

PROBABILITY THEORY: OVERVIEW

In this appendix we introduce some basic concepts and notation of probability theory for use in solving stochastic linear programs.

A.1 BASIC CONCEPTS, EXPECTED VALUE, AND VARIANCE

We begin by paraphrasing some basic definitions and concepts found in W. Feller's book *Introduction to Probability Theory and Applications*.

Definition (Event, Sample Point, Sample Space): An *event* is defined to be the outcome of an experiment or an observation about the state of some system. A simple event, which is a single outcome of a single experiment or observation, is called a *sample point* and will be denoted by ω . The aggregate of all possible sample points is referred to as the *sample space* and will be denoted by the symbol Ω .

Associated with points ω in a sample space is a function $p(\omega) \geq 0$ that is referred to as *the probability of a simple event ω happening*. The sum of these $p(\omega)$ is unity. For example, if we have an urn containing balls labeled with either a 0 or a 1, we might assign the probability of "randomly" choosing a 0 ball as equal to the proportion of 0 balls to the total balls in the urn. In this case, the sample space may be thought of as consisting of two points lying on a line such that one point has coordinate 0 and the other point has coordinate 1 with probabilities $p(0) = \alpha$, $p(1) = \beta$, where $\beta = 1 - \alpha$.

Definition: A *discrete* sample space is one consisting of only a finite number of points, say n .

Another example of a discrete sample space is the toss of a coin, is where the probability of observing a head is $1/2$ and the probability of observing a tail is $1/2$. We ignore the very remote possibility of a coin landing on its edge.

The probability of an event being observed in a subset of the sample space is assumed to be simple additions of probabilities in the subset of the sample points associated with the event.

Definition (Random Variable): A *random variable* X is not a variable but a function defined over a sample space.

In particular, a discrete random variable X is a function that takes on a discrete set of values with values x_1, x_2, \dots, x_n , where x_i can be a sample point ω itself, an event, or, more generally, it can be a function of a set of sample points. The probability that a discrete random variable X can take on a value x_j is denoted by $P[X = x_j] = p_j$.

Definition (Probability Distribution): The set of these probabilities for all the outcomes of a discrete random variable X is called a *probability distribution* (or *density*) of the random variable. Clearly,

$$p_j \geq 0, \quad \sum_{j=1}^n p_j = 1. \quad (\text{A.1})$$

Definition (Expected Value or Mean): The *expected value* or *mean* of a discrete random variable X is

$$E[X] = \sum_{j=1}^n p_j x_j. \quad (\text{A.2})$$

Often the expected value is denoted by μ_x or simply μ when the association with X is clear. In general, the expected value of a function of X , say $h(X)$, is

$$E[h(X)] = \sum_{j=1}^n p_j h(x_j). \quad (\text{A.3})$$

Definition (Variance and Standard Deviation): The *variance* of a random variable X is denoted by σ_x^2 or simply σ^2 when the association with X is clear. It is defined as the expected squared deviations of an observation x_j from its expected values; i.e.,

$$\sigma_x^2 = \sum_{j=1}^n p_j (x_j - E[X])^2 = E[X^2] - (E[X])^2. \quad (\text{A.4})$$

Its positive square root, denoted by σ_x , is called the *standard deviation*.

Definition (Independent Random Variables): If $P[X = x_i] = p_i$ and $P[Y = y_j] = q_j$, then random variables X, Y are said to be *independent* if their joint probability distribution $P[(X, Y) = (x_i, y_j)] = p_i q_j$.

Definition (Correlation Coefficient): The *correlation coefficient* between the two variables X and Y is defined to be

$$\rho_{XY} = \frac{E[(X - E[X])(Y - E[Y])]}{\sigma_X \sigma_Y} = \frac{E[XY] - E[X]E[Y]}{\sigma_X \sigma_Y}. \quad (\text{A.5})$$

The correlation coefficient satisfies $-1 \leq \rho_{XY} \leq +1$. If the random variables X and Y are independent $\rho_{XY} = 0$. The *covariance* between random variables X and Y is defined to be

$$\sigma_{XY} = \sigma_X \sigma_Y \rho_{XY}. \quad (\text{A.6})$$

In a more general setting, a random variable V can be a vector function consisting of d components $V = (V_1, V_2, \dots, V_d)$ with outcomes $v^\omega = (v_1^\omega, v_2^\omega, \dots, v_d^\omega)$. The probabilities associated with v^ω will be denoted by $p(v^\omega)$ or simply by $p(v)$.

Definition (Independent Components): If the joint density probability distribution $p(v_1, v_2, \dots, v_n)$ satisfies

$$p(v_1, v_2, \dots, v_n) = p_1(v_1)p_2(v_2) \cdots p_n(v_n), \quad (\text{A.7})$$

the components of the random variable V are said to be *independent*.

If the components of the random variable V are *independent*, the sample space Ω is obtained by crossing the sets of outcomes for each component of the vector entry, i.e.,

$$\Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_d. \quad (\text{A.8})$$

Then the expectation of a function $h(V)$ is of the form:

$$E[h(V)] = \sum_{v_1} \sum_{v_2} \cdots \sum_{v_d} h(v) p_1(v_1) p_2(v_2) \cdots p_n(v_n). \quad (\text{A.9})$$

A random variable that is continuous is treated in much the same way except that we now use integrals.

Definition (Probability Density Function): If X is a continuous random variable, we associate with it a *probability density function* $f(x)$ with the property that

$$f(x) \geq 0, \quad \int_{\Omega} f(x) dx = 1. \quad (\text{A.10})$$

We shall often use the shorter form *density function* to mean a probability density function.

In the general case of a continuous random vector V , the expectation of a function $h(V)$ is given by

$$E[h(V)] = \int_{\Omega_1} \int_{\Omega_2} \cdots \int_{\Omega_l} h(v)f(v)dv_1dv_2 \cdots dv_l. \quad (\text{A.11})$$

Sometimes an expectation is defined in terms of a Stieltjes integral; that is, the expected value of a function $g(x)$ of a random variable X , which is a combination of continuous and discrete elements, can be expressed as

$$E[X] = \int_{-\infty}^{\infty} g(x)dF(x) \quad (\text{A.12})$$

where $F(x) = P[X \leq x]$ is the cumulative density function.

Stochastic linear programs are hard to solve, in part because of the expense and difficulty involved in evaluating multiple integrals or multiple sums. The computation of the expected value typically involves a very large number of function evaluations, and each function evaluation may require solving a very large linear program.

A.2 NORMAL DISTRIBUTION AND THE CENTRAL LIMIT THEOREM

Many techniques in statistics are based on the normal probability distribution.

Definition (Normal Distribution): A random variable X is said to be distributed *normally* if its density function $f(x)$ is a *normal distribution*, i.e., it is of the form:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad (\text{A.13})$$

where $-\infty \leq \mu \leq \infty$ is the mean and $\sigma > 0$ is the standard deviation of the normal distribution. The normal distribution of (A.13) is often denoted by $\mathcal{N}(\mu, \sigma)$.

Definition (Standardized Normal Random Variable): A normally distributed random variable with mean $\mu = 0$ and standard deviation $\sigma = 1$, is called a *standardized* normal random variable. A standardized normal distribution is denoted by $\mathcal{N}(0, 1)$.

Normal distributions play an important role when sampling techniques are used because, according to the Central Limit Theorem, to be stated soon, the distribution of the mean of a sample of size n approaches a normal distribution as $n \rightarrow \infty$. For this purpose, we need to define what is meant by a random sample, sample mean, and sample variance.

Definition (Random Sample): The random variables X_1, X_2, \dots, X_n are defined to be a *random sample* from a population with probability density $f(x)$ if X_1, X_2, \dots, X_n are independent identically distributed random variables; i.e., if their joint probability density function satisfies

$$g(x_1, x_2, \dots, x_n) = f(x_1)f(x_2) \cdots f(x_n), \quad (\text{A.14})$$

where $f(x_i)$ is the density function of each of the random variables X_i .

Often the form of the density function of a random variable is known; for example, we may know that it is normally distributed. However, the density function may have parameters such as μ and σ , which are not known. The problem is then to estimate these unknown parameters through the use of a *statistic*.

Definition: A *statistic* is a function of a random sample of size n that is used to estimate an unknown parameter of a density function. For example, $\bar{X} = (1/n) \sum_{i=1}^n X_j$ is a statistic used to estimate the mean of a distribution.

LEMMA A.1 (Mean and Variance of a Random Sample) *Consider a population with density function $f(x)$, which has mean μ and variance σ^2 . Let X_1, X_2, \dots, X_n be a random sample from this population and let $\bar{X} = (1/n) \sum_{i=1}^n X_j$. Then*

$$E[\bar{X}] = \mu_{\bar{X}} = \mu \quad \text{and} \quad \text{Var}[\bar{X}] = \sigma_{\bar{X}}^2 = \frac{1}{n} \sigma^2. \quad (\text{A.15})$$

▷ **Exercise A.1** Prove Lemma A.1.

When sampling, we often can guess at the form of the probability density function but do not know its parameters, nor do we know its mean or variance. We can use one of several different ways to estimate the parameters; however, we typically would like the expected value of an estimator of a parameter to be equal to the true value of the parameter. Estimators that satisfy this property are called *unbiased estimators*.

Definition (Unbiased Estimator): An *unbiased estimator* γ of a function $g(\theta)$ of a parameter θ has the property $E[\gamma] = g(\theta)$. For example, the mean μ and variance σ^2 are parameters of a distribution. An unbiased estimator \bar{X} of μ is the mean of a sample of size n :

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i. \quad (\text{A.16})$$

Definition (Sample Variance): The *sample variance* of a random sample of size n is by definition:

$$\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2. \quad (\text{A.17})$$

A biased estimator of σ^2 is the sample variance because

$$E \left[\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \right] = \frac{n-1}{n} \sigma^2 \neq \sigma^2. \quad (\text{A.18})$$

Therefore an unbiased estimator of σ^2 is

$$\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2. \quad (\text{A.19})$$

Definition (Unbiased Sample Variance): We shall refer to (A.19) as the *unbiased sample variance* for estimating σ^2 .

▷ **Exercise A.2** Prove (A.18).

THEOREM A.2 (Chebyshev's Weak Law of Large Numbers) *Let \bar{X} be the sample mean of a random sample of size n drawn from a probability density $f(x)$ with mean μ and variance σ^2 . Then in order for*

$$P[-\epsilon < \bar{X} - \mu < \epsilon] \geq 1 - \delta \quad (\text{A.20})$$

for some specified $0 < \delta < 1$ and $\epsilon > 0$, choose sample size $n > \sigma^2/\epsilon^2\delta$.

The *weak law of large numbers* tells us how large we must take the sample size n in order to have a probability greater than $1 - \delta$ for $|\bar{X} - \mu| < \epsilon$, where \bar{X} is the sample mean.

Example A.1 (Sample Size Using Weak Law of Large Numbers) How large must the size n of the sample drawn from a population with known mean $\mu = 0$ and standard deviation $\sigma = 1000$ be in order that the probability is $\geq 1 - \delta = 0.95$ that the observed sample mean will be within $\epsilon = 10$ of the true mean 0? According to the weak law of large numbers, the sample size n should be chosen as $n = 100^2/10^2(0.5) = 200,000$.

THEOREM A.3 (Central Limit Theorem) *Let X_1, X_2, \dots, X_n be a random sample from a probability density function $f(x)$, which has mean μ and variance σ^2 . Define the random variable Z_n as a function of the sample mean \bar{X} as:*

$$Z_n = \frac{\bar{X} - E[\bar{X}]}{\sqrt{\text{Var}[\bar{X}]}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}.$$

Then the distribution of Z_n approaches that of a standardized normal distribution as $n \rightarrow \infty$.

Example A.2 (Sample Size Using the Central Limit Theorem) If in Example A.1,

$$Z_n = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{X}}{1000/\sqrt{n}},$$

then since

$$\int_{-1.96}^{+1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt = 0.95,$$

we have

$$P[-10 \leq \bar{X} \leq 10] = P\left[\frac{-10}{1000/\sqrt{n}} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq \frac{10}{1000/\sqrt{n}}\right] \geq 0.95.$$

Then $\frac{10}{1000/\sqrt{n}} = 1.96$ or $\sqrt{n} \geq 196$ or sample size should be chosen $n \geq 196^2 = 38,416$. The sample size n obtained using the Central Limit Theorem is much smaller than that obtained using Chebyshev's Weak Law of Large Numbers.

The Central Limit Theorem is one of the most important theorems in probability and statistics. It basically tells us that as the sample size grows the distribution of the sample mean \bar{X} approaches that of a normally distributed random variable.

A.3 CHI-SQUARE DISTRIBUTION, STUDENT'S t -DISTRIBUTION, AND CONFIDENCE INTERVALS

The “solution” of a stochastic linear program will be a decision x that is feasible. Its associated objective cost z is a random variable that depends on x and on the outcome of a random event ω . The problem is to choose $x = \hat{x}$ such that the *expected value* of $z_{\hat{x}}$ is minimum. If sample space Ω is too large, so that it is not possible to evaluate all $\omega \in \Omega$, then our goal will be given ϵ and δ to choose $x = \hat{x}$ and a sample size n such that $P[E[z_{\hat{x}}] < E[z_{\hat{x}}] + \epsilon] \geq 1 - \delta$. With this in mind we define the chi-square distribution, Student's t -distribution, and confidence intervals.

A.3.1 CHI-SQUARE DISTRIBUTION

A probability distribution, called the *chi-square distribution* plays an important role in determining the distribution of the unbiased sample variance.

Definition (Chi-Square Distribution): A random variable X is said to have a *chi-square distribution with k degrees of freedom* if its density function is of the form

$$f(x | k) = \frac{1}{\Gamma(k/2)} \left(\frac{1}{2}\right)^{k/2} x^{k/2-1} e^{-x/2}, \quad 0 < x < \infty. \tag{A.21}$$

where $\Gamma(u)$ is the gamma function:

$$\Gamma(u) = \int_0^{\infty} x^{u-1} e^{-x} dx \quad \text{for } u > 0 \quad (\text{A.22})$$

Integrating by parts results in the property

$$\Gamma(u + 1) = u\Gamma(u).$$

Hence if u is an integer, say $u = n$, then $\Gamma(n + 1) = n!$. Note that $\Gamma(u)$ is defined by (A.22) for any u , integer or noninteger, for example,

$$\Gamma(n + 1/2) = \frac{1 \cdot 3 \cdot 5 \cdots (2n - 1)}{2^n} \sqrt{\pi}$$

and $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(3/2) = (1/2)\sqrt{\pi}$. A random variable with a chi-square distribution with k degrees of freedom has $E[X] = k$ and $\text{Var}[X] = 2k$.

▷ **Exercise A.3** Show that $E[X] = k$ and $\text{Var}[X] = 2k$ for a random variable X that has a chi-square distribution with k degrees of freedom.

Next we discuss some properties of random samples.

LEMMA A.4 (Sample Mean from a Normal Distribution) *Let \bar{X} be the sample mean of a random sample X_1, X_2, \dots, X_n drawn from a normal distribution $\mathcal{N}(\mu, \sigma^2)$. Then \bar{X} is itself normal with mean μ and variance σ^2/n .*

LEMMA A.5 (Chi-Square Distribution) *Let U be a random variable defined by:*

$$U = \sum_{i=1}^k \left(\frac{X_i - \mu_i}{\sigma_i} \right)^2$$

where the X_i are normally and independently distributed with means μ_i and variance σ_i^2 . Then U has a chi-square distribution with k degrees of freedom.

COROLLARY A.6 *Let $S_{\bar{X}}$ be the unbiased sample variance (A.19) of a random sample X_1, X_2, \dots, X_n from a normal distribution with mean μ and variance σ^2 . Then*

$$U = \frac{(n-1)S_{\bar{X}}^2}{\sigma^2}$$

has a chi-square distribution with $(n-1)$ degrees of freedom.

A.3.2 STUDENT'S *t*-DISTRIBUTION

The Student's *t*-distribution also plays a very important role in estimation.

Definition (Student's t-Distribution): A random variable X is said to have a *Student's t-distribution* with k degrees of freedom if its probability density function is of the form

$$stu(x | k) = \frac{\Gamma([k + 1]/2)}{\Gamma(k/2)} \frac{1}{\sqrt{k\pi}} \frac{1}{(1 + x^2/k)^{(k+1)/2}}, \tag{A.23}$$

where Γ is the gamma function defined by (A.22).

LEMMA A.7 (Student's *t*-Distribution) *Let Z be a random variable with a standardized normal distribution and let U be an independent random variable with a chi-square distribution with k degrees of freedom. Then the random variable*

$$X = \frac{Z}{\sqrt{U/k}} \tag{A.24}$$

*has a Student's *t*-distribution with k degrees of freedom, $stu(x | k)$.*

Let X_1, X_2, \dots, X_n be a random sample from a normal probability density function with mean μ and variance σ^2 . Define

$$Z = \frac{(\bar{X} - \mu)}{\sigma/\sqrt{n}} \tag{A.25}$$

$$U = \left(\frac{1}{\sigma^2}\right) \sum_{i=1}^n (X_i - \bar{X})^2. \tag{A.26}$$

Then, from Lemma A.4, it is easy to see that Z has a standardized normal distribution and, from Corollary A.6, U has a chi-square distribution with $n - 1$ degrees of freedom.

LEMMA A.8 (Independence of Z and U) *The random variables Z defined by (A.25) and U defined by (A.26) are independent.*

Because Z and U are independent by Lemma A.8, from Lemma A.7 it follows that

$$\frac{(\bar{X} - \mu)/(\sigma/\sqrt{n})}{\sqrt{(1/\sigma^2) \sum_{i=1}^n (X_i - \bar{X})^2 / (n - 1)}} = \frac{\sqrt{n(n - 1)}(\bar{X} - \mu)}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}} \tag{A.27}$$

has a Student's *t*-distribution with $n - 1$ degrees of freedom, $stu(x | n - 1)$.

A.3.3 CONFIDENCE INTERVALS

Before formally defining a confidence interval, we illustrate the subject of confidence intervals through an example.

