 \int_0^1 \left[ \sum_{k=1}^{\infty} a_k \cos(k\pi x) \right] \, dx
\]

\[
= \frac{1}{2} \sum_{k=1}^{\infty} \frac{a_k^2}{\mu_k}
\]

\[
\leq \frac{1}{2\mu_1} \sum_{k=1}^{\infty} a_k^2
\]

\[
= \frac{1}{\mu_1} \int_0^1 w_{xx}^2 \, dx = \frac{1}{\pi^2} \int_0^1 w_{xx}^2 \, dx,
\]

which is (A4.1); \( \mu_1 \) is the smallest positive eigenvalue \( \mu_k \) for all \( k \).
The proof of the general result (A4.2) simply mirrors the one-dimensional scalar version.

Again let the sequence \( \{ \phi_k(r) \} \), \( k = 0, 1, 2, \ldots \) be the orthonormal eigenvector functions of

\[
\nabla^2 w + \mu w = 0,
\]

where \( w(r) \) is a vector function of the space variable \( r \) and \( \mu \) is the general eigenvalue. Let the corresponding eigenvalues for the \( \{ \phi_k \} \) be the sequence \( \{ \mu_k \} \), \( k = 0, 1, \ldots \), where they are so ordered that \( \mu_0 = 0, 0 < \mu_1 < \mu_2 \cdots \). Note in this case also that \( \phi_0 = \text{constant} \).

Let \( w(r) \) be a function defined for \( r \) in the domain \( B \) and satisfying the zero-flux conditions \( n \cdot \nabla w = 0 \) for \( r \) on \( \partial B \). Then we can write

\[
\nabla^2 w = \sum_{k=0}^{\infty} a_k \phi_k(r),
\]

\[
a_k = \int_B \langle \nabla^2 w, \phi_k \rangle \, dr,
\]

(A4.8)

\[
a_0 = \langle \phi_0, \int_B \nabla^2 w \, dr \rangle = \langle \phi_0, \int_{\partial B} \nabla w \, d\mathbf{r} \rangle = 0.
\]

Here \( \langle \cdot \rangle \) denotes the inner (scalar) product. Integrating \( \nabla^2 w \) twice we get

\[
w(r) = \sum_{k=1}^{\infty} -\frac{a_k}{\mu_k} \phi_k(r) + b_0 \phi_0,
\]

where \( b_0 \) and \( \phi_0 \) are constants. With this expression together with that for \( \nabla^2 w \) we have, on integrating by parts,

\[
\int_B \| \nabla w \|^2 \, dr = \int_{\partial B} \langle w, n \cdot \nabla w \rangle \, d\mathbf{r} - \int_B \langle w, \nabla^2 w \rangle \, dr
\]

\[
= \sum_{k=1}^{\infty} \frac{a_k^2}{\mu_k}
\]

\[
\leq \frac{1}{\mu_1} \sum_{k=1}^{\infty} a_k^2
\]

\[
= \frac{1}{\mu_1} \int_B | \nabla^2 w |^2 \, d\mathbf{r},
\]

which gives the result (A4.2) since \( \mu_1 \) is the least positive eigenvalue.
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Index

Acetabularia, 141, 142, 180
  hair patterning, 180
  regeneration, 181
  whorl regeneration, 181
Actin alignment, 482
  wound model prediction, 489
Actin cable
  formation, 471
  role in wound healing, 468
  time of formation, 468
Actin density function, 473
Actin distribution
  wound model prediction, 488
Actin filament
  density function, 486
  stress-induced alignment, 474
Activator, 76
Activator–inhibitor, 87
  kernels, 624
  kinetics, 77
  neural model, 614, 623–624
  reaction diffusion system, 80, 135
  robustness, 113
Actomyosin, 369, 370
Adelson, D.A., 350, 382
Adzick, N.S., 470
Agladze, K.I., 4, 59, 60
Agladze, K.L., 259
Alberch, P., 354, 400–402, 404, 405, 408, 412, 413
Alberts, B., 317, 473
Algae, 180
Allessie, M.A., 55, 56
Alligator
  default colour, 200
  dental determinant, 209
  effect of temperature on stripes, 193
  genetics, 193
  growth data, 198
  melanin, 195–196
  order of tooth appearance, 209
  pattern and sex, 194
  shadow stripes, 193, 203
  shift experiments, 194
  stripe distribution, 198
  stripe number, 195
  stripe pattern, 193
  stripes, 194, 200
  surgical manipulation, 192
  teeth sequence, 209
  time of stripe formation, 196
Alligator growth
  effect of temperature, 193
  Alligator mississippiensis, 192
Alligator stripes
  effect of temperature, 193, 195
Alligator teeth
  comparison with data, 226
  virtual experiments, 228
Alt, W., 260
Alter, M., 360
Alvord, E.C., 538, 539, 540, 542, 543, 545
Amberger, V.R., 550
Ambystoma mexicanum (salamander), 406
  foreleg, 406, 411, 413
Amemiya, T., 54
Amplitude equations
  bacteria model, 291
Anaesthetic
  mediaeval recipe, 442
Anderson, A.R.A., 418, 612–613
Anderson, R.M., 681, 684
Anderson, S.C., 383
Andersson, J., 161
Ando, J., 418
Andral, L., 685, 694, 695
Angelfish stripes, 204
Angiogenesis, 417
  cell-matrix interactions, 417
  endostatin, 416
Angiogenesis (continued)
mechanical model, 419
model analysis, 427
model network patterns, 433
parameter domain for pattern, 432
parameter values, 425
Angiogenesis model
parameter domain for pattern, 429
Angrist, S.E., 185
Animal coat patterns, 141
computed patterns, 147
legs, 146
polymorphism, 154
size, 145
tail, 147
terratologies, 156
variation, 154
Anteater (Tamandua tetradactyl), 153
Anti-angiogenesis, 416
Apical ectodermal ridge, 350
Aragón, J.L., 191, 237
Archer, C., 403
Arcuri, P., 79, 110–112, 119, 120, 398
Aris, R., 260
Armadillo, 383
Arrhenius temperature variation, 183
Arriaza, B.T., 536
Artois, M., 683
Asaad, G., 655
Asai, R., 204
Atherosclerosis, 491
Atrial flutter, 42
Aubert, M., 672
Aubert, M.F.A., 683
Aubin, J.-P., 71
Autocatalysis, 77
Axial condensation (cells), 351–352
Axon, 614