Example A.3 (Illustration of Confidence Intervals) Suppose that we would like to estimate the mean μ of a normal distribution with known standard deviation $\sigma = 1$. We can sample from the distribution and construct the sample mean \bar{X} as an estimate of the unknown mean μ . However, often we are more interested in constructing an interval that is guaranteed to cover the unknown mean μ with specified probability even though we do not know what the fixed value of μ is. For example, we may be interested in the interval that gives a probability of 0.9554 of covering the unknown mean μ . In order to do this, observe that, for a sample of size n , the random variable

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{X} - \mu}{1/\sqrt{n}} \quad (\text{A.28})$$

has a standardized normal distribution. Now from the probability tables we know that

$$P[-2 < Z < 2] = \int_{-2}^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = 0.9554 \quad (\text{A.29})$$

to four decimal places. Substituting (A.28) into (A.29) we obtain

$$\begin{aligned} 0.9554 &= P[-2 < Z < 2] = P\left[-2 < \frac{\bar{X} - \mu}{1/\sqrt{n}} < 2\right] \\ &= P[\bar{X} - 2/\sqrt{n} < \mu < \bar{X} + 2/\sqrt{n}]. \end{aligned} \quad (\text{A.30})$$

In particular, for $n = 100$,

$$P[\bar{X} - 0.2 < \mu < \bar{X} + 0.2] = 0.9554 \quad (\text{A.31})$$

Then $P[\bar{X} - 0.2 < \mu < \bar{X} + 0.2] = 0.9554$ measures our “confidence” that the unknown true mean lies in the *open interval* $(\bar{X} - 2/\sqrt{n}, \bar{X} + 2/\sqrt{n})$, which is called the 95.54% confidence interval. Notice that the interval $(\bar{X} - 0.2, \bar{X} + 0.2)$ is a random interval that covers the unknown true mean μ with probability 0.9554. That is, if random samples of size 100 were repeatedly drawn from the population and the random intervals $(\bar{X} - 0.2, \bar{X} + 0.2)$ repeatedly computed, then the fraction of times that the interval actually covered the mean would approach 0.9554 as the number of repetitions tends toward infinity. At this point we would also like to point out that often a particular realization of an interval (i.e., for one sample size of 100) is also called a *confidence interval for estimating μ* .

We now formalize the definition of a confidence interval.

Definition (Confidence Interval, Confidence Limit, Confidence Level): Let X_1, X_2, \dots, X_n be a random sample from a probability density function $f(x; \theta)$ parameterized by a constant θ whose unknown value we wish to estimate. Let $L \leq H$, where $L = L(X_1, X_2, \dots, X_n)$, $H = H(X_1, X_2, \dots, X_n)$, be statistics for θ such that $P[L < \theta < H] = \gamma$ where γ does not depend on θ . Then (L, H) is defined to be a 100γ percent confidence interval for θ and γ is called

the *confidence level*. Furthermore, the random variable L is called the *lower confidence limit* and the random variable H is called the *upper confidence limit* for θ . A particular realization (l, h) of (L, H) is also called a 100γ percent *confidence interval* for estimating θ . In a similar manner one can define upper and lower *one-sided confidence intervals*.

In order to derive a confidence interval for estimating the unknown mean μ of a distribution whose variance is also unknown, we note that

$$\frac{(\bar{X} - \mu)/(\sigma/\sqrt{n})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2/(n-1)\sigma^2}} = \frac{\bar{X} - \mu}{S_{\bar{X}}^2/\sqrt{n}}$$

has a Student's t -distribution with $(n-1)$ degrees of freedom, $stu(x | n-1)$. For some choice of q_1 and q_2 , typically $q_1 = q_2$, let

$$P\left[-q_1 < \frac{\bar{X} - \mu}{S_{\bar{X}}^2/\sqrt{n}} < q_2\right] = \gamma.$$

Then $(\bar{X} - q_2(S_{\bar{X}}^2/\sqrt{n}), \bar{X} + q_1(S_{\bar{X}}^2/\sqrt{n}))$ is a 100γ percent confidence interval for μ .

In a similar manner we can derive a confidence interval for estimating the unknown variance σ^2 of a distribution whose mean is also unknown. In order to do this we note that

$$\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2} = \frac{(n-1)S_{\bar{X}}^2}{\sigma^2}$$

has a chi-square distribution with $n-1$ degrees of freedom, $f(x | n-1)$. For some choice of q_1 and q_2 , let

$$P\left[q_1 < \frac{(n-1)S_{\bar{X}}^2}{\sigma^2} < q_2\right] = \gamma.$$

Then $((n-1)S_{\bar{X}}^2/q_2, (n-1)S_{\bar{X}}^2/q_1)$ is a 100γ percent confidence interval for the variance σ^2 .

A.4 NOTES & SELECTED BIBLIOGRAPHY

For an introduction to probability theory, see Feller [1957, 1969].

REFERENCES

A

- Abadie, J. (1965). Verbal Communication to Dantzig.
- Abrahamson, P.G. (1983). "A Nested Decomposition Approach for Solving Staircase Linear Programs," Technical Report SOL 83-4, Department of Operations Research, Stanford University, Stanford, CA.
- Ackoff, R.L. and Rivett, P. (1963). *A Manager's Guide to Operations Research*, Wiley, New York.
- Adler, I. (1976). "Equivalent Linear Programs," Department of IE & OR, University of California, Berkeley, California.
- Adler, I., Karmarkar, N., Resende, M.G.C. and Veiga, G. (1990). "Data Structures and Programming Techniques for the Implementation of Karmarkar's Algorithm," *ORSA Journal on Computing* **1**, 84–106.
- Adler, I., Resende, M.G.C., Veiga, G., and Karmarkar, N. (1989). "An Implementation of Karmarkar's Algorithm for Linear Programming," *Mathematical Programming* **44**, 297–336.
- Adler, I. and Ülkücü, A. (1973). "On the Number of Iterations in the Dantzig-Wolfe Decomposition," in D. M. Himmelblau (ed.), *Decomposition of Large Scale Problems*, North-Holland, Amsterdam, the Netherlands, 181–187.
- Ahuja, R.K., Magnanti, T.L., and Orlin, J.B. (1991). "Network Flows," in G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd (eds.), *Handbooks in Operations Research and Management Science, Vol 1: Optimization*, North-Holland, Amsterdam, the Netherlands, 211–369.
- Ahuja, R.K., Magnanti, T.L., and Orlin, J.B. (1993). *Network Flows: Theory, Algorithms, and Applications*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Ahuja, R.K., Mehlhorn, K., Orlin, J.B., and Tarjan, R.E. (1990). "Faster Algorithms for the Shortest Path Problem," *Journal of the Association for Computing Machinery* **37**, 211–369.
- Ali, A.I., Helgason, R.V., Kennington, J.L., and Lall, H.S. (1978). "Primal Simplex Network Codes: State of the Art Implementation Technology," *Networks*, **8**, 315–339.

- Andersen, E.D. and Ye, Y. (1995). "Combining Interior-Point and Pivoting Algorithms for Linear Programming," working paper, Department Sciences, The University of Iowa, Iowa City, Iowa; to appear in *Management Science*.
- Anstreicher, K. M. (1986). "A Monotonic Projective Algorithm for Fractional Linear Programming," *Algorithmica* **1**, 483–498.
- Anstreicher, K.M. (1989). "A Worst-Case Step in Karmarkar's Algorithm," *Mathematics of Operations Research* **14**, 294–302.
- Anstreicher, K.M. (1990). "On Long Step Path Following and SUMT for Linear and Quadratic Programming," Technical Report, Yale School of Management, Yale University, New Haven, CT.
- Anstreicher, K.M. and Watteyne, P. (1993). "A Family of Search Directions for Karmarkar's Algorithm," *Operations Research* **41**, 759–767.
- Arid, T.J., and Lynch, J.E. (1975). "Computable Accurate Upper and Lower Error Bounds for Approximate Solutions of Linear Algebra Systems," *ACM Transactions on Mathematical Software* **1**, 217–231.
- Arntzen, B.C., Brown, G.G., Harrison, T.P., and Trafton, L.L. (1995). *Interfaces* **25:1**, 69–93.
- Avi-Itzhak, H. (1994). "High Accuracy Correlation Based Pattern Recognition," Ph.D. thesis, Electrical Engineering, Stanford University, Stanford, CA.
- Avi-Itzhak, H.I., Van Mieghen, J.A., and Rub, L. (1995). "Multiple Subclass Pattern Recognition—A Max-Min Approach," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **17**, 418–431.
- Avriel, M. (1976). *Nonlinear Programming: Analysis and Methods*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Axelsson, O. and Munksgaard, N. (1983). "Analysis of Incomplete Factorizations with Fixed Storage Allocation," in D. Evans (ed.), *Preconditioning Methods Theory and Applications*, Gordon and Breach, New York, 219–241.

B

- Bahn, O., Goffin, J.L., Vial, J. P., and Merle, O.D. (1994). "Implementation and Behavior of an Interior Point Cutting Plane Algorithm for Convex Programming: An Application to Geometric Programming," *Discrete Applied Mathematics* **49**, 3–23; previously published in 1991 as a working paper, University of Geneva, Geneva, Switzerland.
- Bailey, D. (1988). "Extra High Speed Matrix Multiplication on the Cray-2," *SIAM Journal on Scientific and Statistical Computing* **9**, 603–607.
- El-Bakry, A., Tapia, R., and Zhang, Y. (1991). "Numerical Comparisons of Local Convergence Strategies for Interior-Point Methods in Linear Programming," Technical Report TR 91-18, Department of Computational and Applied Mathematics, Rice University, Houston, TX.
- Balanski, M.L. and Gomory, R.E. (1963). "A Mutual Primal-Dual Simplex Method," in R.L. Graves and P. Wolfe (eds.), *Recent Advances in Mathematical Programming*, McGraw-Hill, New York, 17 - 26.

- Barnes, E.R. (1986). "A Variation on Karmarkar's Algorithm for Linear Programming," *Mathematical Programming* **36**, 174–182.
- Bartels, R.H. (1971). "A Stabilization of the Simplex Method," *Numerische Mathematik* **16**, 414–434.
- Bartels, R.H. and Golub, G.H. (1969). "The Simplex Method of Linear Programming Using the LU Decomposition," *Communications of the Association for Computing Machinery* **12**, 266–268.
- Bartels, R.H., Stoer, J. and Zenger, C. (1971). "A Realization of the Simplex Method Based on Triangular Decompositions," in J.H. Wilkinson and C. Reinsch (eds.), *Contributions I/II in Handbook for Automatic Computation, Volume II: Linear Algebra*, Springer-Verlag, Berlin and New York, 152–190.
- Bastian, M. (1984). "Implicit Representation of Generalized Variable Upper Bounds Using the Elimination Form of the Inverse on Secondary Storage," *Mathematical Programming* **30**, 357–361.
- Bazaraa, M.S., Jarvis, J.J., and Sherali, H.D. (1990). *Linear Programming and Network Flows*, John Wiley and Sons, New York.
- Beale, E.M.L. (1954a). "Linear Programming by the Method of Leading Variables," *Report of the Conference on Linear Programming*, (May), arranged by Ferranti Ltd., London.
- Beale, E.M.L. (1954b). "An Alternative Method for Linear Programming," *Proceedings of the Cambridge Philosophical Society* **50**, 513–523.
- Beale, E.M.L. (1955a). "On Minimizing a Convex Function Subject to Linear Inequalities," *Journal of the Royal Statistical Society, Series B*, **17**, 173–184.
- Beale, E.M.L. (1955b). "Cycling in the dual simplex algorithm," *Naval Research Logistics Quarterly*, **2**, 269–275.
- Beale, E.M.L. (1970). "Matrix Generators and Output Analyzers," in H.W. Kuhn (ed.), *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton University Press, Princeton, New Jersey, 25–36.
- Beale, E.M.L., Hughes, P.A.B., and Small, R. (1965). "Experiences in Using a Decomposition Program," *Computer Journal* **8**, 13–18.
- Bell, C.G. and Newell, A. (1971). *Computer Structures: Readings and Examples*, McGraw-Hill, New York.
- Bellman, R. E. (1957). *Dynamic Programming*, Princeton University Press, Princeton, New Jersey.
- Bellman, R.E. (1958). "On a Routing Problem," *Quarterly of Applied Mathematics* **1958**, 87–90.
- Bellman, R.E. and Dreyfus, S. (1962). *Applied Dynamic Programming*, Princeton University Press, Princeton, New Jersey.
- Ben-Dov, Y., Hayre, L., and Pica, V. (1992). "Mortgage Valuation Models at Prudential Securities," *Interfaces* **22:1**, 55–71.
- Benders, J.F. (1962). "Partitioning Procedures for Solving Mixed-Variables Programming Problems," *Numerische Mathematik* **4**, 238–252.

- Benichou, M., Gauthier, J.M., Hentges, G., and Ribière, G. (1977). "The Efficient Solution of Large-Scale Linear Programming Problems—Some Algorithmic Techniques and Computational Results," *Mathematical Programming* **13**, 280–322.
- Bennett (1963). "An Approach to Some Structured Linear Programming Problems," Basar Computing Department, School of Physics, University of Sydney, Australia.
- Bertsekas, D.P. (1988). "The Auction Algorithm: A Distributed Relaxation Method for the Assignment Problem," *Annals of Operations Research* **14**, 105–123.
- Berge, C. (1963). *Topological Spaces*, Oliver & Boyd Ltd., Edinburgh.
- Bertsekas, D.P. (1991). *Linear Network Optimization: Algorithms and Codes*, The MIT Press, Cambridge, Massachusetts.
- Birge, J.R. (1980). "Solution Methods for Stochastic Dynamic Linear Programs," Ph.D. thesis. Technical Report SOL 80-29, Department of Operations Research, Stanford University, Stanford, CA.
- Birge, J.R. (1985a). "Aggregation in Stochastic Linear Programming," *Mathematical Programming* **31**, 25–41.
- Birge, J.R. (1985b). "Decomposition and Partitioning Methods for Multi-Stage Stochastic Linear Programming," *Operations Research* **33**, 989–1007.
- Birge, J.R., Dempster, M.A., Gassman, H.I., Gunn, E.A., King, A.J., and Wallace, S.W. (1987). "A Standard Input Format for Multi-Period Stochastic Linear Programs," *Mathematical Programming Society COAL Newsletter* **17**, 1–20.
- Birge, J.R. and Holmes, D.F. (1992). "Efficient Solution of Two-Stage Stochastic Linear Programs Using Interior Point Methods," *Operations Research Letters* **11**, 245–276.
- Birge, J.R. and Louveaux, F. (1997). *Introduction to Stochastic Programming*, Springer-Verlag, Berlin and New York.
- Birge, J.R. and Wallace, S.W. (1988). "A Separable Piecewise Linear Upper Bound for Stochastic Linear Programs," *SIAM Journal on Control and Optimization* **26**, 725–739.
- Birge, J.R. and Wets, R.J. (1986). "Designing Approximate Schemes for Stochastic Optimization Problems, in Particular for Stochastic Programs with Recourse," *Mathematical Programming Study* **27**, 54–102.
- Birge, J.R. and Wets, R.J. (1987). "Computing Bounds for Stochastic Programming Problems by Means of a Generalized Moment Problem," *Mathematics of Operations Research* **12**, 49–162.
- Birge, J.R. and Wets, R.J. (1989). "Sublinear Upper Bounds for Stochastic Programming Problems with Recourse," *Mathematical Programming* **43**, 131–149.
- Birkhoff, G. (1946). "Three Observations on Linear Algebra," *Rev. Univ. Nac. Tucumán, Ser. A.*, Vol. 5, 147–151.
- Bisschop, J. and Meeraus, A. (1981). "Towards Successful Modeling Applications in a Strategic Planning Environment," in G.B. Dantzig, M.A.H. Dempster, and M.J. Kallio. (eds.), *Large-Scale Linear Programming*, Vol. 2, CP-81-51, IIASA Collaborative Proceedings Series, Laxenberg, Austria, 712–745.
- Bisschop, J. and Meeraus, A. (1982). "On the Development of a General Algebraic Modeling System," *Mathematical Programming Study* **20**, 1–29.

- Bixby, R.E. (1981). "Hidden Structure in Linear Programs," in H. Greenberg and J. Maybee (eds.), *Computer Assisted Analysis and Model Simplification*, Academic Press, London and New York, 327–360.
- Bixby, R.E. (1984). "Recent Algorithms for Two Versions of Graph Realization and Remarks on Applications to Linear Programming," in W.R. Pulleybank (ed.), *Progress in Combinatorial Optimization*, Academic Press, Canada, 39–67.
- Bixby, R.E. (2002). "Solving Real-World Linear Programs: A Decade and More of Progress," *Operations Research Letters* **50**, 3–15.
- Bixby, R.E. and Cunningham, W.H. (1980). "Converting Linear Programs to Network Problems," *Mathematics of Operations Research* **5**, 321–357.
- Bixby, R.E. and Fourer, R. (1988). "Finding Embedded Network Rows in Linear Programs I. Extraction Heuristics," *Management Science* **34**, 342–376.
- Bixby, R.E., Gregory, J.W., Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1992). "Very Large Scale Linear Programming: A Case Study in Combining Interior Point and Simplex Methods," *Operations Research* **40**, 885–897.
- Bixby, R.E. and Saltzman, M.J. (1994). "Recovering an Optimal LP Basis from the Interior Point Solution," *Operations Research Letters* **15**, 169–178; previously published in 1992 in Technical Report 607, Department of Mathematical Sciences, Clemson University, Clemson, SC.
- Bixby, R.E. and Wagner, D.K. (1987). "A Note on Detecting Simple Redundancies in Linear Systems," *Operations Research Letters* **6**, 15–17.
- Bixby, R.E. and Wagner, D.K. (1988). "An Almost Linear-Time Algorithm for Graph Realization," *Mathematics of Operations Research* **13**, 99–123.
- Björck, Å. (1967). "Solving Linear Least Squares Problems by Gram-Schmidt Orthogonalization," *BIT* **7**, 1–21.
- Björck, Å. (1987). "Least Square Methods," working paper, Department of Mathematics, Linköping University, S-581 83 Linköping, Sweden.
- Björck, Å., Plemmons, R.J., and Schneider, H. (1981). *Large-Scale Matrix Problems*, North-Holland, Amsterdam, the Netherlands.
- Bland, R.G. (1977). "New Finite Pivoting Methods for the Simplex Method," *Mathematics of Operations Research* **2**, 103–107.
- Blue, J.L. (1975). "Automatic Numerical Quadrature—DQUAD," *Bell Laboratories Report*, Murray Hill, NJ
- Borel, E. (1921). "La Théorie du Jeu et les Équations Intégrales à Noyau Symétrique," *Comptes Rendus de l'Académie des Sciences U.S.S.R.* **173**, 1304–1307; translated by Leonard J. Savage in *Econometrica*, **21**, 97–100, January 1953.
- Borel, E. (1924). "Sur les Jeux où Interviennent le Hasard et l'habileté des Joueurs," *Théorie des Probabilités*, Librairie scientifique, Hermann, Paris, 204–224; translated by Leonard J. Savage in *Econometrica*, **21**, 101–115, January 1953.
- Borel, E. (1927). "Sur les Systèmes de Formes Linéaires à Déterminant Symétrique Gauches et la Théorie Générale du Jeu," from "Algèbre et Calcul des Probabilités," *Comptes Rendus de l'Académie des Sciences U.S.S.R.* **184**, 52–53; translated by Leonard J. Savage in *Econometrica*, **21**, 101–115, January 1953.

- Borgwardt, K.H. (1982a). "The Average Number of Pivot Rules Required by the Simplex Method is Polynomial," *Zeitschrift für Operations Research* **26**, 157–177.
- Borgwardt, K.H. (1982b). "Some Distribution Independent Results About the Asymptotic Order of the Average Number of Pivot Steps in the Simplex Method," *Mathematics of Operations Research* **7**, 441–462.
- Borgwardt, K.H. (1987a). *The Simplex Method: A Probabilistic Analysis*, Springer-Verlag, Berlin and New York.
- Borgwardt, K.H. (1987b). "Probabilistic Analysis of the Simplex Method," in *Operations Research Proceedings 16th Dgor Meeting*, 564–576.
- Bradley, G.H., Brown, G.G., and Graves, G.W. (1977). "Design and Implementation of Large Scale Primal Transshipment Algorithms," *Management Science* **24**, 1–34.
- Bradley, S.P., Hax, A.C., and Magnanti, T.L. (1977). *Applied Mathematical Programming*, Addison-Wesley Publishing Company, Reading, Massachusetts.
- Brearley, A.L., Mitra, G., and Williams, H.P. (1975). "Analysis of Mathematical Programs Prior to Applying the Simplex Algorithm," *Mathematical Programming* **8**, 54–83.
- Brent, R. P. (1973a). "Reducing the Retrieval Time of Scatter Storage Techniques," *Communications of the Association for Computing Machinery* **16:2**, 105–109.
- Brent, R.P. (1973b). *Algorithms for Finding Zeros and Extrema of Functions Without Calculating Derivatives*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Brickman, L. (1989). *Mathematical Introduction to Linear Programming and Game Theory*, Springer-Verlag, Berlin and New York.
- Brooke, A., Kendrick, D., and Meeraus, A. (1988). *GAMS: A User's Guide*, Scientific Press, South San Francisco, California.
- Brown, G.G. and McBride, R.D. (1984). "Solving Generalized Networks," *Management Science* **30**, 1497–1523.
- Brown, G.G. and Thomen, D.S. (1980). "Automatic Identification of Generalized Upper Bounds in Large-Scale Optimization Models," *Management Science* **26**, 1166–1184.
- Brown, G.G. and Wright, W. (1981). "Automatic Identification of Embedded Structure in Large-Scale Optimization Models," in H. Greenberg and J. Maybee (eds.), *Computer Assisted Analysis and Model Simplification*, Academic Press, London and New York, 369–388.
- Brown, G.G. and Wright, W. (1984). "Automatic Identification of Embedded Network Rows in Large-Scale Optimization Models," *Mathematical Programming* **29**, 41–56.
- Brumelle, S.L. and McGill, J.I. (1993). "Airline Seat Allocation with Multiple Nested Fare Classes," *Operations Research* **41**, 127–137.
- Broyden, C.G. (1973). "Some Condition Number Bounds for the Gaussian Elimination Process," *Journal of the Institute of Mathematics and Its Applications* **12**, 273–286.
- Bunch, J.R. and Parlett, B.N. (1971). "Direct Methods for Solving Symmetric Indefinite Systems of Linear Equations," *SIAM Journal on Numerical Analysis* **8**, 639–655.
- Bunch, J.R. and Kaufman, L.C. (1977). "Some Stable Methods for Calculating Inertia and Solving Symmetric Linear Equations," *Linear Algebra and Its Applications* **34**, 341–370.

- Burks, A.W., Goldstine, H.H., and von Neumann, J. (1946). "Planning and Coding of Problems for an Electronic Computing Instrument," Parts 1 and 2, Institute for Advanced Study, Princeton, New Jersey. Reprinted in *Datamation* **8**:9, 24–31, September, 1962 and *Datamation* **8**:10, 36–41, October, 1962.
- Businger, P.A. (1968). "Matrices Which can be Optimally Scaled," *Numerische Mathematik* **12**, 346–348.
- Buzbee, B.L. (1986). "A Strategy for Vectorization," *Parallel Computing* **3**, 187–192.

C

- Cariño, D.R., Kent, T., Myers, D.H., Stacy, C., Sylvanus, M., Turner, A.L., Watanabe, K., and Ziemba, W.T. (1994). "The Russel-Yasuda Kasai Model: An Asset/Liability Model for a Japanese Insurance Company Using Multistage Stochastic Programming," *Interfaces* **14**:1, 29–49.
- Carpenter, T.J., Lustig, I.J., Mulvey, J.M., and Shanno, D.F. (1993). "Higher Order Predictor-Corrector Interior Point Methods with Applications to Quadratic Objectives," *SIAM Journal on Optimization* **3**, 696–725.
- Carpento, G., Martello, S., and Toth, P. (1988). "Algorithms and Codes for the Assignment Problem," in B. Simeone, P. Toth, G. Gallo, F. Maffioli, and S. Pallotino (eds.), *FORTTRAN Codes for Network Optimization, Annals of Operations Research* **13**, 193–224.
- Chambers, J.M. (1977). *Computational Methods for Data Analysis*, John Wiley and Sons, New York.
- Chan, T.F. (1985). "On the Existence and Computation of LU Factorizations with Small Pivots," *Mathematics of Computation* **42**, 535–548.
- Charnes, A. (1952). "Optimality and Degeneracy in Linear Programming," *Econometrica*, **20**, 160–170.
- Charnes, A., Cooper, W.W., and Mellon, B. (1952). "Blending Aviation Gasoline—A Study of Programming Interdependent Activities in an Integrated Oil Company," *Econometrica*, **20**, 135–139.
- Charnes, A. and Kortanek, K.O. (1963). "An Opposite Sign Algorithm for Purification to an Extreme Point Solution," Memorandum No. 89, Office of Naval Research.
- Charnes, A. and Lemke, C. (1960). "Multi-Copy Generalized Networks and Multi-Page Programs," R.P.I. Math Report No. 41, Rensselaer Polytechnic Institute; Troy, New York.
- Cheng, M.C. (1985). "Generalized Theorems for Permanent Basic and Nonbasic Variables," *Mathematical Programming* **31**, 229–234.
- Cheng, M.C. (1987). "General Criteria for Redundant and Nonredundant Linear Inequalities," *Journal of Optimization Theory and Applications* **53**, 37–42.
- Cherkassky, B.V., Goldberg, A.V., and Radzik, T. (1996). "Shortest Path Algorithms: Theory and Experimental Evaluation," *Mathematical Programming* **73**, 129–174.
- Choi, I.C., Monma, C.L., and Shanno, D.F. (1990). "Further Developments of a Primal-Dual Interior Point Method," *Operations Research* **40**, 885–897.

- Chvátal, V. (1983). *Linear Programming*, W. H. Freeman and Company, New York.
- Clements, D.W. and Reid, R.A. (1994). "Analytical MS/OR Tools Applied to a Plant Closure," *Interfaces* **24:2**, 1–12.
- Cline, A.K., Conn, A.R., and van Loan, C. (1982). "Generalizing the LINPACK Condition Estimator," in J.P. Hennart (ed.), *Numerical Analysis*, Lecture Notes in Mathematics No. 909, Springer-Verlag, Berlin and New York, 73–83.
- Cline, A.K., Moler, C.B., Stewart, G.W., and Wilkinson, J.H., (1979). "An Estimate for the Condition Number of a Matrix," *SIAM Journal on Numerical Analysis* **16**, 368–375.
- Cline, A.K. and Rew, R.K. (1983). "A Set of Counter Examples to Three Condition Number Estimators," *SIAM Journal on Scientific and Statistical Computing* **4**, 602–611.
- Cody, W.J. (1974). "The Construction of Numerical Subroutine Libraries," *SIAM Review* **16**, 36–46.
- Cohen, A.M. (1974). "A Note on Pivot Size in Gaussian Elimination," *Linear Algebra and Its Applications* **8**, 361–368.
- Coleman, T.F. and More, J.J. (1983). "Estimation of Sparse Jacobian Matrices and Graph Coloring Problems," *SIAM Journal on Numerical Analysis* **20**, 187–209.
- Cope, J.E. and Rust, B.W. (1979). "Bounds on Solutions of Systems with Accurate Data," *SIAM Journal on Numerical Analysis* **16**, 950–963.
- Cornfield, J. (1940). Verbal Communication to G.B. Dantzig.
- Cottle, R.W. (1974). "Manifestations of the Schur Complement," *Linear Algebra and Its Applications* **8**, 189–211.
- Courant, R. (1943). "Variational Methods for the Solution of Problems of Equilibrium and Vibrations," *Bulletin of the American Mathematical Society* **49**, 1–23.
- Cowell, W. R. (ed.), (1983). *Sources and Development of Mathematical Software*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Cox, L.A., Kuehner W., Parrish, S.H., and Qiu, Y. (1993). "Optimal Expansion of Fiber-Optic Telecommunications Networks in Metropolitan Areas," *Interfaces* **23:2**, 35–48.
- Cryer, C.W. (1968). "Pivot Size in Gaussian Elimination," *Numerische Mathematik* **12**, 335–345.
- Cunningham, W.H. (1979). "Theoretical Properties of the Network Simplex Method," *Mathematics of Operations Research* **4**, 196–208.
- Cunningham, W.H. and Klincewicz, J.G. (1983). "On Cycling in the Network Simplex Method," *Mathematical Programming* **26**, 182–189.
- Curtis, A.R., Powell, M.J.D. and Reid, J.K. (1974). "On the Estimation of Sparse Jacobian Matrices," *Journal of the Institute of Mathematics and Its Applications* **13**, 117–119.
- Curtis, A.R. and Reid, J.K. (1971). "The Solution of Large Sparse Unsymmetric Systems of Linear Equations," *Journal of the Institute of Mathematics and Its Applications* **8**, 344–353.

D

- Dahl, O., Dijkstra, E., and Hoare, C.A.R. (1972). *Structured Programming*, Academic Press, London and New York
- Dahlquist, G. and Björk, Å. (1974). *Numerical Methods*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Dantzig, G.B. (1939). "On a Class of Distributions that Approach the Normal Distribution Function," *Annals of Mathematical Statistics* **10**, September, 247–253.
- Dantzig, G.B. (1940). "On the Non-Existence of Tests of Students' Hypothesis Involving Power Functions Independent of Sigma," *Annals of Mathematical Statistics* **11**, June, 186–192.
- Dantzig, G.B. (1948, January 5). "A Theorem on Linear Inequalities," unpublished report.
- Dantzig, G.B. (1949). "Programming in a Linear Structure," Report of the September 9, 1948 meeting in Madison, *Econometrica*, **17**, 73–74.
- Dantzig, G.B. (1951a). "Maximization of a Linear Function of Variables Subject to Linear Inequalities," in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 339–347.
- Dantzig, G.B. (1951b). "Application of the Simplex Method to the Transportation Problem," in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 359–373.
- Dantzig, G.B. (1951c). "Programming of Interdependent Activities, II: Mathematical Model," *Econometrica*, **17**, 200–211; also in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 19–32.
- Dantzig, G.B. (1951d). "A Proof of the Equivalence of the Programming Problem and the Game Problem, in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 330–335.
- Dantzig, G.B. (1951e). "Linear Programming," in *Problems for the Numerical Analysis of the Future*, Proceedings of Symposium on Modern Calculating Machinery and Numerical Methods, UCLA, July 29–31, 1948, Appl. Math. Series 15, National Bureau of Standards, June 1951, 18–21.
- Dantzig, G.B. (1954a). "The Dual Simplex Algorithm (Notes on Linear Programming: Part VII)," *RM-1270*, The RAND Corporation, July.
- Dantzig, G.B. (1954b). "Upper Bounds, Secondary Constraints, and Block Triangularity in Linear Programming (Notes on Linear Programming: Part VIII, IX, X)," *RM-1367*, The RAND Corporation, October.
- Dantzig, G.B. (1954c). "Notes on Linear Programming: Part XI, Composite Simplex-Dual Simplex Algorithm—I," *RM-1274*, The RAND Corporation, April.

- Dantzig, G.B. (1954d). "A Comment on Eddie's 'Traffic Delays at Toll Booths,'" *Journal of Operations Research Society of America* **2**, 339–341.
- Dantzig, G.B. (1955a). "Linear Programming under Uncertainty," *Management Science* **1**, 197–206. Also in A.F. Veinott (ed.), *Mathematical Studies in Management Science*, Macmillan, New York, 1965, 330–339.
- Dantzig, G.B. (1955b). "Optimal Solution of a Dynamic Leontief Model with Substitution (Notes on Linear Programming: Part XIII)," *RM-1281-1*, The RAND Corporation, April. Also *Econometrica*, **23**, 151–176.
- Dantzig, G.B. (1955c). "Developments in Linear Programming," Proceedings Second Symposium on Linear Programming, National Bureau of Standards and Comptroller, U.S.A.F. Headquarters, January, 667–685.
- Dantzig, G.B. (1956a). "Constructive Proof of the Min-Max Theorem," *Pacific Journal of Mathematics* **6**, 25–33.
- Dantzig, G.B. (1956b). "Recent Advances in Linear Programming," *Management Science* **2**, 131–144.
- Dantzig, G.B. (1956c). "Formulating a Linear Programming Model," *P-893*, The RAND Corporation, July; also in *Linear Programming and Extensions*, Princeton University Press, Princeton, NJ, 1963, Chapter 3.
- Dantzig, G.B. (1956d). "Note on Klein's 'Direct Use of Extremal Principles in Solving Certain Problems Involving Inequalities,'" *Operations Research* **4**, 247–249.
- Dantzig, G.B. (1957a). "Thoughts on Linear Programming and Automation," *Management Science* **3**, 131–139; also in *P-824*, The RAND Corporation, March 2, 1956.
- Dantzig, G.B. (1957b). "Concepts, Origins and Uses of Linear Programming," in Davis, Edison, and Page (eds.), *Proceedings of First International Conference on Operations Research*, Operations Research Society of America, Baltimore, December, 100–108.
- Dantzig, G.B. (1957c). "Discrete Variable Extremum Problems," *Operations Research* **5**, 226–277.
- Dantzig, G.B. (1957d). "On The Status of Multi-Stage Linear Programs," *RM-1028*, The RAND Corporation, February; also in A.F. Veinott (ed.), *Mathematical Studies in Management Science*, Macmillan, New York, 1965, Chapter 6, Section III: Topics in Linear Programming, 303–320; also in *Management Science*, 1959, 71–90.
- Dantzig, G.B. (1958a). "Solving Two-Move Games with Perfect Information," *P-1459*, The RAND Corporation, August.
- Dantzig, G.B. (1958b). "On Integer and Partial Integer Linear Programming Problems," *P-1410*, The RAND Corporation, June.
- Dantzig, G.B. (1958c). "Chemical Equilibrium in Complex Mixtures," *Journal of Chemical Physics* **28**, 751–755.
- Dantzig, G.B. (1959). "Note on Solving Linear Programs in Integers," *Naval Research Logistics Quarterly*, **6**, 75–76.
- Dantzig, G.B. (1960a). "Inductive Proof of the Simplex Method," *IBM Journal of Research and Development* **4**, 505–506; also in *P-1851*, The RAND Corporation, December 28, 1959.

- Dantzig, G.B. (1960b). "On the Shortest Route through a Network," *Management Science* **6**, 187–190. Also in D.R. Fulkerson (ed.), *Some Topics in Graph Theory*, MAA Studies, No. 11, 1975, 89–93.
- Dantzig, G.B. (1960c). "On the Significance of Solving Linear Programming Problems with Some Integer Variables," *Econometrica*, **28**, 30–44.
- Dantzig, G.B. (1960d). "General Convex Objective Forms," in K. Arrow, S. Karlin, and P. Suppes (eds.), *Mathematical Methods in the Social Sciences*, Stanford University Press, Stanford, California, 151–158.
- Dantzig, G.B. (1960e). "A Machine-Job Scheduling Model," *Management Science* **6**, 191–196.
- Dantzig, G.B. (1961a). "Future Developments of Operations Research," Operations Research Center, University of California, Berkeley, Summary Report 1, 1961; also in *Proceedings Thirteenth Annual Industrial Engineering Institute*, University of California, February 1961, 25–28, Special Topic; also in *Decentralized Planning*, Journal of Operations Research Society, Switzerland, 1962, 53–55.
- Dantzig, G.B. (1961b). "Quadratic Programming—A Variant of the Wolfe-Markowitz Algorithms," Technical Report RR-2, Operations Research Center, University of California, Berkeley; also in *Linear Programming and Extensions*, Princeton University Press, Princeton, NJ, 1963, Chapter 24-4.
- Dantzig, G.B. (1962a). "A Proof of a Property of Leontief and Markov Matrices," Technical Report RR-25, Operations Research Center, University of California, Berkeley.
- Dantzig, G.B. (1962a). "Compact Basis Triangularization for the Simplex Method," Technical Report RR-28, Operations Research Center, University of California, Berkeley; also in R.L. Graves and P. Wolfe (eds.), *Recent Advances in Mathematical Programming*, McGraw-Hill, New York, 125–133.
- Dantzig, G.B. (1963). *Linear Programming and Extensions*, Princeton University Press, Princeton, New Jersey, August. Revised edition Fall 1966; fourth printing, 1968, 621 pages. [Japanese translation, Tutte-Mori, Inc., Tokyo, 1983.]
- Dantzig, G.B. (1964a). "New Mathematical Methods in the Life Sciences," *The American Mathematical Monthly* **71**, 4–15. Also in Stacy and Waxman (eds.), *Computers and Biomedical Research 1*, Academic Press, New York, March 1966.
- Dantzig, G.B. (1964b). "Research Problems," *Bulletin of the American Mathematical Society* **70**, 499–501.
- Dantzig, G.B. (1964c). "Linear Control Processes and Mathematical Programming," Technical Report 64-13, Operations Research Center, University of California, Berkeley; also in *SIAM Journal on Control and Optimization* **4**, 1966, 56–60; also in G.B. Dantzig, G.B. and A.F. Veinott, Jr. (eds.), *Mathematics of the Decision Sciences Part 2*, the American Mathematical Society Summer Seminar, Providence, RI, 1968, 31–36; also in J. Abadie (ed.), *Nonlinear Programming*, North-Holland, Amsterdam, the Netherlands, 1967, 283–286; also in A.K. Aziz (ed.), *Lecture Series in Differential Equations 1*, Van Nostrand Reinhold Publishing Co., 1969, 1–7.
- Dantzig, G.B. (1965a). "Operations Research in the World of Today and Tomorrow," Technical Report 65-7, Operations Research Center, University of California, Berkeley; also in Technion Yearbook; also in commemorative volume for Professor Keller, Institut für Ökonometri; presidential address, TIMS 1966, entitled "Management

- Science in the World of Today and Tomorrow” in *Management Science* **13**, February 1967, pages C-107–C-111.
- Dantzig, G.B. (1965b). “Large-Scale System Optimization: A Review,” Technical Report 65-9, Operations Research Center, University of California, Berkeley.
- Dantzig, G.B. (1965c). “Optimization in Operations Research,” Technical Report 65-10, Operations Research Center, University of California, Berkeley; also in Wayne Kalenich (ed.), *Proceedings, International Federation for Information Processing Congress, 1965*, May.
- Dantzig, G.B. (1965c). “The Simplex Method,” in R. Machol (ed.), *Engineering Handbook*, McGraw-Hill, New York, June 1965, Chapter 25, 10 pages.
- Dantzig, G.B. (1966a). “Linear Programming and its Progeny,” *Naval Research Reviews* **XIX** (6), June 1966, 1; also in E.M.L. Beale (ed.), *Application of Mathematical Programming Techniques*, English Universities Press, Ltd., London, 1970, 3–16.
- Dantzig, G.B. (1966b). “All Shortest Routes in a Graph,” Technical Report 66-3, Department of Operations Research, Stanford University, Stanford, CA, November; also in *Théorie des Graphes, International Symposium, Rome, Italy, July 1966*, 91–92, published by DUNOD, Paris.
- Dantzig, G.B. (1966c). “On Positive Principal Minors,” Technical Report 67-1, Department of Operations Research, Stanford University, Stanford, CA, January.
- Dantzig, G.B. (1967). “Large-Scale Linear Programming,” Technical Report 67-8, Department of Operations Research, Stanford University, Stanford, CA, November; also in “Large-Scale Systems and the Computer Revolution,” in H.W. Kuhn (ed.), *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton University Press, Princeton, New Jersey, August 1967, 51–72. Also in G.B. Dantzig, and A. F. Veinott, Jr. (eds.), *Mathematics of the Decision Sciences*, the American Mathematical Society Summer Seminar, Providence, RI, 1968, 77–92.
- Dantzig, G.B. (1969a). “Complementary Spanning Trees,” Technical Report CS 126, Department of Computer Science, Stanford University, Stanford, CA, March; also in J. Abadie (ed.), *Integer and Nonlinear Programming*, North-Holland, Amsterdam, the Netherlands, 1970, 499–505.
- Dantzig, G.B. (1969b). “A Hospital Admission Problem,” Technical Report 69-15, Department of Operations Research, Stanford University, Stanford, CA, December.
- Dantzig, G.B. (1970a). “On a Model for Computing Round-Off Error of a Sum,” Technical Report STAN-CS-70-156, Department of Computer Science, Stanford University, Stanford, CA.
- Dantzig, G.B. (1970b). “A Control Problem of Bellman,” Technical Report 70-15, Department of Operations Research, Stanford University, Stanford, CA, September. Also in *Management Science: Theory* **16**, May 1971, 542–546.
- Dantzig, G.B. (1972a). “Health Care in Future Cities,” Technical Report 72-22, Department of Operations Research, Stanford University, Stanford, CA, September.
- Dantzig, G.B. (1972b). “On the Relation of Operations Research to Mathematics,” Technical Report 72-23, Department of Operations Research, Stanford University, Stanford, CA, October.