Bacillus pestis (plague), 665
Bacon, P.J., 675
Bacteria
Bacillus subtilis, 306
background, 253
chemotactic response function, 262
chemotaxis, 253
dense-branching patterns, 309
diffusion coefficient, 261
experimental results, 253
experiments, 254
ice minus, 21
liquid medium model, 265
liquid model simulations, 275
mathematical model, 264, 265
model simulations and experiments, 299
nonlinear analysis, 287
pattern experiments, 254
pattern formation analysis, 267
proliferation, 263
semi-solid medium model, 264, 279
semi-solid model analysis, 281
simulation results for semi-solid models, 292
spatial patterns, 257
stochastic fluctuation, 307
swarm ring patterns, 299
swarm ring stability, 303
two-dimensional swarm ring results, 305
values of diffusion coefficient, 262
word equation model, 260
yield coefficient, 265
Bacterial patterns
analysis interpretation, 274
dense-branching, 309
experimental initial conditions, 297
intuitive explanation, 266
linear problem, 286
liquid model simulations, 275
liquid phase analysis, 267
model simulations, 297
numerical solutions, 274
swarm ring model results, 299
Bacterial territories, 57
Bagnara, J.T., 236
Bankivia fasciata, 650
Bard, J.B.L., 144, 148, 149, 358, 518
Barinaga, M., 311
Barlow, F.S., 13
Barnes, R.D., 639
Barocas, V.H., 312, 426, 497, 501, 531
Baron, A., 152
Barrat, J., 672
Barrio, R.A., 100
Basal lamina, 367
Basic reproduction rate (epidemic), 662
Basin of attraction, 99
Begg, D., 375
Belousov–Zhabotinskii reaction
travelling wavefront, 35, 39
wavespeed, 39
Belted Galloway cows, 152
Bement, W.M., 482
Ben-Jacob, E., 259
Ben-Yu, G., 130
Bentil, D.E., 224, 312, 332, 362, 363, 364, 391, 684
Berding, C., 101, 103, 139
Bereiter-Hahn, J., 446
Berg, H.C., 253, 255, 257, 258, 261, 264
Bertolami, C.N., 467, 494, 498
Biochemical switch, 165
Biomaterials, 502
Biomechanics evolving tissue, 502
Black Death, 661 spread, 664, 665
Blankenburg, F.G., 549
Boegel, K., 684
Boehm, T., 417
Bonotto, S., 180, 181
Born, W., 410
Bosch, Hieronymus, 409, 536
Brady, R.H., 71
Brain coronal section, 567 cross-section, 540 hallucination patterns, 627 human, 541 human tumour invasion, 566 McConnel Brain Imaging Centre, 545 Web database, 564
Brain surgery historical, 536
Brain tumour, 539 anaplastic astrocytomas, 539 astrocytomas, 539 best prognostic position, 573 chemotherapy, 594 gliomas, 539 grade, 539 human, 563 model limitations, 578 rat, 559 recurrence, 592 survival time, 547, 580, 582 therapy, 580 three dimensions, 576 treatment difficulties, 539 untreated survival time, 571 virtual tumour, 567 with spatial heterogeneity, 544 worst prognostic position, 573
Brain tumour model parameter estimation, 550 BrainWeb database, 564
Bridge, J.F., 185
Britton, N.F., 49
Brown, K.C., 16, 17 Brown, M.D., 418 Brungal, G., 451 Bryant, S., 490
Butterfly (and moth) wing patterns, 161 buckeye (Precis coenia), 163, 174, 179 cautery experiments, 169, 180 Cethosis, 179 Crenidomimas cocordiae, 179 dependent patterns, 170, 173 Dichorragia nesimachus, 161 eyespots (see also ocelli), 163, 174 Hamanumida daedalus, 179 Iterus zalmoixis, 174 Lynantria dispar, 162 Mycalesis maura, 176 ocelli, 162, 174 Precis coenia: see buckeye, 163 Psodos koracina, 170 Stichophthalma camadeva, 161 Taeneris domitilla, 176 temperature effects, 177 transplant experiments, 174 Troides haliphron, 173 Troides hypolitus, 173 Troides prattorum, 173, 174 wing venation, 163 Butterfly eyespot growth buckeye, 177 Byrne, H.M., 613
Calcium conservation equation (cytogel model), 372 effect on Acetabularia hair spacing, 190 stimulated calcium release mechanism, 371 threshold kinetics, 371 waves on eggs, 373 California king snake, 73
CAM
  cell adhesion molecule, 387
teeth, 214
Campbell, J., 649, 650
Camar, A., 665
Cancer
  melanoma, 613
Cantrell, R.S., 549
Carnivores, 142
Carpenter, G.A., 43
Carroll, S.B., 163
Cartilage
  abnormal patterns, 407
  condensations, 350
  morphogenetic rules, 355
Cartilage patterns
  effect of colchicine, 405
Castets, V., 101
Cats (Felidae), 148
  coat patterns, 148
Cell
  aggregations, 350–351
  cell contact inhibition, 327
  chemotactic flux, 323