- Dantzig, G.B. (1972c). "The Price Lecture on Compact City," Price Lecture Series University of Pittsburgh.
- Dantzig, G.B. (1973a). "The ORSA New Orleans Address on Compact City," *Management Science* **19**, 1151-1161.
- Dantzig, G.B. (1973b). "Solving Staircase Linear Programs by a Nested Block-Angular Method," Technical Report 73-1, Department of Operations Research, Stanford University, Stanford, CA, January.
- Dantzig, G.B. (1973c). "Drews' Institutionalized Divvy Economy," Technical Report 73-7, Department of Operations Research, Stanford University, Stanford, CA, September. Revised, International Institute for Applied Systems Analysis, December 1973. Revised 1974, as Technical Report TR 74-14. Also in "An Institutionalized Divvy Economy," Technical Report SOL 75-17 (revision of TR 74-14), Department of Operations Research, Stanford University, Stanford, CA. Also in *Journal of Economic Theory* **11**, 1975, 372-384.
- Dantzig, G.B. (1974a). "On a Convex Programming Problem of Rozanov," *Applied Mathematics and Optimization* **1**, 189-192; also entitled "A Generalized Programming Solution to a Convex Programming Problem with a Homogeneous Objective," IIASA Research Report RR-73-21, December 1973; also in *Symposia Mathematica*, Monograf, Italy, Vol. XIX, Academic Press, 1976, 209-214.
- Dantzig, G.B. (1974b). "On the Reduction of an Integrated Energy and Interindustry Model to a Smaller Linear Program," Technical Report SOL 74-20, Department of Operations Research, Stanford University, Stanford, CA, December; also in *The Review of Economics and Statistics* **LVIII**, May 1976, 248-250.
- Dantzig, G.B. (1974c). "Formulating a PILOT Model for Energy in Relation to the National Economy," Technical Report SOL 75-10, Department of Operations Research, Stanford University, Stanford, CA, April.
- Dantzig, G.B. (1975). "Note on the Objective Function for the PILOT Model," Technical Report SOL 75-20, Department of Operations Research, Stanford University, Stanford, CA, August; also in A. Prekopa (ed.), *Survey of Mathematical Programming*, Proceedings, IX International Symposium on Mathematical Programming, Publishing House of the Hungarian Academy of Sciences, Budapest, 1980, 325-328.
- Dantzig, G.B. (1976). "Linear Programming: Its Past and Its Future," in Salkovitz, Edward I. (ed.), *Science Technology, and the Modern Navy, Thirtieth Anniversary*, ONR-37, Office of Naval Research, 84-95.
- Dantzig, G.B. (1977). "Large-Scale Systems Optimization with Application to Energy," Technical Report SOL 77-3, Department of Operations Research, Stanford University, Stanford, CA, April; also in Proceedings of Symposia in Applied Mathematics, American Mathematical Society, Providence, RI.
- Dantzig, G.B. (1978). "Are Dual Variables Prices? If Not, How to Make them More So," Technical Report SOL 78-6, Department of Operations Research, Stanford University, Stanford, CA, March; also in Franco Angeli (ed.), *Mathematical Programming and its Economics Applications*, Milano, Italy, 1981, 135-148.
- Dantzig, G.B. (1979a). "The Role of Models in Determining Policy for Transition to a More Resilient Technological Society," IIASA Distinguished Lecture Series /1, Vienna, June 12, International Institute for Applied Systems Analysis, Laxenburg, 1979.

- Dantzig, G.B. (1979b). "Comments on Khachian's Algorithms for Linear Programming," Technical Report SOL 79-22, Department of Operations Research, Stanford University, Stanford, CA, November. Also in *SiamNews* **13**, October 1980.
- Dantzig, G.B. (1980a). "Expected Number of Steps of the Simplex Method for a Linear Program with a Convexity Constraint," Technical Report SOL 80-3, Department of Operations Research, Stanford University, Stanford, CA, March (revised October 1980).
- Dantzig, G.B. (1980b). "Time-Staged Methods in Linear Programming; Comments and Early History, Technical Report SOL 80-18, Department of Operations Research, Stanford University, Stanford, CA, June; also in in G.B. Dantzig, M.A.H. Dempster, and M.J. Kallio. (eds.), *Large-Scale Linear Programming*, Vol. 2, CP-81-51, IIASA Collaborative Proceedings Series, Laxenberg, Austria, 1981, 3-16; also in "Large Scale Systems.," in Haims, Y.Y. (ed.), *Studies in Management Science and Systems*, Vol. 7, North-Holland Publishing Company, Amsterdam, 1982, 19-30.
- Dantzig, G.B. (1980c). "Time-Staged Linear Programs," Technical Report SOL 80-28, Department of Operations Research, Stanford University, Stanford, CA, October.
- Dantzig, G.B. (1981a). "Reminiscences About the Origins of Linear Programming," Technical Report SOL 81-5, Department of Operations Research, Stanford University, Stanford, CA, April; also in Contemporary Mathematics, American Mathematical Society; also in R.W. Cottle, M.L. Kelmanson, B. Korte (eds.), *Mathematical Programming*, Proceedings of the International Congress on Mathematical Programming, Rio de Janeiro, Brazil, April 6-8, 1981, North-Holland Publishing Co., Amsterdam, 1984, 105-112; also in *Operations Research Letters* **1**, 43-48, April 1982. Also in in A. Bachem, M. Grottschel, and B. Korte (eds.), *Mathematical Programming: The State of the Art, Bonn 1982*, Springer-Verlag, Berlin and New York, 78-86. Also in *Australian Society for OR Bulletin*, 1986.
- Dantzig, G.B. (1981b). "Concerns About Large-Scale Models," Technical Report SOL 81-27, Department of Operations Research, Stanford University, Stanford, CA, December; also in *Large-Scale Energy Models*, AAAS Selected Symposia Series 73, Westview Press, Inc. Boulder, CO, for the Amer. Assoc. for the Adv. of Sc., Washington, DC, 15-20, 1983.
- Dantzig, G.B. (1981c). "The PILOT Energy-Economic Model for Policy Planning," Technical Report SOL 81-26, Department of Operations Research, Stanford University, Stanford, CA, December; also in in T. Nejat Veziroglu (ed.), *Energy Programs Policy Economics*, Proceedings of the 4th Miami International Conference on Alternative Energy Sources, Volume 8, Ann Arbor Science Publishers, Ann Arbor Michigan, 1982, 409-415.
- Dantzig, G.B. (1981d). "Contributions of Mathematics to Planning During and Immediately After World War II," working paper, November. Also in *History of Mathematics in World War II*, MAA Series.
- Dantzig, G.B. (1982a). "Time-Staged Methods in Linear Programs," in Y.Y. Haims (ed.), *Studies in Management Science*, Vol. 7: *Large-Scale Systems*, North-Holland, Amsterdam, the Netherlands, 19-30.
- Dantzig, G.B. (1982b). "Mathematical Programming and Decision Making in a Technological Society," Technical Report SOL 82-11, Department of Operations Research, Stanford University, Stanford, CA, August; also in *Information Processing Society of Japan* **24**, May 1983 (in Japanese).

- Dantzig, G.B. (1983). "Can Leontief and P-Matrices be Rescaled Positive Definite," Technical Report SOL 83-23, Department of Operations Research, Stanford University, Stanford, CA, November.
- Dantzig, G.B. (1984). "Economic Growth and Dynamic Equilibrium," Technical Report SOL 84-8, Department of Operations Research, Stanford University, Stanford, CA, October.
- Dantzig, G.B. (1985a). "Deriving a Utility Function for the Economy," Technical Report SOL 85-6, Department of Operations Research, Stanford University, Stanford, CA, June; revised April 1986.
- Dantzig, G.B. (1985b). "Impact of Linear Programming on Computer Development," Technical Report SOL 85-7, Department of Operations Research, Stanford University, Stanford, CA. Revised version, Stanford University, July 1986, in D.V. Chudnovsky and R.D. Jenks (eds.), *Proceedings Computers in Mathematics*, Lecture Notes in Pure and Applied Mathematics, Marcel Dekker, Inc., 1990, 233–240; also in *ORMS Today* **14**, August 1988, 12–17.
- Dantzig, G.B. (1986). "Need to do Planning under Uncertainty and the Possibility of Using Parallel Processors for this Purpose," Technical Report SOL 86-11, Department of Operations Research, Stanford University, Stanford, CA, April.
- Dantzig, G.B. (1987a). "Simplex Method for Solving Linear Programs," *The New Palgrave: A Dictionary of Economic Theory and Doctrine*, Macmillan, London.
- Dantzig, G.B. (1987b). "Linear Programming," *The New Palgrave: A Dictionary of Economic Theory and Doctrine*, Macmillan, London, to appear.
- Dantzig, G.B. (1987c). "Planning Under Uncertainty Using Parallel Computing," Technical Report SOL 87-1, Department of Operations Research, Stanford University, Stanford, CA, January; also in *Annals of Operations Research* **14**, 1988, 1–16.
- Dantzig, G.B. (1987d). "Origins of the Simplex Method," Technical Report SOL 87-5, Department of Operations Research, Stanford University, Stanford, CA, May; also in Nash, S.G. (ed.), *Proceedings of the ACM Conference on a History of Scientific Computing*, ACM Press, Addison-Wesley Publishing Company, 1990, 141–151.
- Dantzig, G.B. (1988a). "Dikin's Interior Method for Solving LP," manuscript, Department of Operations Research, Stanford University, Stanford, CA.
- Dantzig, G.B. (1988b). "Making Progress During a Stall in the Simplex Algorithm," Technical Report SOL 88-5, Department of Operations Research, Stanford University, Stanford, CA, February; also in *Linear Algebra and Its Applications* **114/115**, 1989, 251–259.
- Dantzig, G.B. (1989). "Decomposition Techniques for Large-Scale Electric Power Systems Planning Under Uncertainty," in R. Sharda, B.L. Golden, E. Wasil, O. Balci, and W. Steward (eds.), *Impact of Recent Computer Advances on Operations Research* North Holland, 3–20.
- Dantzig, G.B. (1990). "The Diet Problem," *Interfaces* **20**:4, July/Aug, 43–47.
- Dantzig, G.B. (1991). "Converting a Converging Algorithm into a Polynomial Bounded Algorithm," Technical Report SOL 91-5, Department of Operations Research, Stanford University, Stanford, CA, March.

- Dantzig, G.B. (1992a). "An ϵ -Precise Feasible Solution to a Linear Program with a Convexity Constraint in $1/\epsilon^2$ Iterations Independent of Problem Size," Technical Report SOL 92-5, Department of Operations Research, Stanford University, Stanford, CA, October.
- Dantzig, G.B. (1992b). "Bracketing to Speed Convergence Illustrated on the von Neumann Algorithm for Finding a Feasible Solution to a Linear Program with a Convexity Constraint," Technical Report SOL 92-6, Department of Operations Research, Stanford University, Stanford, CA, October.
- Dantzig, G.B. (1995). Working paper.
- Dantzig, G.B., Cottle, R.W., Eaves, B.C., Golub, G.H., Hillier, F.S., Manne, A.S., Wilde, D.J., and Wilson, R.B. (1973). "On The Need for a Systems Optimization Laboratory," in T.C. Hu and S.M. Robinson (eds.), *Mathematical Programming*, Proceedings of an Advanced Seminar Conducted by the Mathematics Research Center, University of Wisconsin, September 1972, Academic Press, London and New York, 1-31; previously published in September 1972 in Technical Report 72-11, Department of Operations Research, Stanford University, Stanford, CA, September.
- Dantzig, G.B. and Adler, I. (1971). "Maximum Diameter of Abstract Polytopes," Technical Report 71-12, Department of Operations Research, Stanford University, Stanford, CA, August; also in Harry Williams (ed.), *IDA Economics Papers*, January 1972; also in *Mathematical Programming Study* 1, 1974, 20-40.
- Dantzig, G.B., Adler, I., and Murty, K. (1970). "Existence of x-Paths in Abstract Polytopes," Technical Report 70-1, Department of Operations Research, Stanford University, Stanford, CA, March.
- Dantzig, G.B., Adler, I., and Murty, K. (1974). "Existence of A-Avoiding Paths in Abstract Polytopes," *Mathematical Programming Study* 1, 41-42.
- Dantzig, G.B., Avi-Itzhak, B., Connolly, T.J., McAllister, P.H., and Winkler, W.D. (1982a). "A Dynamic Equilibrium Model for Energy-Economic Planning," Technical Report SOL 82-2, Department of Operations Research, Stanford University, Stanford, CA, March.
- Dantzig, G.B., Avi-Itzhak, B., Connolly, T.J., McAllister, P.H., and Winkler, W.D. (1982b). "Mathematical Appendix: A Dynamic Equilibrium Model for Energy-Economic Planning," Technical Report SOL 82-3, Department of Operations Research, Stanford University, Stanford, CA, March.
- Dantzig, G.B., Avi-Itzhak, and Iusem, A. (1983). "The Consumers Energy Services Model of the PILOT System," in Benjamin Lev (ed.), *Energy Models and Studies*, Studies in Management Science and Systems 9, North-Holland Publishing Co., Amsterdam, 195-220.
- Dantzig, G.B., Beale, E.M.L., Watson, R.D. (1986). "A First Order Approach to a Class of Multi-Time-Period Stochastic Programming Problems," *Mathematical Programming Study* 27, 103-117.
- Dantzig, G.B. and Avriel, M. (1976). "Determining Prices and Monetary Flows of the PILOT Energy Model," Technical Report SOL 76-28, Department of Operations Research, Stanford University, Stanford, CA, October.