  chemotaxis model, 403
  chondrocyte, 350
  chromatoblasts, 236
  conservation equation, 320
  contact guidance, 518
  convective flux, 321
  dermal, 317
  differentiation, 74
  embryonic, 317
  epidermal, 317, 447
  epithelial, 639
  excitatory, 629
  fibroblast, 317
  galvanotaxis flux, 323
  guidance cues, 323
  haptotactic flux, 323
  inhibitory, 629
  matrix field equations, 328
  matrix mechanical interaction equation, 324
  membrane, 374
  mesenchymal, 318, 320
  motile, 318
  neuronal, 614
  pattern bifurcations, 351, 352
  pigment, 144, 171, 640
  proliferation rate, 320
  random dispersal, 321
  retinal, 622
  secretory, 639
  traction force, 318, 320, 324, 326
  transport, 321
Cell adhesion molecule mechanism, 387
Cell adhesion molecules
  N-CAMs, 387
Cell chemotaxis model
  analytical results, 251
Cell traction
  experimental data, 326
  stress forms, 327
Cell-chemotaxis model, 238
  propagating pattern generator, 248
Cellular automata, 144, 638, 639
Central ganglion, 639
Central symmetry patterns, 162–164, 168, 169
  experiments, 168, 169
  generating mechanism, 164
  scale and geometry effects, 168, 170, 171
Centre National d’Études sur la Rage, 673, 674, 675
Cerebrum cortex
  waves, 55
Chaos
  spatial, 65
  wave induced, 41
Chaplain, M.A.J., 141, 418, 613
  characteristic polynomial, 8, 84, 140
  neural activity (shell) model, 644
  reaction diffusion system, 84
Cheetah
  coat pattern abnormality, 158
  Cheetah (Acinonyx jubatis), 147, 148
Chemical prepattern, 74, 312
  animal coat markings, 144
  comparison with mechanochemical pattern generation, 315
Chemoattractant
  aspartate, 254
  succinate, 254
Chemotaxis
  bacteria, 253
  bacteria-nutrient system, 68
  cell, 319
  effect of parameter variation, 243
  response function, 262
Chemotherapy, 541, 594
  medical details, 595
  numerical methods, 600
  numerical results, 600
  patient data, 595
  two-cell type model, 598
Chen, W.F., 473, 505
Chick limb chondrogenesis, 351
Chicoine, M.R., 539, 540, 541, 543, 544, 550, 551, 552, 553, 554, 555, 559
Children’s scribblings, 656
Deer (continued)
survival, 751
winter yards, 728
Defaure, J.P., 239
Defoe, Daniel, 667
delta function, 220
Delvoye, P., 534
Dendrite, 614
Dentition, 205
human vs alligator, 207
Denton, E.J., 205
Dependent patterns, 170, 173, 174
Dermal papillae, 347
Dermal wound
area, 493
basic healing scenario, 493
mechanical properties, 494
shape, 494
tension, 494
Dermal wounds
background, 491
Dermatoglyphics, 358
Dermatoglyphs
model comparison with experiment, 364
Determination stream, 162, 163, 169
hypothesis, 162
Development
limb, 118
pattern and form, 72
sequence, 113
Developmental
bifurcation programme, 396
constraint, 400, 407, 411, 414
laws, 414
Developmental biology
survey of unanswered questions, 311
Dhouailly, D., 346, 381, 402
Dictyostelium discoideum, 4, 5, 61
spatial patterns, 137
spiral patterns, 57
Diffusing morphogen
gene activation system, 164
Diffusion
anisotropic, 138
cross, 11
field, 164, 178
fox, 717
in oriented environments, 518
in a strained field, 520
unidirectional, 520
Diffusion coefficient
anisotropic, 180
critical ratio, 85
FHN (Fitzhugh–Nagumo) system, 41
long range, 323, 654
Diffusion damping, 132
Diffusion-driven instability, 82
boundary conditions, 84
continuous eigenvalues, 90
different case scenarios, 87
general conditions, 82, 87
initial conditions, 89
parasite analogy, 88
predator–prey analogies, 88
schematic illustration, 88
Digital arch, 403
Dilation (matrix), 325
Dillon, R., 353, 358
Dispersion relation, 63, 86, 90, 93, 95, 96, 103,
104, 617, 645
complex (mechanical models), 334
fast focusing, 339
infinite range of unstable wavenumbers, 342,
620
mechanical (cell–matrix) model, 329,
332
mode selection, 113
neural activity (shell) model, 644
sol–gel mechanochemical model, 376
spiral wave, 62
Dowson, T.A., 657
Drosophila melanogaster (fruit fly), 141
Drug
hallucinogenic, 627
Dufaure, J.P., 236
Duffy, M.R., 53, 61, 64
Dunbar, S.R., 5, 7, 9
Dunn, M.G., 510, 534
E. coli patterns, 257
ECM (extracellular matrix), 317
adhesive sites, 323
displacement, 321
effective strain, 513
plastic respone in wounds, 504
zero stress state, 513
ECM plasticity, 513
ECM remodelling, 503
Ecological
control, 125, 130
Edelman, G.M., 214, 389
Edelstein, B.B., 166
Edelstein-Keshet, L., 365, 500–501, 530
Edmund, A.G., 213
Effective strain matrix, 515
Eigenfunction
1-dimensional, 90–91
2-dimensional, 92
axisymmetric, 136
linearly unstable, 91
Ferguson, M.W.J., 192–194, 197, 198, 201, 206, 209, 214, 219, 236, 470, 489
Feroe, J.A., 43, 66
Ferrenq, I., 312, 319, 326, 423, 497, 531, 534
Fibre alignment, 325
Fibrillation (cardiac), 42
spiral waves, 56
Fibroblasts
development of curved ridges, 364
human, 364
Field, R.J., 49
Fife, P.C., 2
Filopodia, 317
Fingerprint, 622
Fingerprints
chromosomal aberrations, 359
comparison algorithms, 360
dermis-epidermis interaction, 360
formation, 358
model comparison with experiment, 364
unusual patterns, 359
Firing rate, 629
Fish communication, 205
Fish pigmentation patterns, 205
Fisher–Kolmogoroff
diffusion estimate, 556
Fishing zone, 136
Fitzhugh–Nagumo model
piecewise linear, 69
Focal condensation (cells), 355
Folkman, J., 318, 324, 416, 417, 446
Ford, R.M., 262
Forrester, J.S., 491
Fowler, A.C., 720
Fox
epizootic, 675
immunity, 710
population in England, 705
rabies vaccination, 717
Frantz, J.M., 448, 452
Fremuth, F., 447
French, V., 162, 163, 174
Frenzen, C.L., 210
Frerichs, R.R., 719
Frog (Xenopus laevis), 405, 406
Fruit fly (Drosophila melanogaster), 141
Fulica atra (common coot), 383
Fung, Y.C., 418, 426, 502
Furnas, D.W., 538
Gáspár, V., 4
Gabbiani, G., 491
Gale, E., 404, 405
Galen, 441
Gallin, W.J., 387
Galvanotaxis, 319, 323
Garnerin, P., 716
Gaszik, L.E., 579
Genes, 312
Genet (Genetta genetta), 147, 148
Genetic modification
animals, 21
Genetic mutation, 411
Genetically engineered microbes
patch size effects on invasion, 34
Genetically engineered organisms
containment, 25, 27
invasion conditions, 27
risks, 21
spatial spread, 18
stability and diffusion, 34
Geneticaly engineered organisms, 18
Genetics
role in pattern, 193
Geoffroy St.-Hilaire, L., 71, 410
Geographic spread of epidemics, 661
Geometry
effect on pattern, 103
Gerber, A., 383
Gibbs, R.J., 41
Gierer, A., 77, 79, 113
Giese, A., 543, 550, 555, 562, 579, 592
Gilligan, C.A., 668
Giraffe
coat patterns, 143, 150
embryo, 150
Giraffa camelopardis, 150
Giraffa camelopardis reticulata, 150
Giraffa camelopardis rothschildi, 150
Giraffa camelopardis tippelskirchi, 143
Glioma, 538
basic model, 542
Glioma cell
diffusion in vitro, 550
motility, 550
Gliomas, 539
Glyptodon (armadillo), 383
Goethe, J. von, 71
Goldie, J.H., 598
Goldschmidt, R., 163
Goldstein, S., 12
Gompertz, 210
Goodwin, B.C., 181, 182, 187
Gould, S.J., 312
Green, H., 360, 361, 467, 532
Greenberg, J.M., 61
Greenblatt, S.H., 536
Greene, H.W., 235
Gregg, C.T., 667
Grindrod, P., 54, 55, 241
Janney, P.A., 471
Jennings, R.W., 503
Jester, J.V., 491
Johnson, D.R., 328, 350, 355, 402, 403
Jordan, D.W., 6
Jowett, A.K., 210
Krinsky (Krinskii), V.I., 4, 54, 56, 59, 60, 61
Kronmiller, J.E., 210
Kruuk, H., 73, 143, 147
Kuhn, A., 162–164, 168, 169
Kulesa, P.M., 117, 208, 210, 211, 213, 217, 219, 224, 227, 228, 231–234
Kuramoto, K., 4, 61, 64, 65
λ–ω system, 50
polar form, 62
spiral waves, 61, 63, 64
wavetrain solutions, 49, 52
Lamellapodia, 318
Landau equations, 291
Landau, L., 324, 325, 506
Lane, D.C., 374
Langer lines, 513
Langer, J.S., 117
Langer, K., 513
Langer, W.L., 665
Lapidus, R., 262
Laplacian operator
general results, 757
Lara-Ochoa, F., 87, 633
Lateral geniculate nucleus, 622
Lauffenburger, D.A., 260, 262
Le Douarin, N.M., 238
Lee, B., 426
Lefever, R., 5
Lejeune, O., 5
Lemke, L., 73, 235
Lenoir, R., 71
Leopard (Panthera pardus), 73, 142, 143
cost patterns, 143
tail patterns, 148
Lepidoptera (see also butterfly, moth)
generalised wing, 163
Leslie matrix, 16
Levin, S.A., 130
Levinson, N., 28
Levinton, J., 411
Leviton, A.E., 383
Lewis, J., 468, 469, 475, 479, 484, 485, 488, 530
Lewis, M.A., 34, 312, 723, 743, 753, 754
Lewis-Williams, J.D., 657
Liang, B.C., 504
Lifshitz, E., 324, 325, 506
Limb bud, 351, 354
Lindley, S., 21
Lindquist, G., 448, 452
Lions, P.L., 124
Little, C., 418, 421
Liu, S.Q., 418, 502