- Dantzig, G.B., Bigelow, J., Golub, G., Gordon, R., Montalbano, M., Pinsky, P., Sahberwal, F., Wirth, N., and Witzgall, C. (1967). "Mathematical Programming Language," Technical Report 67-4, Department of Operations Research, Stanford University, Stanford, CA, June.
- Dantzig, G.B., Bigelow, J., Golub, G., Gordon, R., Montalbano, M., Pinsky, P., Sahberwal, F., Wirth, N., and Witzgall, C. (1968). "Mathematical Programming Language," Technical Report CS 119, Department of Computer Science, Stanford University, Stanford, CA.
- Dantzig, G.B. and Blattner, W.O., and Rao, M.R. (1966a). "Finding a Cycle in a Graph with Minimum Cost to Time Ratio with Application to a Ship Routing Problem," Technical Report 66-1, Department of Operations Research, Stanford University, Stanford, CA, November; also in *Theorie Des Graphes*, International Symposium, Rome, Italy, July 1966, 77-84, published by DUNOD, Paris.
- Dantzig, G.B. and Blattner, W.O., and Rao, M.R. (1966b). "All Shortest Routes from a Fixed Origin in a Graph," Technical Report 66-2, Department of Operations Research, Stanford University, Stanford, CA, November; also in *Theorie Des Graphes*, International Symposium, Rome, Italy, July 1966, 85-90, published by DUNOD, Paris.
- Dantzig, G.B., Connolly, T.J., and Parikh, S.C. (1977). "Stanford PILOT Energy/Economic Model," Technical Report SOL 77-19, Department of Operations Research, Stanford University, Stanford, CA, July; also in *Advances in the Economics of Energy and Resources, Volume 1 - The Structure of Energy Markets*, JAI Press, 1979, 77-103; also in El Mallakh, R. and El Mallakh, D.H. (eds.), *Proceedings of the 4th International Conference on Energy Options and Conservation*, October 17-19, 1977, The International Research Center for Energy and Economic Development, Boulder, Colorado, 1978, 87-119. *Policy Analysis and Information Systems* 2, 1978, 23-51.
- Dantzig, G.B., Connolly, T.J., and Parikh, S.C. (1978). "Stanford PILOT Energy/Economic Model," EA-626, Volumes 1 and 2, Interim Report, Electric Power Research Institute, Palo Alto, California, May.
- Dantzig, G.B., Connolly, T.J., Parikh, S.C., Riddel, J.M. (1978). "A Description and Demonstration of the Stanford PILOT Energy/Economic Model," *Stanford PILOT Energy/Economic Model*, EA-626, Volumes 1 & 2, Electric Power Research Institute, May, 1-40; also in *Proceedings of Second US-USSR Symposium on Econometric Modeling*, Skyland, Virginia, 1978, University of Maryland, College Park, Maryland 1980.
- Dantzig, G.B. and Cottle, R.W. (1963). "Positive (Semi-) Definite Matrices and Mathematical Programming," Technical Report RR-35, Operations Research Center, University of California, Berkeley; also in "Positive (Semi-) Definite Programming," in J. Abadie (ed.), *Nonlinear Programming*, North-Holland, Amsterdam, the Netherlands, 55-73.
- Dantzig, G.B. and Cottle, R.W. (1967). "Complementary Pivot Theory of Mathematical Programming," Technical Report 67-2, Department of Operations Research, Stanford University, Stanford, CA, April; also in G.B. Dantzig, and A. F. Veinott, Jr. (eds.), *Mathematics of the Decision Sciences*, the American Mathematical Society Summer Seminar, Providence, RI, 1968; also in *Linear Algebra and Its Applications* 1, 103-125; also in G. B. Dantzig and B. C. Eaves (eds.), *Studies in Optimization*,

- MAA Studies in Mathematics, Vol. 10, Mathematical Association of America, 1974, 27–51.
- Dantzig, G.B. and Cottle, R.W. (1968). “A Generalization of the Linear Complementarity Problem,” Technical Report 68-9, Department of Operations Research, Stanford University, Stanford, CA.; also in *Journal of Combinatorial Theory* **8**, January 1970, 79–90.
- Dantzig, G.B. and Cottle, R.W. (1974). “Optimization, Mathematical Theory of (Linear and Nonlinear Programming),” *Encyclopaedia Britannica*, Vol. 13, 628–632.
- Dantzig, G.B., Collen, M.F., Rubin, L., Neyman J., Baer, R.M., and Siegelau, A.B. (1964). “Automated Multiphase Screening and Diagnosis,” *American Journal of Public Health* **54**, 741–750.
- Dantzig, G.B. and DeHaven, J. (1961). “On The Reduction of Certain Multiplicative Chemical Equilibrium Systems to Mathematically Equivalent Additive Systems,” *P-2419*, The RAND Corporation, August; also *Journal of Chemical Physics* **36**, May, 1962, 2620–2627.
- Dantzig, G.B., DeHaven, J., and Sams, C.F. (1960). “A Mathematical Model of the Respiratory System,” in Proceedings, Fourth Air Pollution Medical Research Conference, San Francisco, December, 72–95; also in *P-2048*, The RAND Corporation, July 1960.
- Dantzig, G.B., DeHaven, J., and Sams, C.F. (1961a). “A Mathematical Model of the Chemistry of the External Respiratory System,” in J. Neyman (ed.), *Proceedings 4th Berkeley Symposium on Mathematical Statistics and Probability, 1961*, University of California Press, Berkeley, California, 181–196.
- Dantzig, G.B., DeHaven, J., and Sams, C.F. (1961b). “A Mathematical Model of the Human External Respiratory System,” *Perspectives of Biology and Medicine* **IV** (3), Spring 1961, 324–376.
- Dantzig, G.B., Dempster, M.A.H., and Kallio, M.J. (eds.), (1981). *Large-Scale Linear Programming*, Vol. 1, CP-81-51, IIASA Collaborative Proceedings Series, Laxenberg, Austria.
- Dantzig, G.B., Dempster, M.A.H., and Kallio, M.J., editors (1981). *Large-Scale Linear Programming*, Vol. 2, CP-81-51, IIASA Collaborative Proceedings Series, Laxenberg, Austria.
- Dantzig, G.B. and Eaves, B.C. (1972). “Fourier-Motzkin Elimination and Its Dual,” Technical Report 72-18, Department of Operations Research, Stanford University, Stanford, CA, June 1972; also in *Journal of Combinatorial Theory* **14**, May 1973, 288–297; also in B. Roy (ed.), *Combinatorial Programming: Methods and Applications*, D. Reidel Publishing Co., Boston, 1975, 93–102.
- Dantzig, G.B. and Eaves, B.C., editors (1974). *Studies in Optimization, MAA Studies in Mathematics*, Vol. 10, Mathematical Association of America.
- Dantzig, G.B., Eaves, B.C., and Gale, D. (1976). “An Algorithm for a Piecewise Linear Model of Trade and Production with Negative Prices and Bankruptcy,” Technical Report SOL 76-19, Department of Operations Research, Stanford University, Stanford, CA; also in *Mathematical Programming* **16**, 1979, 190–209.

- Dantzig, G.B., Eaves, B.C., and Rothblum, U. (1983). "A Decomposition and Scaling-inequality for Line-sum-symmetric Nonnegative Matrices," Technical Report SOL 83-21, Department of Operations Research, Stanford University, Stanford, CA, December; also in *SIAM Journal on Discrete Mathematics* **6**, April 1985.
- Dantzig, G.B., Eisenberg, E., and Cottle, R.W. (1962). "Symmetric Dual Nonlinear Programs," Technical Report RR-35, Operations Research Center, University of California, Berkeley; also in *Pacific Journal of Mathematics* **15**, 1965, 809–812.
- Dantzig, G.B., Eisenstat, S., Magnanti, T.L., Maier, S., McGrath, M., Nicholson, V., and Reidl, C. (1970). "MPL: Mathematical Programming Language Specification Manual for Committee Review," Technical Report STAN-CS-70-187, Department of Computer Science, Stanford University, Stanford, CA.
- Dantzig, G.B., Folkman, J., and Shapiro, M. (1965). "On the Continuity of the Minimum Set of a Continuous Function," The RAND Corporation, June; also in *Journal of Mathematical Analysis and Applications* **17**, March 1967, 519–548.
- Dantzig, G.B., Ford, L.R., and Fulkerson, D.R. (1956). "A Primal-Dual Algorithm for Linear Programs," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 171–181; also, *RM-1709*, The RAND Corporation, May.
- Dantzig, G.B., Friel, J., Golightly, R., Harvey, R.P., and McKnight, R.D. (1975). "Solution of a Large-Scale Air Force Ordnance Planning Problem by Mathematical Programming," Proceedings of the Fire Support Requirements Methodology Workshop, Ketron, Inc., August.
- Dantzig, G.B. and Fulkerson, D.R. (1955). "Computation of Maximal Flows in Networks," *Naval Research Logistics Quarterly*, **2**, 277–283.
- Dantzig, G.B. and Fulkerson, D.R. (1954). "Minimizing the Number of Tankers to Meet a Fixed Schedule," *Naval Research Logistics Quarterly*, **1**, 217–222.
- Dantzig, G.B. and Fulkerson, D.R. (1956). "On the Max-Flow Min-Cut Theorems of Networks," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 215–221.
- Dantzig, G.B., Fulkerson, D.R., and Johnson, S.M. (1954). "Solution for a Large-Scale Traveling Salesman Problem," *Journal of Operations Research Society of America* **2**, 393–410.
- Dantzig, G.B., Fulkerson, D.R., and Johnson, S.M. (1959). "On a Linear Programming Combinatorial Approach to the Traveling Salesman Problem," *Operations Research* **7**, 58–66.
- Dantzig, G.B. and Glynn, P.W. (1990). "Parallel Processors for Planning Under Uncertainty," *Annals of Operations Research* **22**, 1–21; also in Technical Report SOL 88-8, Department of Operations Research, Stanford University, Stanford, CA, June.
- Dantzig, G.B., Glynn, P.W., Avriel, M., Stone, J.C., Entriken, R., and Nakayama, M. (1989). "Decomposition Techniques for Multi-Area Transmission Planning Under Uncertainty," Report EL-6484, Electrical Systems Division, Electric Power Research Institute, Palo Alto, California.

- Dantzig, G.B., Harvey, R.P., McKnight, R.D., and Smith S.S. (1969). "Sparse Matrix Techniques in Two Mathematical Programming Codes," Technical Report 69-1, Department of Operations Research, Stanford University, Stanford, CA; also in D.J. Rose and R.A. Willoughby (eds.), *Sparse Matrices and their Applications*, Plenum Press, New York, , Proceedings of the Symposium on Sparse Matrices and Their Applications IBM RA-1, IBM Watson Research Center, September 1968), March 1969, 85–100.
- Dantzig, G.B., Harvey, R.P., Lansdowne, Z.F., Maier, S.F., and Robinson, D.W. (1977a). "Computational Experience with a Continuous Network Design Code," GSBA Working Paper 236, Graduate School of Business Administration, Duke University, NC, December.
- Dantzig, G.B., Harvey, R.P., Lansdowne, Z.F., Maier, S.F., and Robinson, D.W. (1977b). "Formulating and Solving the Network Design Problem by Decomposition," GSBA Working Paper 215, Graduate School of Business Administration, Duke University, Durham, NC, January 1977; also in "A Convex Network Design Model Based on a Decomposition Procedure," *Transportation Research B*, **13B**, 1979, 5–17,
- Dantzig, G.B., Harvey, R.P., Lansdowne, Z.F., and Muth, R. (1979). "Framework for a System of Transportation Spatial Form Research Tools," Report DOT-TSC-RSPA-79-12, Final Report to U.S. Department of Transportation, Washington, DC, April.
- Dantzig, G.B., Harvey, R.P., Lapin, L.L., and Uren, J. (1966). "An Integer Branching Algorithm for Locating Warehouses," Standard Oil Company of California Report, Operations Research Division, October (revised December 1968).
- Dantzig, G.B., Harvey, R.P., and McKnight, R. (1964). "Updating the Product Form of the Inverse for the Revised Simplex Method," Technical Report 64-33, Operations Research Center, University of California, Berkeley; also in Association for Computing Machinery Proceedings, August 1965; and Abstract in Journal of the Association for Computing Machinery, October 1965.
- Dantzig, G.B., Hax, R., Pomeroy, R., Sanderson, R., and van Slyke, R. [with contributions by G. Buerk, I. Durrer, B. Laurent, S. Mukerjee] (1970). "Natural Gas Transmission System Optimization," American Gas Association, Inc., April.
- Dantzig, G.B. and Hirsch, W. (1954). "The Fixed Charge Problem," P-648, The RAND Corporation; also in *Naval Research Logistics Quarterly*, **15**, 413–424.
- Dantzig, G.B., Ho, J.K., and Infanger, G. (1991). "Solving Stochastic Linear Programs on a Hypercube Multicomputer," Technical Report SOL 91-10, Department of Operations Research, Stanford University, Stanford, CA, August.
- Dantzig, G.B. and Hoffman, A. (1956). "Dilworth's Theorem on Partially Ordered Sets," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 207–213; also in "New Directions in Mathematical Programming," RAND Symposium on Mathematical Programming, March 16-20, 1959, RAND R-351, page 1.
- Dantzig, G.B. Hoffman, A.J., and Hu, T.C. (1983). "Triangulations (Tilings) and Certain Block Triangular Matrices," Technical Report SOL 83-17, Department of Operations Research, Stanford University, Stanford, CA, September; also in *Mathematical Programming* **31**, 1985, 1–14.

- Dantzig, G.B. and Holling, C.S. (1974). "Determining Optimal Policies for Ecosystems," Technical Report 74-11, Department of Operations Research, Stanford University, Stanford, CA, August.
- Dantzig, G.B., Holling, C.S., Baskerville, C., Jones, D.D., and Clark, W. C. (1975). "A Case Study of Forest Ecosystem/Pest Management," Prepared for Proceedings International Canadian Conference on Applied Systems Analysis, 1975, WP-75-60, International Institute for Applied Systems Analysis, Laxenburg, Austria, June 1975.
- Dantzig, G.B., Holling, C.S., Clark, W.C., Jones, D.D., Baskerville, G., and Peterman, R.M. (1976). "Quantitative Evaluation of Pest Management Options: The Spruce Budworm Case Study," in D.L. Wood (ed.), *Proceedings of the XVth International Congress of Entomology*, August; also in W.E. Waters (ed.), *Current Topics in Forest Entomology*, U.S. Government Printing Office, Washington, DC, February 1979, 82-102.
- Dantzig, G.B., Holling, C.S., and Winkler, C. (1986). "Determining Optimal Policies for Ecosystems," *TIMS Studies in the Management Sciences* **21**, 1986, 453-473.
- Dantzig, G.B. and Infanger, G. (1992a). "Large-Scale Stochastic Linear Programs: Importance Sampling and Benders Decomposition," in C. Brezinski and U. Kulisch (eds.), *Computational and Applied Mathematics I—Algorithms and Theory*, Proceedings of the 13th IMACS World Congress, Dublin, Ireland, July 22-26, 1991, North Holland, 111-120; also Technical Report SOL 91-4, Department of Operations Research, Stanford University, Stanford, CA.
- Dantzig, G.B. and Infanger, G. (1992b). "Approaches to Stochastic Programming with Applications to Electric Power Systems," in K. Frauendorfer, H. Glavitsch, and R. Bacher (eds.), *Optimization in Planning and Operations of Electric Power Systems*, Lecture Notes of the SVOR/ASRO Tutorial, October 15-16, 1992, Thun, Switzerland, Physica Verlag, 125-138.
- Dantzig, G.B. and Infanger, G. (1993). "Multi-Stage Stochastic Linear Programs for Portfolio Optimization," *Annals of Operations Research* **45**, 59-76; also in Technical Report SOL 91-11, Department of Operations Research, Stanford University, Stanford, CA, September; also in *Proceedings of the Annual Symposium of RAMP (Research Association on Mathematical Programming)*, Tokyo, 1991.
- Dantzig, G.B. and Iusem, A. (1981). "Analyzing Labor Productivity Growth with the PILOT Model," Technical Report SOL 81-4, Department of Operations Research, Stanford University, Stanford, CA, March; also in Energy, Productivity and Economic Growth, A Workshop sponsored by the Electric Power Research Institute, Oelgeschlager, Gunn & Hain, Cambridge, Mass., 1983, 347-366.
- Dantzig, G.B. and Jackson, P. (1979). "Pricing Underemployed Capacity in a Linear Economic Model," Technical Report SOL 79-2, Department of Operations Research, Stanford University, Stanford, CA, February; also in R. W. Cottle, F. Giannessi, and J.L. Lions (eds.), *Variational Inequalities and Complementarity Problems: Theory and Applications*, John Wiley and Sons, Ltd., London, 1980, 127-134.
- Dantzig, G.B. and Johnson, D.L. (1963). "Maximum Payloads per Unit Time Delivered Through an Air Network," Report D1-82-0265, Boeing Scientific Research Laboratories, June; also in *Operations Research* **12**, 230-236.

- Dantzig, G.B. and Johnson, S. (1955). "A Production Smoothing Problem," Proceedings, Second Symposium on Linear Programming, National Bureau of Standards and Comptroller, U.S.A.F. Headquarters, January, 151-176.
- Dantzig, G.B., Johnson, S., and Wayne, W. (1958). "A Linear Programming Approach to the Chemical Equilibrium Problem," *Management Science* **5**, 38-43.
- Dantzig, G.B., Kawaratani, T.K., and Ullman, R.J. (1960). "Computing Tetraethyl-Lead Requirements in a Linear-Programming Format," *Operations Research* **8**, 24-29.
- Dantzig, G.B., Leichner, S.A., and Davis, J.W. (1992). "A Strictly Improving Phase I Algorithm Using Least-Squares Subproblems," Technical Report SOL 92-1, Department of Operations Research, Stanford University, Stanford, CA, April.
- Dantzig, G.B., Levin, S., and Bigelow, J. (1965). "On Steady-State Intercompartmental Flows," Technical Report 65-26, Operations Research Center, University of California, Berkeley; also in *Journal of Colloid and Interface Science* **23**, April 1967, 572-576.
- Dantzig, G.B. and Madansky, A. (1961). "On the Solution of Two-Stage Linear Programs Under Uncertainty," in J. Neyman (ed.), *Proceedings 4th Berkeley Symposium on Mathematical Statistics and Probability, 1961*, University of California Press, Berkeley, California, 165-176; also in *P-2039*, The RAND Corporation.
- Dantzig, G.B., Magnanti, T.L., and Maier, S. (1972). "The User's Guide to MPL/T.1 (Revised)," GSBA Working Paper 76, Graduate School of Business Administration, Duke University, Durham, NC, December 1972.
- Dantzig, G.B., Maier, S.F., and Lansdowne, Z.F. (1976). "The Application of Decomposition to Transportation Network Analysis," DOT Report, Control Analysis Corporation, Palo Alto, California, March.
- Dantzig, G.B. and Manne, A. (1974). "A Complementarity Algorithm for an Optimal Capital Path with Invariant Proportions," International Institute for Applied Systems Analysis, Laxenburg, Austria; also in Technical Report 74-1, Department of Operations Research, Stanford University, Stanford, CA, March; also in *Journal of Economic Theory* **9**, November, 312-323.
- Dantzig, G.B., McAllister, P.H., and Stone, J.C. (1985). "Changes Made for the PILOT-1983 Model," Technical Report SOL 85-12, Department of Operations Research, Stanford University, Stanford, CA, July.
- Dantzig, G.B., McAllister, P.H., and Stone, J.C. (1988a). "Deriving a Utility Function for the U.S. Economy," Technical Report SOL 88-6, Department of Operations Research, Stanford University, Stanford, CA, April. Parts I, II, III, in *Journal for Policy Modeling* **11**, 1989, 391-424 and Parts IV, V in *Journal for Policy Modeling* **11**, 1989, 569-592.
- Dantzig, G.B., McAllister, P.H., and Stone, J.C. (1988b). "Analyzing the Effects of Technological Change: A Computational General Equilibrium Approach," Technical Report SOL 88-12, Department of Operations Research, Stanford University, Stanford, CA, July.
- Dantzig, G.B., McAllister, P.H., and Stone, J.C. (1990). "An Interactive Model Management System: User Interface and System Design," Technical Report SOL 90-3, Department of Operations Research, Stanford University, Stanford, CA, January.