Källén, A., 675, 680, 694
Kamiya, A., 418
Kaplan, C., 675
Kareiva, P., 34, 130
Karev, G.B., 362
Kath, W.L., 168, 172
Kauffman, S.A., 141
Kawasaki, K., 22, 307–310, 549
Keeling, M.J., 668
Keener, J., 43, 45, 47, 48, 54, 61
Keller, E.F., 5, 259
Keller, J.B., 5, 42, 374
Kelley, P.J., 538, 540, 543, 544
Kellogg, R., 656
Kennedy, C.R., 260
Kerbel, R.S., 417
Kernel
function, 629, 631
influence, 615, 616
local activation–long range inhibition, 616, 641
moments, 620
symmetric exponential, 617, 620, 631
Keuck, G., 312
Kevorkian, J., 551
Killer bees, 130, 720
Kinetics
delay, 2
Gierer–Meinhardt, 77, 79
marginal state, 110
Schnakenberg, 76, 79
Thomas, 77, 79
Kingdon, J., 142
Kischer, C.W., 494
Kiviniemi, K., 451
Klagsbrun, M., 417
Klauber, L.M., 235
Klee, M.R., 637
Klingler, M., 638
Klüver, H., 627
Koch, A.J., 77
Koga, S., 4, 61, 63–65
Kolega, J., 473
Kollar, E.J., 210, 360
Koldodney, M.S., 427
Kondo, S., 204
Kopell, N., 49–51, 61
Kreeth, F.W., 538, 540, 542, 581
Moth (see also butterfly)
antenna, 72
black mountain (Clostera curtula), 170
Chocolate chip (Psudos coracina), 170
Ephistia, 164
Ephistia kuhniella, 162, 164, 169
Hyalophora cecropia, 73
outbreak, 129
simulated cautery experiments, 169
wing patterns, 161
Mouse (Mus musculus), 406
Müller, S.C., 4, 54, 58
Mus musculus (mouse), 406
Mycalesis maura, 176
Myerscough, M.R., 116, 117, 197, 239, 242, 245, 248, 249
Myosin, 369
Nagawa, H., 350, 367, 376, 382
Nagorecka, B.N., 100, 350, 383, 386
Nakanishi, Y., 350, 367, 376
Natural selection, 396
Nazarro, J.M., 542, 584
Nelson, M.E., 749
Neo-Darwinism, 397
Neritta turrita, 649
Nerve cells, 614
Network
spatio-temporal evolution, 420
Neural
activity, 627
activity model for shell patterns, 639
instability, 631
stimulation, 639
Neural firing
weighting function, 615
Neural model
dispersion relation, 632, 633
pattern formation, 614, 629
shell pattern, 639, 640
spatial firing, 614
stability analysis, 631, 642
Neural shell model
  continuous time analogue, 651
Neuron, 614, 622
  autonomous firing rate, 614, 615
mantle (mollusc), 639
Neuronal process, 614
Neuwelt, E.A., 542, 584
Newell, P.C., 3, 4, 57
Newell-Morris, L., 360
Noble, J.V., 667
Nonlocal
  dispersion (cells), 322
  elastic interactions, 325
Null clines
  excitable kinetics, 43
FitzHugh–Nagumo system, 43
Gierer–Meinhardt kinetics, 79
Schnakenberg kinetics, 79
Thomas kinetics, 79
Nymphalids, 161, 162
  ocelli, 163
  wing pattern groundplan, 161, 162
O’Reilly, M.S., 416
Obrink, B., 214
Ocelli patterns, 174, 176
  model mechanism, 174
  temporal growth, 177
Ocular dominance stripes, 614, 622
  activation/inhibition domains, 623–624
  activation/inhibition kernels, 624
  effect of domain growth, 626
  generating mechanism, 623
  macaque monkey, 622
Odell, G.M., 312, 368, 373, 402, 473, 474, 482
Ohgiwara, M., 307
Okajima, M., 360
Okubo, A., 13, 16–19, 730, 733
Olsen, L., 500, 531, 534
Ontogeny, 312
Open loop system, 317
Optic nerve, 622
Ortoleva, P.J., 69
Osborn, J.W., 213, 219
Osmotic
  collapse, 357
  pressure, 377
Osteoderm, 383
Oster, G.F., 104, 312, 314, 320, 346, 350, 355, 357, 368, 374, 400–403, 405, 471, 472, 473, 474, 482, 485, 495, 530, 627
Othmer, H.G., 134, 141, 182, 260, 322, 353, 358
Ottaway, J.H., 601
Otto, H., 382
Ouyang, Q., 101
Owen, M.R., 613
Pacemaker, 3–4
  chaotic, 4
Painter, K.J., 204, 237
Pallister, J.-L., 409
Papilionidae (butterflies)
  wing patterns, 173, 174
Papilla, 346, 347, 401
Paré, Ambroise, 409
Parameter space, 106
  parametric method, 105
Partanen, A.M., 210
Pascual, M., 4
Pastoret, P.P., 672
Patan, S., 418
Pate, E., 182
Patou, M., 407
Pattern
Acetabularia hair, 180
  animal coat, 141
  animal leg, 145
  basin of attraction, 392
  belted cows, 152
  bifurcating sequence, 152
  bifurcation, 148, 154
  butterfly eyespot growth, 177
  butterfly wing, 161
  cartilage (limb), 350
  chondrogenic, 352
  complex, 382
  computed, 147
  critical domain size, 121
  dependent, 170
  developmental biology, 94
  doubly periodic tessellation, 629, 634
  dynamics in growing domains, 117
  ecological, 120
  energy function, 103
  finite amplitude, 141
  formation in biology, 71
  generation in single-species models, 120
  hallucinogenic, 627, 628, 636
  heterogeneity function, 103, 131
  hexagonal pattern of feather primordia, 347, 348
  initiation, 90
  initiation trigger, 114
  leopard spot size, 145
  lepidopteran wing, 162
  microvilli, 374
  nonexistence in reaction diffusion systems, 130
  ocular dominance stripe, 622, 626
Pattern formation

bacteria model, 264, 265
fast focusing, 339
in growing domains, 117
sequential, 391
space, 91
tissue interaction, 381

Pattern formation mechanism

animal coat markings, 154
cell(fibroblast)-matrix, 329
cytogel, 369
dependent (butterfly wing), 170
epidermal–dermal tissue interaction, 381
feather germ primordia, 345
initial conditions, 113
interaction models, 387
mechanical models, 312
microvilli, 374
mode selection, 110
neural, 614, 629
neural (shell) model, 639
ocular dominance stripe, 624
robustness, 113
sensitivity, 113
whorl (Acetabularia), 182
wing pattern, 164

Pattern robustness, 391

Patterns

butterfly, 162
E. coli, 257
holograph, 155
Pavelka, M, 414
Peacock, E.E., 493
Pelmont, J., 451
Penrose, R., 345, 358, 365
Pepys, Samuel, 667
Perelson, A.S., 332, 348, 403
Perichondrium, 403
Perumpanani, A.J., 319
Peters, R.P., 727, 747, 749
Peterson, R.O., 726
Petroll, W.M., 491
Petrov, V., 4
Perucelli, R.J., 441
Phalange, 351
Phenomenological pattern, 411
Pheromone, 72
Phillips, B.R., 261
Phosphenes, 655
Phylectic gradualism, 397
Phylogeny, 312
Pigment cells, 144
domain in wing (lepidopteran) patterns, 173
Pilkington, G.J., 550, 605
Placode, 346, 347, 401

Plague

Bacillus pestis, 665
Black Death, 665
current incidence and model, 668
Great Plague (London), 667
residual foci, 667
San Francisco epidemic, 667
septicemic, 666
20th century, 664
Plankton–herbivore system, 7
Plessor, T., 54
Pocock, R.I., 157
Pollard, T.D., 471
Polyclones
transition of dominance, 608
Portmann, A., 142
Positional information, 74, 96
Post-fertilisation (egg) waves, 373
Potten, C.S., 450
Powell, F.C., 527
Prawda, J., 719
Precis coenia (buckeye butterfly), 163
Predator–prey
blow-up, 12
pursuit and evasion, 9
waves, 5
wolf–moose, 12
Pregnancy stretch marks, 526
Prepattern

hair initiation (Acetabularia), 190
morphogen, 144
theory, 101
Price, R.J., 418
Price, T., 414
Rabies
- period of recurring epidemics, 694, 709
- secondary outbreak, 707
- simple model for spatial spread, 673
- SIR model for spatial spread, 681
- spread, 666, 673, 695
- two-dimensional epidemic wavefront, 695, 707, 708
- vaccination control, 709
- wavespeed dependence on carrying capacity, 692

Rabies break
- analytical approximation, 700
- killing, 715
- method comparison, 716

Radice, G., 446
Rabies (canine), 720

Rabies (fox)
- break, 695, 698
- break width, 700, 710
- control strategies, 675, 696, 709
- English 'experiment', 682
- epidemic, 661
- epidemic fluctuations, 679
- epidemic frequency, 693
- epidemic wave, 663, 677, 690
- epidemic wavespeed, 673, 677, 692
- epizootic in England, 706, 708
- estimate of model diffusion coefficient, 694
- fox density (England), 706
- furious, 682
- outbreak in England, 704
- outbreak predictions, 704

Rabies break
- analytical approximation, 700
- killing, 715
- method comparison, 716

Radice, G., 446
Rabies (canine), 720
Restinosis, 491
Retinal ganglion cell, 81
Retino-cortical magnification factor, 628
Retinoic acid, 354, 402
Reynolds, J.C., 12, 17
Rhinoceri, 154
Richardson, M.K., 74, 312
Riddle, R.D., 358
Rinzel, J., 42, 43, 46, 49, 70, 374
Risse, G.B., 667
Ritvo, H., 671
RLU markings
distribution, 727
Robertson, I.M., 656
Robertson, S.C.J., 656
Rock art, 658
Rome, S.C.J., 656
R⪡ose, C., 213
Rowe, D.M., 205
Rytomaa, T., 451
Sage, E.H., 417, 419
Salamander
limb cartilage variant, 412
paedomorphic form, 412
Saliou, P., 672
Savic, D., 144
Scale
critical, 95, 134
effect on pattern, 103
effects, 151
invariant mechanisms, 182
isolation of unstable modes, 111
parameter, 89, 94
Scales, 381
epidermal, 381, 383
Scarring
pathological, 494, 526
Schaap, P., 260
Schaumann, B., 360
Scherer, G.W., 427
Schiller, R., 262
Schmidt, L., 747
Schmidt, S.L., 69
Schmitz, G., 34
Schnakenberg kinetics, 76
Turing space, 107, 108
Schnakenberg, J., 76, 79, 113, 183
Schneider, L.G., 685
Schor, A.M., 418
Schwanwitsch, B.N., 161, 162, 164, 173, 179
Schwartz, V., 164
Scincus officinalis (lizard), 382
Searle, A.G., 142
Segel, L.A., 5, 259, 483
Segmental condensation, 356, 403
Sekimura, T., 162, 518
Self-organisation, 82
Sengel, P., 346, 381
Seward, W.L., 710, 713, 714, 719
Shadow stripes
angelfish, 204
Shamanism, 614, 655
Shapiro, B., 655
Shaw, C-M, 538, 539
Shaw, L.J., 384
Sheep
coat pattern teratology, 160
Sheldon, P.R., 397
Shell (mollusc)
Bankivia fasciata, 649–651
basic elements, 639
basic structure, 639
bifurcation to pattern, 644
Citarium picus, 638
continuous time model, 651
Conus episcopus, 651
Conus marmoreus, 638
Conus textus, 638
Nerita turrita, 649, 650
pattern interaction with geometry, 650
pattern polymorphism, 638
Shell patterns
examples, 638
neural activity model, 638
Sherratt, J.A.S., 4, 262, 327, 444, 446, 447, 451, 452, 458, 462, 463, 468, 469, 470, 475, 479, 481, 483, 484, 486, 487, 495, 500, 530, 531, 613
Shibata, M., 42, 55
Shigesada, N., 22, 28, 306, 309, 549
Shihira-Ishikawa, I., 180
Shochat, E., 595
Showalter, R.K., 4, 41, 54
Shubin, N., 404, 405, 412
Shuster, S., 491
Sibatani, A., 163
Silbergeld, D.L., 538, 539, 540, 541, 543, 544, 545, 550, 552, 553, 559, 560
Simon, R.H., 494, 498, 503
SIR (epidemic) models
spatial spread of rabies among foxes, 681
Skalak, R., 502, 530
Skalak, T.C., 418
Skin (organ) primordia, 319, 401
Skin patterns
snake, 234
Teeth, 192
barrier experiments, 231
CAM, 214
cameral model, 214
epidermal growth factor, 210
homeobox genes, 210
initiation data, 214
model experiments, 229
parameter estimates, 224
prediction experiments, 228
transplant experiments, 229
virtual experiments, 228
Zahnreihe theory, 213