- Dantzig, G.B. and Orchard-Hays, W. (1953). "Alternate Algorithm for the Revised Simplex Method Using Product Form for the Inverse (Notes on Linear Programming: Part V)," *RM-1268*, The RAND Corporation, November.
- Dantzig, G.B. and Orchard-Hays, W. (1954). "The Product Form for the Inverse in the Simplex Method," *Mathematical Tables and Other Aids to Computation VIII*, April, 64-67.
- Dantzig, G.B. and Orden, A. (1952). "A Duality Theorem Based on the Simplex Method," Symposium on Linear Inequalities and Programming, Report 10, Project SCOOP, Planning Research Division, Director of Management Analysis Service, Comptroller, U.S.A.F. Headquarters, April, 51-55.
- Dantzig, G.B. and Orden, A. (1953). "Duality Theorems (Notes on Linear Programming: Part II)," *RM-1526*, The RAND Corporation.
- Dantzig, G.B., Orden, A., and Wolfe, P. (1955). "The Generalized Simplex Method for Minimizing a Linear Form Under Linear Inequality Constraints (Notes on Linear Programming: Part I)," *Pacific Journal of Mathematics* **5**, 183-195; also in *RM-1264*, The RAND Corporation, April 5, 1954.
- Dantzig, G.B. and Pace, N. (1963a). "Molecular-Sized Channels and Flows Against the Gradient," Technical Report 63-14, Operations Research Center, University of California, Berkeley.
- Dantzig, G.B. and Pace, N. (1963b). "A Model for Sodium-Potassium Transport in Red Cells," Technical Report 63-26, Operations Research Center, University of California, Berkeley.
- Dantzig, G.B. and Parikh, S.C. (1975). "On a PILOT Linear Programming Model for Assessing Physical Impact on the Economy of a Changing Energy Picture," Technical Report SOL 75-14, Department of Operations Research, Stanford University, Stanford, CA, June 1975 (revised SOL 75-14R, August 1975); also in F.S. Roberts (ed.), *Energy: Mathematics and Models*, Proceedings of a SIMS Conference on Energy, held at Alta, Utah, July 7-11, 1975, SIAM, 1976, 1-23; also in IASA Conference '76, May 10-13 1976, 183-200; also in Proceedings of Symposia in Applied Mathematics, Vol. 21, American Mathematical Society, 1977, pp. 93-106.
- Dantzig, G.B. and Parikh, S.C. (1976). "Energy Models and Large-Scale Systems Optimization," Technical Report SOL 76-23, Department of Operations Research, Stanford University, Stanford, CA, November; also in W.W. White (ed.), *Computers and Mathematical Programming*, Proceedings of the Bicentennial Conference on Mathematical Programming, November 1976; also in NBS Special Publication 502, February 1978, 4-10.
- Dantzig, G.B. and Parikh, S.C. (1977). "At the Interface of Modeling and Algorithms Research" Technical Report SOL 77-29, Department of Operations Research, Stanford University, Stanford, CA, October. Proceedings of Nonlinear Programming Symposium 3, University of Wisconsin, July 1977, Academic Press, 1978, 283-302.
- Dantzig, G.B. and Parikh, S.C. (1978). "PILOT Model for Assessing Energy-Economic Options," in Bagiotti, T. and Franco, G. (eds.), *Pioneering Economics*, Edizioni Cedam - Padova, Italy, 271-276.
- Dantzig, G.B. and Pereira, M.V.F. (1988). "Mathematical Decomposition Techniques for Power System Expansion Planning," EPRI EL-5299, Volumes 1-5, February 1988, Electric Power Research Institute, Palo Alto, CA.

- Dantzig, G.B. and Perold, A.F. (1978). "A Basic Factorization Method for Block Triangular Linear Programs," SOL78-7, April; also in I.S. Duff and G.W. Stewart (eds.), *Sparse Matrix Proceedings*, SIAM, Philadelphia, 1979, 283–312.
- Dantzig, G.B. and Ramser, J.H. (1959a). "Optimum Routing of Gasoline Delivery Trucks," Proceedings, World Petroleum Congress, Session VIII, Paper 19, 1959.
- Dantzig, G.B. and Ramser, J.H. (1959b). "The Truck Dispatching Problem," *Management Science* **6**, 80–91.
- Dantzig, G.B. and Reynolds, G.H. (1966). "Optimal Assignment of Computer Storage by Chain Decomposition of Partially Ordered Sets," Technical Report 66-6, Operations Research Center, University of California, Berkeley.
- Dantzig, G.B. and Saaty, T.L. (1973). *Compact City*, Freeman, San Francisco.
- Dantzig, G.B. and Sethi, S.P. (1981). "Linear Optimal Control Problems and Generalized Linear Programming," *Journal of the Operational Research Society* **32**, 467–476.
- Dantzig, G.B. and Shapiro, M. (1960). "Solving the Chemical Equilibrium Problem Using the Decomposition Principle," P-2056, The RAND Corporation, August.
- Dantzig, G.B., Stone, J.C., and McAllister, P.H. (1986). "Using the PILOT Model to Study the Effects of Technological Change," Technical Report SOL 86-16, Department of Operations Research, Stanford University, Stanford, CA, December; also in B. Lev, J. Bloom, A. Gleit, F. Murphy, and C. Shoemaker (eds.), *Strategic Planning in Energy and Natural Resources*, Studies in Management Science and Systems, Vol. 15, Proceedings of the 2nd Symposium on Analytic Techniques for Energy, Natural Resources and Environmental Planning April 1986; North-Holland, Amsterdam, 1987, 31–42.
- Dantzig, G.B., Stone, J.C., and McAllister, P.H. (1988). "Formulating an Objective for an Economy," Proceedings of the Martin Beale Memorial Symposium, Mathematical Programming 42, (Series B), 11–32.
- Dantzig, G.B. and Tomlin, J.A. (1987). "E.M.L. Beale, FRS: Friend and Colleague," Technical Report SOL 87-2, Department of Operations Research, Stanford University, Stanford, CA, January 1987; also in *Mathematical Programming* **38**, 117–131.
- Dantzig, G.B. and Van Slyke, R.M. (1964a). "Generalized Upper Bounded Techniques for Linear Programming — I," Technical Report 64-17, Operations Research Center, University of California, Berkeley; also in Proceedings IBM Scientific Computing Symposium, Combinatorial Problems, March 16–18, 1964, 249–261.
- Dantzig, G.B. and Van Slyke, R.M. (1964b). "Generalized Upper Bounded Techniques for Linear Programming — II," Technical Report 64-18, Operations Research Center, University of California, Berkeley.
- Dantzig, G.B. and Van Slyke, R.M. (1967). "Generalized Upper Bounding Techniques," *Journal of Computer and System Science* **1**, 213–226.
- Dantzig, G.B. and Van Slyke, R.M. (1971). "Generalized Linear Programming," in David Wismer (ed.), *Optimization Methods and Applications for Large Systems*, McGraw-Hill, New York, 75–120.
- Dantzig, G.B. and Veinott, A.F. (eds.), (1967). *Mathematics of the Decision Sciences*, Proceedings of the American Mathematical Society Summer Seminar, Providence, RI.

- Dantzig, G.B. and Veinott, A.F., editors (1968a). *Lectures in Applied Mathematics*, Vol. 11, American Mathematics Society, Providence, RI.
- Dantzig, G.B. and Veinott, A.F. (1968b). "Integral Extreme Points," Technical Report 67-7, Department of Operations Research, Stanford University, Stanford, CA, November; also in *SIAM Review* **10**, 371–372.
- Dantzig, G.B. and Veinott, A.F. (1977). "Discovering Hidden Totally Leontief Substitution Systems," Technical Report SOL 77-17, Department of Operations Research, Stanford University, Stanford, CA, June. Revised for *Mathematics of Operations Research* **3**, May 1978, 102–103.
- Dantzig, G.B. and Wald, A. (1951). "On the Fundamental Lemma of Neyman and Pearson," *Annals of Mathematical Statistics* **22**, 87–93.
- Dantzig, G.B. and Wolfe, P. (1960). "Decomposition Principle for Linear Programs," *Operations Research* **8**, 101–111; also in P. Wolfe (ed.), *RAND Symposium on Mathematical Programming*, March 1959, RAND R-351, page 5; also in G. B. Dantzig and B. C. Eaves (eds.), *Studies in Optimization*, MAA Studies in Mathematics, Vol. 10, Mathematical Association of America, 1974.
- Dantzig, G.B. and Wolfe, P. (1961). "The Decomposition Algorithm for Linear Programming," *Econometrica*, **29**, 767–778.
- Dantzig, G.B. and Wolfe, P. (1962). "Linear Programming in a Markov Chain," *Operations Research* **10**, 702–710; also *RM-2957-PR*, The RAND Corporation, April.
- Dantzig, G.B. and Wood, M.K. (1951). "Programming of Interdependent Activities, I: General Discussion," *Econometrica*, **17**, 193–199; also in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 15–18.
- Dantzig, G.B. and Ye, Y. (1990). "A Build-Up Interior Method for Linear Programming: Affine Scaling Form," Technical Report SOL 90-4, Department of Operations Research, Stanford University, Stanford, CA, March.
- Davidon, W.C. (1979). "Variable Metric Methods for Optimization," *Atomic Energy Commission (AEC) Research and Development Report ANL-5990*, Argonne National Laboratory, Argonne, Illinois.
- Davis, K.D. and McKeown, P.G. (1981). *Quantitative Models for Management*, Kent Publishing Company, Boston, Massachusetts.
- Davis, P.J. and Rabinowitz, P. (1984). *Methods of Numerical Integration*, Academic Press, London and New York.
- Day, J. and Peterson, B. (1988). "Growth in Gaussian Elimination," *The American Mathematical Monthly* **95**, 489–513.
- de Boor, C.W. (1971). "CADRE: An Algorithm for Numerical Quadrature," in J. Rice (ed.), *Mathematical Software*, Academic Press, London and New York, 417–449.
- Deák, I. (1988). "Multidimensional Integration and Stochastic Programming," in Y. Ermoliev and R.J.B. Wets (eds.), *Numerical Techniques for Stochastic Optimization*, Springer-Verlag, Berlin and New York, 187–200.
- Dembo, R.S. and Steihaug, T. (1983). "Truncated Newton Methods for Large Scale Unconstrained Optimization," *Mathematical Programming* **26**, 190–212.

- Demmel, J.W. (1987). "On the Distance to the Nearest Ill-Posed Problem," *Numerische Mathematik* **51**, 251–289.
- Denardo, E.V. and Fox, B.L. (1979). "Shortest-Route Methods: 1. Reaching, Pruning, and Buckets," *Operations Research* **27**, 161–186.
- Dennis, J.E., Jr. (1977). "Nonlinear Least Squares," in D. Jacobs (ed.), *The State of the Art in Numerical Analysis*, Academic Press, London and New York, 269–312.
- Dennis, J.E. and Moré, J.J. (1977). "Quasi-Newton Methods, Motivation and Theory," *SIAM Review* **19**, 46–89.
- Dennis, J.E. and Moré, J.J. (1974). "A Characterization of Superlinear Convergence and its Application to Quasi-Newton Methods," *Mathematics of Computation* **28**, 549–560.
- Dial, R.B. (1969). "Algorithm 360: Shortest Path Forest with Topological Ordering," *Communications of the Association for Computing Machinery* **12**, 632–633.
- Dijkstra, E. (1959). "A Note on Two Problems in Connection with Graphs," *Numerische Mathematik* **1**, 269–271.
- Dikin, I.I. (1967). "Iterative Solution of Problems of Linear and Quadratic Programming," *Doklady Akademii Nauk USSR* **174**, 747–748, translated in *Soviet Mathematics Doklady* **8**, 674–675.
- Dikin, I.I. (1974). "On the Convergence of an Iterative Process," *Upravlyaemye Sistemy* **12**, 54–60.
- Dikin, I.I. (1990). "The Convergence of Dual Variables," Technical Report, Siberian Energy Institute, Irkutsk, Russia.
- Dikin, I.I. (1992). "Determination of an Interior Point of one System of Linear Inequalities," *Kibernetika and System Analysis* **1**, 74–96.
- Dikin, I.I. and Zorkaltsev, V.I. (1980). *Iterative Solution of Mathematical Programming Problems: Algorithms for the Method of Interior Points*, Nauka, Novosibirsk, USSR.
- Dilworth, R.P. (1950). "A Decomposition Theorem for Partially Ordered Sets," *Annals of Mathematics* **51**, 161–166.
- Dinic, E.A. (1970). "Algorithm for Solution of a Problem of Maximum Flow in a Network with Power Estimation," *Soviet Mathematics Doklady* **11**, 1277–1280.
- Dixon, L.C.W. (1972a). "Quasi-Newton Methods Generate Identical Points," *Mathematical Programming* **2** 383–387.
- Dixon, L.C.W. (1972b). "Quasi-Newton Methods Generate Identical Points. II. The Proof of Four New Theorems," *Mathematical Programming* **3** 345–358.
- Dodson, D.S. and Lewis, J.G. (1985). "Proposed Sparse Extensions to the Basic Linear Algebra Subprograms," *SIGNUM Newsletter* **20**, 22–25.
- Doig, A.G. and Belz, M.H. (1956). "Report on Trim Problems for May, 1956." Department of Statistics, University of Melbourne, Australia. The report is addressed to Australia Paper Manufacturers, Melbourne, Australia, July 31, 1956.
- Doig, A.G. and Land, A.H. (1960). "An Automatic Method of Solving Discrete Programming Problems," *Econometrica*, **28**, 497–520.
- Dongarra, J.J., Bunch, J.R., Moler, C.B., and Stewart, G.W. (1979). "LINPACK Users Guide," SIAM, Philadelphia.

- Dongarra, J.J., DuCroz, J., Hammarling, S. and Hanson, R.J. (1985). "A Proposal for an Extended Set of Fortran Basic Linear Algebra Subprograms," *SIGNUM Newsletter* **20**, 2–18.
- Dongarra, J.J., DuCroz, J., Hammarling, S. and Hanson, R.J. (1988a). "An Extended Set of Fortran Basic Linear Algebra Subprograms," *ACM Transactions on Mathematical Software* **14**, 1–17.
- Dongarra, J.J., DuCroz, J., Hammarling, S. and Hanson, R.J. (1988b). "Algorithm 656 An Extended Set of Fortran Basic Linear Algebra Subprograms: Model Implementation and Test Programs," *ACM Transactions on Mathematical Software* **14**, 18–32.
- Dongarra, J.J., DuCroz, J., Hammarling, S. and Hanson, R.J. (1988c). "A Set of Level 3 Basic Linear Algebra Subprograms," Report ANL-MCS-TM-88, Argonne National Laboratory, Argonne, Illinois.
- Dongarra, J.J., Duff, I.S., Sorensen, D.C., and van der Vorst, H.A. (1991). *Solving Linear Systems on Vector and Shared Memory Computers*, SIAM, Philadelphia.
- Dongarra, J.J. and Eisenstat, S. (1984). "Squeezing the Most out of an Algorithm in Cray Fortran," *ACM Transactions on Mathematical Software* **10**, 221–230.
- Dongarra, J.J., Gustavson, F.G., and Karp, A. (1984). "Implementing Linear Algebra Algorithms for Dense Vectors on a Vector Pipeline Machine," *SIAM Review* **26**, 91–112.
- Dongarra, J.J. and Hinds, A. (1979). "Unrolling Loops in Fortran," *Software Practice and Experience* **9**, 219–229.
- Duff, I.S. (1976). "A Survey of Sparse Matrix Research," *Report AERE CSS 28*, Atomic Energy Research Establishment, Harwell, England.
- Duff, I.S. (1977). "MA28—A Set of Fortran Subroutines for Sparse Unsymmetric Linear Equations," *Report AERE AERE R8730*, Atomic Energy Research Establishment, Harwell, England.
- Duff, I.S. (1981). "An Algorithm for Obtaining a Maximum Transversal," *ACM Transactions on Mathematical Software* **7**, 315–330.
- Duff, I.S., Erisman, A.M., and Reid, J.K. (1986). *Direct Methods for Sparse Matrices*, Oxford University Press, Oxford and New York.
- Duff, I.S. and Reid, J.K. (1978). "An Implementation of Tarjan's Algorithm for the Block Triangularization of a Matrix," *ACM Transactions on Mathematical Software* **4**, 137–147.
- Duff, I.S. and Stewart, G.W. (eds.) (1979). *Sparse Matrix Proceedings*, SIAM, Philadelphia.
- Duffin, R.J. (1974). "On Fourier's Analysis of Linear Inequality Systems," *Mathematical Programming Study* **1**, 71–95.
- Dupačová, J. (1990). "Stability and Sensitivity Analysis for Stochastic Programming," *Annals of Operations Research* **27**, 115–142.

E

- Eaves, B.C. (1979). "A View of Complementary Pivot Theory (or Solving Equations with Homotopies)," in C.V. Coffman and G.J. Fix (eds.), *Constructive Approaches to Mathematical Models*, 1979, Academic Press, London and New York, 153–170; and in H-O. Peitgen and H-O. Walther (eds.), *Functional Differential Equations and Approximation of Fixed Points*, 1979, Springer-Verlag, Berlin and New York
- Edmonds, J. and Karp, R.M. (1972). "Theoretical Improvements in Algorithmic Efficiency for Network Flow Algorithms," *Journal of the Association for Computing Machinery* **19**, 248–264.
- Edmondson, J.H. (1951). "Proof of the Existence Theorem of an Epsilon Transformation," class exercise dated March 28, 1951, for Department of Agriculture Graduate School course in linear programming given by George Dantzig.
- Egerváry, E. (1931). "Matrixok Kombinatorius Tulajfonságairól," *Matematikai és Fizikai Lapok*, No. 38, 16–28. "On Combinatorial Properties of Matrices," translated by H.W. Kuhn, Paper No. 4, George Washington University Logistics Research Project. Published in *Logistics Papers*, Issue No. 11, Appendix I to Quarterly Progress Report No. 21, (November 16, 1954 to February 15, 1955).
- Elias, P., Feinstein, A., and Shannon, C.E. (1956). "Note on Maximum Flow through a Network," *IRE Transactions on Information Theory*, **IT-2**, 117–119.
- Elmaghraby, S.E. (1977). *Activity Networks: Project Planning and Control by Network Models*, Wiley, New York.
- Entriken, R. (1989). "The Parallel Decomposition of Linear Programs," Ph.D. thesis, Department of Operations Research, Stanford University, Stanford, CA.
- Erisman, A.M., Grimes, R.G., Lewis, J.G., and Poole, W.G., Jr. (1985). "A Structurally Stable Modification of Hellerman-Rarick's P^4 Algorithm for Reordering Unsymmetric Sparse Matrices," *SIAM Journal on Numerical Analysis* **22**, 369–385.
- Erisman, A.M. and Reid, J.K. (1974). "Monitoring the Stability of the Triangular Factorization of a Sparse Matrix," *Numerische Mathematik* **22**, 183–186.
- Ermoliev, Y. (1983). "Stochastic Quasi-Newton Methods and Their Applications to Systems Optimization," *Stochastics*, **9**, 1–36.
- Ermoliev, Y. (1988). "Stochastic Quasi-Gradient Methods," in Y. Ermoliev and R.J.B. Wets (eds.), *Numerical Techniques for Stochastic Optimization*, Springer-Verlag, Berlin and New York, 141–186.
- Ermoliev, Y. and Wets, R.J.B., eds. (1988). *Numerical Techniques for Stochastic Optimization*, Springer-Verlag, Berlin and New York.
- Eisemann, K. (1957). "The Trim Problem," *Management Science* **3**, 279–284.

F

- Fabian, T. (1954). "Process Type Analysis of the Iron and Steel Industry, Part IV—Programming Open Hearth Steel Production," Discussion Paper No. 46, Management Sciences Research Project, University of California, Los Angeles, California, 12 pages.