Temperature shocks (wing patterns), 180
T. mauritanica (lizard), 382

Teratologies, 407
animal coat patterns, 156
magnesium chloride, 410
monster births, 410
Terman, D., 43, 46, 49, 70

Territory formation
single wolf pack, 729

Tessellation patterns
basic units, 343, 634
hexagon, 97, 343, 629, 635, 636
planar, 97, 629
polar form, 97, 343, 627, 635
regular, 343, 627, 629
rhombus, 99, 343, 637
roll/stripes, 100, 634
square, 98, 343, 637

Tetrapod
limb development, 405
vertebrate, 401

Teulières, L., 672

Thalidomide
anti-angiogenesis, 416

Thesleff, L., 210

Thoma, R., 418

Thomas
kinetics, 77
mechanism, 145

Thomas, D., 77, 79, 110–113, 145

Thomas, J., 360, 361, 532

Thompson, D’Arcy, W., 414

Thorogood, P., 536

Thril, C., 350, 354

Tiger (Felis tigris), 143
coat patterns, 143, 149

Tilman, D., 34

Timmenga, E.J.F., 494

Tissue compaction, 473
Tissue dilation, 473
Tissue interaction, 367
CAM mechanism, 387
effects, 388
mechanism, 381, 389
Tissue interaction models, 381
Tissue remodelling, 503

Tlidi, M., 5

Toga, A.W., 561

Toma, B., 685, 695

Tooth
dental determinant, 209, 222
initiation biology, 207
mesenchyme, 207
papilla, 207
primordium, 207
sequence, 211

Tooth primordium initiation
model, 213

Topological index of a pattern, 365


Traction (cell) forces, 326

Tranqui, L., 312, 319, 327, 419, 434, 531, 534

Tranquillo, R.T., 312, 493, 497–499, 500, 501, 531

Transition of dominance
different cancer cell lines, 609, 610

Travelling wave
Belousov–Zhabotinskii, 35
cytogel model, 373
initiation of pattern, 114
mechanical model, 337, 340
microorganisms, 68
pulse, 44, 46
trains, 2, 49

Treadwell, R.W., 237

Trepanning, 536

Treplication
Peru, 536

Trepining, 536

Tribal survival, 755

Trifurcating pattern, 407
Trifurcation, 402

Trinkaus, J.P., 319, 446

Troides
haliphron, 173
hypolitus, 173
prattorum, 174
Index 809

Tsujikawa, T., 61, 259

Tumour
  background, 538
  cell mutation, 605
  chemotherapy, 541
  chemotherapy treatment, 594
  detectable size, 548
  glioma, 538
  model uses, 580
  rat brain parameters, 559
  resection, 539
  resection treatment, 580
  treatment scenarios, 579

Tumour area
  measurement from CT scans, 595

Tumour biopsies
  failure, 611

Tumour cell diffusion
  grey and white matter, 558

Tumour facilitation
  human corpus collosum, 562

Tumour invasion
  dependence on grade, 568
  human brain, 563
  position dependence, 567
  rat brain, 559

Tumour model
  multi-cell polyclonality, 612
  with spatial heterogeneity, 544

Tumour predictions
  comparison with data, 581

Tumour recurrence
  analytical solution, 584

Tumour resection
  patient survival times, 581
  with spatial heterogeneity, 588

Tumour spread
  in vitro, 550
  one-dimensional analysis, 549
  parameter estimation, 550

Turing
  instability, 82
  mechanism, 75
  patterns, 101
  space, 91, 105, 107, 110
  structures, 101

Turing, A.M., 74–76, 141

Turner, L., 262

Turner, W., 409

Tyson, J., 2, 43, 54, 56, 61

Tyson, R., 5, 259, 261, 267, 275, 284, 287, 292, 439

Ulmus (bone), 351

Unanswered questions in development, 311

Vainio, S., 210

Valais goat (Capra aegragrus hircus), 152

Valenstein, E.S., 536

Vampires, 671

van Ballenberghe, V., 724

Van den Brenk, H.A.S., 447, 451–452

Vascular network
  dispersion relation, 430
  evolution, 421
  experimental model, 419
  model, 420

Vascularisation, 416

Vasculogenesis, 416
  model analysis, 427
  model network patterns, 433

Vasculogenesis network
  model patterns, 434, 435
  open problems, 439

Verano, J.W., 536

Vernon, R.B., 417, 421, 523

Vertebrate
  cartilage morphogenetic rules, 402
  limb construction scenario, 414
  limb development, 355
  skin, 346

Vibrations, 155
  plate, 155

Virtual tumour, 567
  initial location, 567

Viscosity
  bulk, 325
  shear, 325

Vision
  monocular, 623

Visual cortex, 614, 623
  basic geometric patterns, 627
  geometry of basic patterns, 629
  patterns, 630, 634
  stripe pattern formation mechanism, 622

Visuo-cortical transformation, 628, 630

Von Engelhardt, A., 162–164, 168, 169

Waddington, C.H., 638

Walbot, V., 317, 353

Walker, A.E., 536

Walter, M., 144

Wandeler, A., 684, 685, 696, 710, 715

Warrell, D.A., 671

Wasoff, F., 358

Watt, F.M., 324, 462

Wave
  chaotic, 4
  epidemic, 663
  epizootic, 675, 678, 685
Wave (continued)
epizootic speed of propagation, 685
evasion, 5
excitable media, 41
induced chaos, 41
invasion, 14
logic gates, 49
Lotka–Volterra, 5
multi-species, 1
muscle tissue, 42
oscillatory kinetics, 49
plague, 667
post-fertilisation, 374
pursuit, 5, 9
rabies epidemic, 663, 673
small amplitude wavetrain, 52
spiral, 4
spreading depression, 55
two-dimensional, 4
trains, 49
two-dimensional epizootic (foxes), 704
Wave length
critical, 90
hair spacing (Acetabularia), 186
variation with morphogen (calcium) concentration, 190
Wave vector, 90
Wavefront
Belousov–Zhabotinskii reaction, 35
Wavenumber, 84
critical, 105
discrete, 91
Weil, M., 579
Welch, M.P., 503
Wellcome Trust, 537
Wellmann, K.F., 655, 657
Welsh, B.J., 4, 59
Werner, S., 447
Wessells, N., 346
Westergaard, B., 206, 209, 214, 219
Westergaard, J.M., 696
Westphal, M., 562, 579
Whitby, D.J., 489
White, K.A.J., 733, 747, 748–750, 752
White-tailed deer, 725
Wiesel, T.N., 622
Wildlife Society, 724
Williams, S.K., 427
Williamson, M.H., 12, 16, 17
Williamson, P., 397, 411
Winfrey, A.T., 54, 56
Winters, K.W., 239
Wittenberg, R., 134
Woerdeman, M.W., 213
Wolf
probability density functions, 738
territory formation, 729
Wolf movement
chemotaxis, 742
Wolf pack
buffer zone, 739
splitting, 746
territories, 728
territory size, 734
Wolf reintroduction, 751
Wolf territoriality
effect of deer, 747
Wolf territory, 728, 729
single pack, 729
Wolf–deer interaction
deer extinction, 747
deer reproduction, 747
Wolf–deer model, 745
parameter estimates, 747
Wolfram, S., 638, 649
Wollkind, D.J., 100, 101
Wolpert, L., 74, 75, 164, 213, 314, 351–353
Wolves
Isle Royale, 724, 726
Minnesota, 724, 725
movement switching, 752
multi-pack model, 734
RLU influence, 741
role of seasonality, 727
scent marking switching, 752
two-pack model equations, 738
warning systems, 725
World Health Organisation (WHO), 667, 684, 695
Wound
burns, 491
corneal, 491
dermal, 491
dermal healing scenario, 495
embryo, 468
epidermal model results, 451
residual strain, 503
Wound dermal
animal-human differences, 498
basic model, 495
comparison with experiment, 507
ECM-cell interactions, 505
effective strain, 515
elastoplastic stress, 505
finite strain model, 521
healing quantification, 492
matrix degradation, 515
matrix secretion, 515
one-dimensional model, 526
pathological scarring, 500
pathological scars, 494
plastic response of ECM, 504
plasticity, 513
questions, 493
residual stresses, 502
review of developments, 500
small strain model, 525
strain matrix, 513
with tissue remodelling, 503
Wound dermal model
initial conditions, 524
Wound embryo
comparison with data, 479
critical parameter, 479
data retraction, 487
parameter interpretation, 481
stress alignment model, 482
two-dimensional model, 482
Wound epidermal
actin conservation, 473
carcinoma, 466
marsupials, 470
Wound healing
concluding remarks, 533
effect of geometry, 462
epidermal model, 447, 449, 451
history, 441
introduction, 444
logic, 495
open problems, 530
time prediction, 463
topical applications, 462
Wound model
with ECM structure, 521
finite deformations, 524
Wound repair
fetal, 206
Wound repair model
clinical implications, 461
Wounds
epidermal, 444
fetal, 470
scarless, 470
Wright, N.A., 451
Wyatt, T., 5
Wysolmerski, R.B., 427
Xenopus laevis (frog), 405, 406
Xu, Y., 155, 156
Y-(or branching) bifurcation, 404
Yachi, S., 716
Yagasiti, H., 4, 66
Yamaguchi, T., 447
Yamaguti, M., 11
Yoshikawa, A., 11
Young, D.A., 144
Yount, G.L., 580
Zaikin, A.N., 2
Zebra, 144
care pattern teratology, 159
care patterns, 143, 148, 149
carembryo, 149
Equus burchelli, 149
Equus grevyi, 143
Equus zebra, 149
gestation, 144
scapular stripes, 149
stripe pattern, 149, 622
Zehr, D.R., 236
Zhabotinskii, A.M., 2
Zhu, M., 87, 100, 101, 284, 287, 433, 439, 627
Zieske, J.D., 447, 452
Zimen, E., 696
Zone of influence (cells), 403
Zone of recruitment (cells), 403
Zonurus cordylus (lizard), 382
ZPA (zone of polarising activity), 352
Zweifel, R.G., 235
Zykov, V.S., 43, 54