- Fabian, T. (1955). "Process Analysis of the Iron and Steel Industry: A Model," *Econometrica*, **23**, 347–348 (abstract); also Research Report No. 47, Management Sciences Research Project, University of California, Los Angeles, California.
- Fabian, T. (1958). "A Linear Programming Model of Integrated Iron and Steel Production," *Management Science* **4**, 425–449.
- Faddeev, D.K., and Faddeeva, V.N. (1963). *Computational Methods in Linear Algebra*, Freeman, San Francisco; translated from the Russian version.
- Farkas, J. (1902). "Über die Theorie der einfachen Ungleichungen," *Journal für die Reine and Angewandte Mathematik* **124**, 1–24.
- Faulkner, J.C. (1988). "Bus crew scheduling and the set partitioning model," Ph.D. thesis, Department of Theoretical and Applied Mechanics, University of Auckland, Auckland, New Zealand.
- Feinstein, C.D. and Thapa, M.N. (1993). "A Reformulation of a Mean-Absolute Deviation Portfolio Optimization Model," *Management Science* **39**, 1552–1553.
- Feller, W. (1957). *An Introduction to Probability Theory and Its Applications*, Volume I, John Wiley and Sons, New York.
- Feller, W. (1969). *An Introduction to Probability Theory and Its Applications*, Volume II, John Wiley and Sons, New York.
- Fenner, T. and Loizou, G. (1974). "Some New Bounds on the Condition Numbers of Optimally Scaled Matrices," *Journal of the Association for Computing Machinery* **21**, 514–524.
- Ferguson, A.R. and Dantzig, G.B. (1955). "Notes on Linear Programming: Part XVI—The Problem of Routing Aircraft—a Mathematical Solution," *Aeronautical Engineering Review* **14**, 51–55; also in *RM-1369*, The RAND Corporation, September 1, 1954, and *P-561*, The RAND Corporation, 1954.
- Ferguson, A.R. and Dantzig, G.B. (1956). The Allocation of Aircraft to Routes—An Example of Linear Programming under Uncertain Demand," *Management Science* **3** 1, 45–73; also in *P-727*, The RAND Corporation, December 7, 1956; also in Bowman and Fetter (eds.), *Analysis of Industrial Operations*, Richard D. Irwin, Inc., Homewood, Illinois, 1959.
- Fisher, W.D. and Schruben, L.W. (1953). "Linear Programming Applied to Feed-Mixing under Different Price Conditions," *Journal of Farm Economics* **35**, 471–483.
- Fiacco, A.V. and McCormick, G.P. (1968). *Nonlinear Programming: Sequential Unconstrained Minimization Techniques*, John Wiley and Sons, New York.
- Fletcher, R. (1985). "Degeneracy in the Presence of Round-Off Errors," Technical Report NA/89, Department of Mathematical Sciences, University of Dundee, Dundee.
- Fletcher, R. (1987). "Recent Developments in Linear and Quadratic Programming", in A. Iserles and M.J.D. Powell (eds.), *The State and Art in Numerical Analysis*, Oxford University Press, Oxford and New York, 213–243.
- Fletcher, R., and Mathews, S.P.J. (1984). "Stable Modification of Explicit LU Factors for Simplex Updates," *Mathematical Programming* **30**, 267–284.
- Ford, L.R., Jr., and Fulkerson, D.R. (1956). "Maximal Flow Through a Network," *Canadian Journal of Mathematics* **8**, 399–404. This appeared first as the RAND Corporation Research Memorandum RM-1400, November 19, 1954.

- Ford, L.R., Jr., and Fulkerson, D.R. (1957). "A Simple Algorithm for Finding Maximal Network Flows and an Application to the Hitchcock Problem," *Canadian Journal of Mathematics* **9**, 210–218.
- Ford, L.R., Jr., and Fulkerson, D.R. (1958a). "Constructing Maximal Dynamic Flows from Static Flows," *Operations Research* **6**, 419–433.
- Ford, L.R., Jr., and Fulkerson, D.R. (1958b). "Suggested Computation for Maximal Multi-Commodity Network Flows," *Management Science* **5**, 97–101.
- Ford, L.R., Jr., and Fulkerson, D.R. (1962). *Flows in Networks*, Princeton University Press, Princeton, New Jersey.
- Forrest, J.J.H. and Goldfarb, D. (1992). "Steepest-edge Simplex Algorithms for Linear Programming," *Mathematical Programming* **57**, 341–376.
- Forrest, J.J.H. and Tomlin, J. A. (1972). "Updating Triangular Factors of the Basis to Maintain Sparsity in the Product Form Simplex Method," *Mathematical Programming* **2**, 263–278.
- Forsgren, A.L. and Murray, W. (1990). "Newton Methods for Large-Scale Linearly Constrained Minimization," Technical Report SOL 90-6, Department of Operations Research, Stanford University, Stanford, CA.
- Forsythe, G.E. (1960). "Crout with Pivoting," *Communications of the Association for Computing Machinery* **3**, 507–508.
- Forsythe, G.E. (1970). "Pitfalls in Computation, or Why a Math Book Isn't Enough," *The American Mathematical Monthly* **9**, 931–956.
- Forsythe, G.E., Malcolm, M.A., and Moler, C.B. (1977). *Computer Methods for Mathematical Computations*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Forsythe, G.E. and Moler, C.B. (1967). *Computer Solution of Linear Algebraic Systems*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Forsythe, G.E. and Wastow, W.R. (1960). *Finite Difference Methods for Partial Differential Equations*, John Wiley and Sons, New York.
- Foster, L.V. (1986). "Rank and Null Space Calculations Using Matrix Decomposition Without Column Interchanges," *Linear Algebra and Its Applications* **74**, 47–71.
- Fourer, R. (1979). "Sparse Gaussian Elimination of Staircase Linear Systems," Technical Report SOL 79-17, Department of Operations Research, Stanford University, Stanford, CA.
- Fourer, R. (1982). "Solving Staircase Linear Programs by the Simplex Method, 1: Inversion," *Mathematical Programming* **23**, 274–313.
- Fourer, R. (1983a). "Solving Staircase Linear Programs by the Simplex Method, 2: Pricing," *Mathematical Programming* **25**, 251–292.
- Fourer, R. (1983b). "Modeling Languages versus Matrix Generators for Linear Programming," *ACM Transactions on Mathematical Software* **9**, 143–183.
- Fourer, R. (1984). "Staircase Systems and Matrices," *SIAM Review* **26**, 1–70.
- Fourer, R. (1985). "A Simplex Algorithm for Piecewise-Linear Programming I: Derivation and Proof," *Mathematical Programming* **33** 204–233.
- Fourer, R., Gay, D.M., and Kernighan, B.W. (1992). *AMPL: A Modeling Language for Mathematical Programming*, Scientific Press, South San Francisco, California.

- Fourer, R. and Mehrotra, S. (1992). "Performance of an Augmented System Approach for Solving Least-Squares Problems in an Interior-Point Method for Linear Programming," *Mathematical Programming Society COAL Newsletter* **19**, 26–31; previously published in 1991 as a technical report, Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, IL.
- Fourer, R. and Mehrotra, S. (1993). "Solving Symmetric Indefinite Systems in an Interior-Point Method for Linear Programming," *Mathematical Programming* **62**, 15–39; previously published in 1992 as Technical Report 92-01, Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, IL.
- Fourier, J.B.J. (1826). "Solution d'une question particulière du calcul des inégalités," 1826, and extracts from "Histoire de l'Académie," 1823, 1824, *Oeuvres* II, 317–328.
- Fourier, J.B.J. (1890). "Second Extrait," in G. Darboux (ed.), *Oeuvres*, Gauthiers-Villars, Paris, 325–328; English translation by Kohler [1973].
- Fox, L. (1973). *Introduction to Numerical Linear Algebra*, Oxford University Press, Oxford and New York; originally published in 1964.
- Fraley, C. and Vial, J.-Ph. (1989). "Numerical Study of Projective Methods for Linear Programming," in S. Dolecki (ed.), *Optimization Proceedings of the 5th French-German Conference in Castel-Novel, Varetz, France, October 1988*, 25–38.
- Frauendorfer, K. (1988). "Solving SLP Recourse Problems with Arbitrary Multivariate Distributions — The Dependent Case," *Mathematics of Operations Research* **13**, 3, 377–394.
- Frauendorfer, K. (1992). *Stochastic Two-Stage Programming*, Lecture Notes in Economics and Mathematical Systems 392, Springer-Verlag, Berlin and New York.
- Frauendorfer, K., and Kall, P. (1988). "Solving SLP Recourse Problems with Arbitrary Multivariate Distributions — The Independent Case," *Problems of Control and Information Theory* **17** (4), 177–205.
- Fredman, M.L. and Willard, D.E. (1994). "Trans-dichotomous Algorithms for Minimum Spanning Trees and Shortest Paths," *Journal of Computer and System Sciences* **48**, 533–551.
- Freund, R. (1988a). "Projective Transformation for Interior Point Methods, Part I: Basic Theory and Linear Programming," working paper OR 179-88, Operations Research Center, Massachusetts Institute of Technology.
- Freund, R. (1988b). "An Analog of Karmarkar's Algorithm for Inequality Constrained Linear Programs, with a 'New' Class of Projective Transformations for Centering a Polytope," *Operations Research Letters* **7**, 9–14; previously published in 1987 as a working paper OR 1921-87, Operations Research Center, Massachusetts Institute of Technology.
- Freund, R. (1991a). "Theoretical Efficiency of a Shifted Barrier Function Algorithm for Linear Programming," *Linear Algebra and Its Applications* **152**, 19–41.
- Freund, R. (1991b). "Polynomial-Time Algorithms for Linear Programming Based only on Primal Scaling and Projected Gradients of a Potential Function," *Mathematical Programming* **51**, 203–222.
- Frisch, K.R. (1955). *The Logarithmic Potential Method of Convex Programs*, Memorandum of May 13, 1955, UIEOSLO.

- Frisch, K.R. (1957). *Linear Dependencies and a Mechanized Form of the Multiplex Method Linear Programming*, Memorandum of September, 1957, University Institute of Economics, Oslo, Norway.
- Fulkerson, D.R., and Dantzig, G.B. (1955). "Computations of Maximal Flows in Networks," *Naval Research Logistics Quarterly*, **2**, 277–283.
- ## G
- Gabow, H.N. and Tarjan, R.E. (1989). "Faster Scaling Algorithms for Network Problems," *SIAM Journal on Computing* **18**, 1013–1036.
- Gainen, L. (1955). "Linear Programming in Bid Evaluations," in H.A. Antosiewicz (ed.), *Proceedings of the Second Symposium in Linear Programming*, Vol. 2, National Bureau of Standards and Directorate of Management Analysis, DCS/Comptroller, USAF, Washington, DC, 29–38.
- Gaivoronski, A. (1988). "Implementation of Stochastic Quasi-Gradient Methods," in Y. Ermoliev and R.J.B. Wets (eds.), *Numerical Techniques for Stochastic Optimization*, Springer-Verlag, Berlin and New York, 313–352.
- Gale, D.H. (1951). "Convex Polyhedral Cones and Linear Inequalities," in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 287–297.
- Gale, D.H., Kuhn, H.W., and Tucker, A.W. (1951). "Linear Programming and the Theory of Games," in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 317–329.
- Galil, Z. (1978). "A New Algorithm for the Maximal Flow Problem," *Proceedings of the Nineteenth Annual Symposium on Foundations of Computer Science, IEEE*, 231–245.
- Gallivan, K., Jalby, W., Meier, U., and Sameh, A.H. (1988). "Impact of Hierarchical Memory Systems on Linear Algebra Algorithm Design," *International Journal of Supercomputer Applications* **2**, 12–48.
- Gallo, G. and Pallottino, S. (1988). "Shortest Path Algorithms," *Annals of Operations Research* **13**, 3–79.
- Garbow, B.S., Boyle, J.M., Dongarra, J.J., and Moler, C.B. (1977). *Matrix Eigensystem routines—EISPACK Guide Extensions, Lecture Notes in Computer Science* **51**, Springer-Verlag, Berlin and New York.
- Garvin, W.W., Crandall H.W., John, J.B., and Spellman, R.A. (1957). "Applications of Linear Programming in the Oil Industry," *Management Science* **3**, 407–430.
- Gass, S.I. (1991). "Model World: Models at the OK Corral," *Interfaces* **21**:6, 80–86.
- Gass, S.I. (1985). *Linear Programming: Methods and Applications*, McGraw-Hill, New York.
- Gass, S.I., and Saaty, T.L. (1955a). "The Computational Algorithm for the Parametric Objective Function," *Naval Research Logistics Quarterly*, **2**, 39–45.

- Gass, S.I., and Saaty, T.L. (1955b). "The Parametric Objective Function, Part 1," *Operations Research* **2**, 316–319.
- Gass, S.I., and Saaty, T.L. (1955c). "The Parametric Objective Function, (Part 2)—Generalization," *Operations Research* **3**, 395–401.
- Gassner, B.J. (1964). "Cycling in the Transportation Problem," *Naval Research Logistics Quarterly*, **11**, 43–58.
- Gay, D.M. (1978). "On Combining the Schemes of Reid and Saunders for Sparse LP Bases," in I.S. Duff and G.W. Stewart (eds.), *Sparse Matrix Proceedings*, SIAM, Philadelphia, 313–334.
- Gay, D.M. (1985). "Electronic Mail Distribution of Linear Programming Test Problems," *Mathematical Programming Society COAL Newsletter* **13**, 10–12.
- Gay, D.M. (1987). "A Variant of Karmarkar's Linear Programming Algorithm for Problems in Standard Form," *Mathematical Programming* **37**, 81–90.
- Gentleman, M. (1973). "Least Squares Computations by Givens Transformations Without Square Roots," *Journal of the Institute of Mathematics and Its Applications* **12**, 329–336.
- Geoffrion, A.M. (1970). "Elements of Large-Scale Mathematical Programming," *Management Science* **16**, 652–691.
- George, A. and Liu, J. (1981). *Computer Solution of Large Sparse Positive Definite Systems*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- George, A. and Ng, E. (1984). "Symbolic Factorization for Sparse Gaussian Elimination with Partial Pivoting," Technical Report CS-84-43, Department of Computer Science, University of Waterloo, Waterloo, Ontario, Canada.
- de Ghellinck, G. and Vial, J.-Ph. (1986). "A Polynomial Newton Method for Linear Programming," special issue of *Algorithmica* **1**, 425–453.
- Gill, P.E., Golub, G.H., Murray, W. and Saunders, M.A. (1974). "Methods for Modifying Matrix Factorizations," *Mathematics of Computation* **28**, 505–535.
- Gill, P. E., and Murray, W. (1974a). *Safeguarded Steplength Algorithms for Optimization Using Descent Methods*, Report NAC 37, National Physical Laboratory, England.
- Gill, P.E. and Murray, W. (1974b). "Newton Type Methods for Unconstrained and Linearly Constrained Optimization," *Mathematical Programming* **7**, 311–350.
- Gill, P.E. and Murray, W. (1974c). "Quasi-Newton Methods for Linearly Constrained Optimization," in P.E. Gill and W. Murray (eds.), *Numerical Methods for Unconstrained Optimization*, Academic Press, London and New York, 67–92.
- Gill, P.E. and Murray, W. (1974d). *Numerical Methods for Constrained Optimization*, Academic Press, London and New York.
- Gill, P.E. and Murray, W. (1977). "Linearly Constrained Problems Including Linear and Quadratic Programming," in D. Jacobs (ed.), *The State of the Art in Numerical Analysis*, Academic Press, London and New York, 313–363.
- Gill, P.E. and Murray, W. (1978). "Algorithms for the Solution of the Nonlinear Least Squares Problem," *SIAM Journal on Numerical Analysis* **15**, 977–992.

- Gill, P.E. and Murray, W. (1979). *Conjugate-Gradient Methods for Large Scale Nonlinear Optimization*, Technical Report SOL 79-15, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W. and Picken, S.M. (1972). "The Implementation of Two Modified Newton Algorithms for Unconstrained Optimization," Report NAC 11, National Physical Laboratory, England.
- Gill, P.E., Murray, W., Picken, S.M., and Wright, M.H. (1979). "The Design and Structure of a FORTRAN Program Library for Optimization," *ACM Transactions on Mathematical Software* **5**, 259–283.
- Gill, P.E., Murray, W., Ponceleón, D.B., and Saunders, M.A. (1994). "Solving reduced KKT systems in barrier methods for linear programming," in J.H. Watson and D. Griffiths (eds.), *Numerical Analysis 1993*, Pitman Research Notes in Mathematics 303, Longmans Press, 89–104; originally published in 1991 as "Solving Reduced KKT Systems in Barrier Methods for Linear and Quadratic Programming," Technical Report SOL 91-7, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W., Ponceleón, D.B., and Saunders, M.A. (1995). "Primal-dual methods for linear programming," *Mathematical Programming* **70**, 251–277; originally published in 1991, Technical Report SOL 91-3, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W., and Saunders, M.A. (1988). "Interior-Point Methods for Linear Programming: A Challenge to the Simplex Method," Technical Report SOL 88-14, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W., Saunders, M.A., Tomlin, J.A. and Wright, M.H. (1986). "On Projected Newton Barrier Methods for Linear Programming and an Equivalence to Karmarkar's Projective Method," *Mathematical Programming* **36**, 183–209.
- Gill, P.E., Murray, W., Saunders, M.A. and Wright, M.H. (1981a). "A Procedure for Computing Forward Finite-Difference Intervals for Numerical Optimization," Technical Report SOL 81-25, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H. (1981b). "QP-based Methods for Large Scale Nonlinearly Constrained Optimization," in O.L. Mangasarian, R.R. Meyer, and S.M. Robinson (eds.), *Nonlinear Programming 4*, Academic Press, London and New York, 57–98.
- Gill, P.E., Murray, W. Saunders, M.A. and Wright, M.H. (1983). "On the Representation of a Basis for the Null Space," Technical Report SOL 83-19, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W., Saunders, M.A. and Wright, M.H. (1984a). "Sparse Matrix Methods in Optimizations," *SIAM Journal on Scientific and Statistical Computing* **5**, 562–589.
- Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H. (1984b). "Software and its Relationship to Methods," Technical Report SOL 84-10, Department of Operations Research, Stanford University, Stanford, CA.

- Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H. (1984c). "Model Building and Practical Implementation Aspects in Nonlinear Programming," Presented at the NATO Advanced Study Institute on *Computational Mathematical Programming* Bad Windsheim, July 23–August 2, 1984.
- Gill, P.E., Murray, W., Saunders, M.A. and Wright, M.H. (1985). "LUSOL User's Guide," Working Technical Report, Department of Operations Research, Stanford University, Stanford, CA.
- Gill, P.E., Murray, W., Saunders, M.A. and Wright, M.H. (1987). "Maintaining LU Factors of a General Sparse Matrix," *Linear Algebra and Its Applications* **88/99** 239–270.
- Gill, P.E., Murray, W., Saunders, M.A. and Wright, M.H. (1989). "A Practical Anti-Cycling Procedure for Linearly Constrained Optimization," *Mathematical Programming* **45**, 437–474.
- Gill, P.E., Murray, W., and Wright, M.H. (1981). *Practical Optimization*, Academic Press, London and New York.
- Gill, P.E., Murray, W., and Wright, M.H. (1991). *Numerical Linear Algebra and Optimization*, Addison-Wesley Publishing Company, Reading, Massachusetts.
- Gille, P. and Loute, E. (1982). "Updating the LU Gaussian Decomposition for Rank-One Corrections; Application to Linear Programming Basis Partitioning Techniques," *Cahier No. 8201, Séminaire de Mathématiques Appliquées aux Sciences Humaines, Facultés Universitaires Saint-Louis, Brussels, Belgium*.
- Givens, W. (1954). "Numerical Computation of the Characteristic Values of a Real Symmetric Matrix," *Technical Report ORNL-1574*, Oak Ridge National Laboratory, Oak Ridge, Tennessee.
- Glassey, C.R. (1971). "Dynamic LP's for Production Scheduling," *Operations Research* **19**, 45–56.
- Glover, F., Karney, D., and Klingman, D. (1972). "The Augmented Predecessor Index Method for Locating Stepping Stone Paths and Assigning Dual Prices in Distribution Problems," *Transportation Science* **6**, 171–180.
- Glover, F., Karney, D., and Klingman, D. (1973). "A Note on Computational Studies for Solving Transportation Problems," *Proceedings of the ACM Annual Conference*, Atlanta, Georgia, 7–11..
- Glover, F., Karney, D., and Klingman, D. (1974). "Implementation and Computational Comparisons of Primal, Dual, and Primal-Dual Computer Codes for Minimum Cost Flow Network Problems," *Networks*, **4**, 191–212.
- Glover, F., Karney, D., Klingman, D., and Napier, A. (1974). "A Computational Study on Start Procedures, Basis Change Criteria and Solution Algorithms for Transportation Problems," *Management Science* **20**, 793–813.
- Glynn, P.W. and Iglehart, D.L. (1989). "Importance Sampling for Stochastic Simulation," *Management Science* **35**, 1367–1392.
- Goffin, J.L. and Vial, J.-Ph. (1990). "Cutting Plane and Column Generation Techniques with the Projective Algorithm," *Journal of Optimization Theory and Applications* **65**, 409–429.
- Goldberg, A.V. (1993). "Scaling Algorithms for the Shortest Path Problems," in *Proceedings 4th ACM-SIAM Symposium on Discrete Algorithms*, 222–231.

- Goldberg, A.V. and Radzik, T. (1993). "A Heuristic Improvement of the Bellman-Ford Algorithm," *Applied Mathematics Letters* **6**, 3–6.
- Goldfarb, D. and Reid, J.K. (1977). "A Practical Steepest-Edge Simplex Algorithm," *Mathematical Programming* **12**, 361–371.
- Goldman, A.J. (1956). "Resolution and Separation Theorems for Polyhedral Convex Sets," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 41–51.
- Goldman, A.J. and Tucker, A.W. (1956a). "Polyhedral Convex Cones," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 19–39.
- Goldman, A.J. and Tucker, A.W. (1956b). "Theory of Linear Programming," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 53–97.
- Goldstein, L. (1952). "Problem of Contract Awards," in A. Orden and L. Goldstein (eds.), *Symposium on Linear Inequalities and Programming*, Project SCOOP No. 10, Planning Research Division, Director of Management Analysis Service, Comptroller, USAF, Washington, DC, April, 147–154.
- Golub, G.H. and Van Loan, C.F. (1989). *Matrix Computations*, John Hopkins University Press, Baltimore.
- Gonzaga, C.C. (1989). "Conical Projection Algorithms for Linear Programming," *Mathematical Programming* **43**, 151–173.
- Gonzaga, C.C. (1991). "Large Steps Path-Following Methods for Linear Programming, Part II: Potential Reduction Method," *SIAM Journal on Optimization* **1**, 280–292.
- Gonzaga, C.C. (1992). "Path Following Methods for Linear Programming," *SIAM Review* **34**, 167–227.
- Goodman, S.E. and Hedetniemi, S.T. (1985). *Introduction to the Design and Analysis of Algorithms*, McGraw-Hill, San Francisco, California.
- Gordan, P. (1873). "Über die Auflösung linearer Gleichungen mit reellen Koeffizienten," *Mathematical Annals* **6**, 23–28.
- Gould, N. (1991). News Clip in *SiamNews*, January, 9.
- Grandzol, J.R. and Traaen, T. (1995). "Using Mathematical Programming to Help Supervisors Balance Workloads," *Interfaces* **25**:4, 92–103.
- Graves, G.W. (1965). "A Complete Constructive Algorithm for the General Mixed Linear Programming Problem," *Naval Research Logistics Quarterly*, **12**, 1–34.
- Grcar, J.F. (1990). "Matrix Stretching for Linear Equations," Report SAND90-8723, Sandia National Laboratories, Albuquerque, NM.
- Greenberg, H.J. (ed.), (1978a). *Design and Implementation of Optimization Software*, Sijthoff and Noordhoff, Alpen aan den Rijn.
- Greenberg, H.J. (1978b). "A Tutorial on Matricial Packing," in H.J. Greenberg (ed.), *Design and Implementation of Optimization Software*, Sijthoff and Noordhoff, Alpen aan den Rijn, 109–142.

- Greenberg, H.J. (1978c). "Pivot Selection Tactics," in H.J. Greenberg (ed.), *Design and Implementation of Optimization Software*, Sijthoff and Noordhoff, Alphen aan den Rijn, 143–178.
- Greenberg, H.J. and Kalan, J. (1975). "An Exact Update for Harris' TREAD," *Mathematical Programming Study* **4**, 26–29.
- Greenstadt, J.L. (1967). "On the Relative Efficiencies of Gradient Methods," *Mathematics of Computation* **24**, 145–166.
- Grimes, R.G. and Lewis, J.G. (1981). "Condition Number Estimation for Sparse Matrices," *SIAM Journal on Scientific and Statistical Computing* **2**, 384–388.
- Grötschel, M., Lovász, L., and Schrijver, A. (1988). *Geometric Algorithms and Combinatorial Optimization*, Springer-Verlag, Berlin and New York.
- Grunbaum, B. (1967). *Convex Polytopes*, John Wiley and Sons, New York.
- Gunderman, R.E. (1973). "A Glimpse into Program Maintenance," *Datamation* **19** 99–101.

H

- Hadley, G. (1972). *Linear Programming*, Addison-Wesley Publishing Company, Reading, Massachusetts.
- Hall, P. (1985). "Rates of Convergence in the Central-Limit Theorem," *Bulletin of the London Mathematical Society* **17**, 151–156; previously published in 1982, *Research Notes in Mathematics*, **62**.
- Hall, L.A. and Vanderbei, R.J. (1993). "Two-Thirds is Sharp for Affine Scaling," *Operations Research Letters* **13**, 197–201.
- Hammersly, J.M. and Handscomb, D.C. (1964). *Monte Carlo Methods*, Matheun, London.
- Harris, P.M.J. (1975). "Pivot Selection Codes in the Devex LP Code," *Mathematical Programming Study* **4**, 30–57.
- Hellerman, E. and Rarick, D. (1971). "Reinversion in the Preassigned Pivot Procedure," *Mathematical Programming* **1**, 195–216.
- Hellerman, E. and Rarick, D. (1972). "The Partitioned Preassigned Pivot Procedure," in D.J. Rose and R.A. Willoughby (eds.), *Sparse Matrices and their Applications*, Plenum Press, New York, 67–76.
- Henderson, A. and Schlaifer, R. (1954). "Mathematical Programming," *Harvard Business Review*, **32**, May-June, 73–100.
- Hertog, D.D. and Roos, C. (1991). "A Survey of Search Directions in Interior Point Methods for Linear Programming," *Mathematical Programming* **52**, 481–509.
- Hestenes, M.R. and Stiefel, E. (1952). "Methods of Conjugate Gradients for Solving Linear Systems," *Journal of Research of the National Bureau of Standards* **49**, 409–436.
- Higham, N.J. (1985). "Nearness Problems in Numerical Algebra," Ph.D. thesis, University of Manchester, Manchester, England.
- Higham, N.J. (1987). "A Survey of Condition Number Estimation for Triangular Matrices," *SIAM Review* **29**, 575–596.

- Higham, N.J. and Higham, D.J. (1989). "Large Growth Factors in Gaussian Elimination with Pivoting," *SIAM Journal on Matrix Analysis and Applications* **10**, 155–164.
- Higle, J.L. and Sen, S. (1991). "Stochastic Decomposition: An Algorithm for Two Stage Linear Programs with Recourse," *Mathematics of Operations Research* **16/3**, 650–669.
- Hillier, F.S., and Lieberman, G.J. (1995). *Introduction to Operations Research*, McGraw-Hill, San Francisco, California.
- Hitchcock, F.L. (1941). "The Distribution of a Product from Several Sources to Numerous Localities," *Journal of Mathematical Physics* **20**, 224–230.
- Hirsch, W.M. (1957). "Hirsch Conjecture," verbal communication to Dantzig.
- Ho, J.K. (1974). "Nested Decomposition for Large-Scale Programs with the Staircase Structure," Technical Report SOL 74-4, Department of Operations Research, Stanford University, Stanford, CA.
- Ho, J.K. (1984). "Convergence Behaviour of Decomposition Algorithms for Linear Programs," *Operations Research Letters* **3**, 91–94.
- Ho, J.K. and Louie, E. (1981). "An Advanced Implementation of the Dantzig-Wolfe Decomposition Algorithm for Linear Programming," *Mathematical Programming* **20**, 303–326.
- Ho, J.K. and Manne, A. (1974). "Nested Decomposition for Dynamic Models," *Mathematical Programming* **6**, 121–140.
- Hoffman, A.J. (1953). "Cycling in the Simplex Algorithm," *National Bureau of Standards Report No. 2974*.
- Hoffman, A.J. and Hirsch, W. (1961). "Extreme Varieties, Concave Functions and the Fixed Charge Problem," *Communications in Pure and Applied Mathematics* **14**, 355–369.
- Hoffman, A.J., Mannos, M., Sokolowsky, D., and Wiegmann, N. (1953). "Computational Experience in Solving Linear Programs," *Journal of the Society for Industrial and Applied Mathematics*, **1**, 17–33.
- Hockney, R.W. and Jesshope, C.R. (1988). *Parallel Computers 2*, Adam Hilger, Bristol and Philadelphia.
- Holmes, D. (1994). "A Collection of Stochastic Programming Problems," Technical Report 94-11, Department of Industrial Engineering and Operations Research, University of Michigan, Ann Arbor, Michigan.
- Hooker, J.N. (1986). "Karmarkar's Linear Programming Algorithm," *Interfaces* **16:4**, 75–90.
- Householder, A.S. (1964). *The Theory of Matrices in Numerical Analysis*, Dover Publications, New York.
- Huang, C.C., Ziemba, W.T., and Ben-Tal, A. (1977). "Bounds on the Expectation of a Convex Function with a Random Variable with Applications to Stochastic Programming," *Operations Research* **25**, 315–325.
- Huard, P. (1970). "A Method of Centers by Upper-Bounding Functions with Applications," in J.B. Rosen, O.L. Mangasarian, and K. Ritter (eds.), *Nonlinear Programming: Proceedings of a Symposium held at the University of Wisconsin, Madison, May 1970*, Academic Press, London and New York, 1–30.

I

- Ignizio, J.P. (1976). *Goal Programming and Extensions*, Heath, Lexington, Massachusetts.
- Infanger, G. (1991). "Monte Carlo (Importance) Sampling within a Benders Decomposition Algorithm for Stochastic Linear Programs Extended Version: Including Results of Large-Scale Problems," Technical Report SOL 91-6, Department of Operations Research, Stanford University, Stanford, CA; shorter version in *Annals of Operations Research* **39**, 69–95.
- Infanger, G. (1994). *Planning Under Uncertainty: Solving Large-Scale Stochastic Linear Programs*, Boyd and Fraser Publishing Company, Massachusetts.
- International Business Machines Corporation (1978). *Mathematical Programming System Extended and Generalized Upper Bounding (GUB)*, IBM Manual SH20-0968-1, White Plains, New York.
- Isaacson, E. and Keller, H.B. (1966). *Analysis of Numerical Methods*, John Wiley and Sons, New York.

J

- Jack, C., Kai, S., and Shulman, A. (1992). "NETCAP—An Interactive Optimization System for GTE Telephone Network Planning," *Interfaces* **22**:1, 72–89.
- Jackson, J.R. (1957). "Simulation Research on Job Shop Production," *Naval Research Logistics Quarterly*, **1**, 287.
- Jacobs, W.W. (1954). "The Caterer Problem," *Naval Research Logistics Quarterly*, **1**, 154–165.
- Jankowski, M. and Wozniakowski, M. (1977). "Iterative Refinement Implies Numerical Stability," *BIT* **17**, 303–311.
- Jewell, W.S. (1958). "Optimal Flows Through Networks," Interim Technical Report No. 8, on Fundamental Investigations in Methods of Operations Research, Massachusetts Institute of Technology, Cambridge, Massachusetts.
- Johnson, D.B. (1977). "Efficient Algorithms for Shortest Paths in Sparse Networks," *Journal of the Association for Computing Machinery* **24**, 1–13.
- Jones, M.T. and Plassmann, P.E. (1995). "An Improved Incomplete Cholesky Factorization," *ACM Transactions on Mathematical Software* **21**, 5–17.
- Judin, D.B., and Nemirovskii, A.S. (1976a). "Estimation of the Informational Complexity of Mathematical Programming Problems," *Ekonomika i Matematicheskie Metody*, **12**, 128–142. (English translation in *Matekon: Translation of Russian and East European Mathematical Economics*, **13**, 3–25, 1977.)
- Judin, D.B., and Nemirovskii, A.S. (1976b). "Informational Complexity and Effective Methods for the Solution of Convex Extremal Problems," *Ekonomika i Matematicheskie Metody*, **12**, 357–369. (English translation in *Matekon: Translation of Russian and East European Mathematical Economics*, **13**, 25–45, 1977.)
- Judin, D.B., and Nemirovskii, A.S. (1976c). "Informational Complexity of Strict Convex Programming," *Ekonomika i Matematicheskie Metody*, **12**, 550–559.

K

- Kahan, W. (1966). "Numerical Linear Algebra," *Canadian Mathematical Bulletin* **9**, 757–801.
- Kahaner, D., Moler, C.B., and Nash, S. (1988). *Numerical Methods and Software*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Kalaba, R.E. and Juncosa, M.L. (1956). "Optimal Design and Utilization of Communications Networks," *Management Science* **3**, 33–44.
- Kalan, J.E. (1971). "Aspects of Large-Scale In-Core Linear Programming," *Proceedings of the ACM Annual Conference*, 304–313.
- Kalan, J.E. (1976). "Machine Inspired Enhancements of the Simplex Algorithm," Technical Report CS75001-R, Computer Science Department, Virginia Polytechnical University, Blacksburg, Virginia.
- Kall, P. (1974). "Approximations to Stochastic Programs with Complete Fixed Recourse," *Numerische Mathematik* **22**, 333–339.
- Kall, P. (1979). "Computational Methods for Two Stage Stochastic Linear Programming Problems," *Zeitschrift für angewandte Mathematik und Physik* **30**, 261–271.
- Kall, P. and Stoyan, D. (1982). "Solving Stochastic Programming Problems with Recourse Including Error Bounds," *Mathematische Operationsforschung und Statistik, Series Optimization* **13**, 431–447.
- Kall, P. and Wallace, S.W. (1994). *Stochastic Programming*, John Wiley and Sons, New York.
- Kantorovich, L.V. (1939). "Mathematical Methods in the Organization and Planning of Production," Publication House of the Leningrad State University; translated in *Management Science* **6**, 1960, 366–422.
- Kantorovich, L. V. (1942). "On the Translocation of Masses," *Comptes Rendus de l'Académie des Sciences U.S.S.R.* **37**, 199–201.
- Kantorovich, L.V. and Gavurin, M.K. (1949). "The Application of Mathematical Methods to Freight Flow Analysis," (translation), in: Collection of Problems of Raising the Efficiency of Transport Performance, Akademiia Nauk SSSR, Moscow-Leningrad, 1949, 110–138.
- Karmarkar N. (1984). "A New Polynomial-Time Algorithm for Linear Programming," *Combinatorica* **4**, 373–395.
- Katz, P., Sadrian, A., and Tendick, P. (1994). "Telephone Companies Analyze Price Quotations with Bellcore's PDSS Software," *Interfaces* **24:1**, 50–63.
- Kaul, R.N. (1965). "An Extension of Generalized Upper Bounding Techniques for Linear Programming," Technical Report ORC 65-27, Operations Research Center, University of California, Berkeley.
- Kennedy, W.J., Jr. and Gentle, J.E. (1980). *Statistical Computing*, Marcel Dekker, Inc. New York and Basel.
- Kennington, J.L. and Wang, Z. (1991). "An Empirical Analysis of the Dense Assignment Problem: Sequential and Parallel Implementations," *ORSA Journal on Computing* **3**, 299–306.

- Kernighan, B.W. and Plauser, P.J. (1970). *The Elements of Programming Style*, McGraw-Hill, New York.
- Khachian, L.G. (1979). "A Polynomial Algorithm for Linear Programming," *Doklady Akademii Nauk USSR* **244**, 1093–1096. [English translation: *Soviet Mathematics Doklady* **20**, 191–194].
- Khan, M.R. and Lewis, D.A. (1987). "A Network Model for Nurse Staff Scheduling," *Zeitschrift für Operations Research* **31**, B161–B171.
- Kotiah, T.C.T. and Steinberg, D.I. (1977). "Occurrences in Cycling and Other Phenomena Arising in a Class of Linear Programming Models," *Communications of the Association for Computing Machinery* **20**, 107–112.
- Kotiah, T.C.T. and Steinberg, D.I. (1978). "On the Possibility of Cycling with the Simplex Method," *Operations Research* **26**, 374–375.
- Kim, K. and Nazareth, J.L. (1994). "A Primal Null-Space Affine Scaling Method," *ACM Transactions on Mathematical Software* **20**, 373–392.
- Klee, V.L. and Minty, G.J. (1972). "How Good is the Simplex Algorithm?" in O. Shisha (ed.), *Inequalities III*, Academic Press, London and New York, 159–175.
- Klee, V.L. and Walkup, D.W. (1967). "The d -Step Conjecture for Polyhedra of Dimension $d < 6$," *Acta Mathematica* **117**, 53–78.
- Klingman, D., Phillips, N., Steiger, D., Wirth, R., and Young, W. (1986). "The Challenges and Success Factors in Implementing an Integrated Products Planning System for Citgo," *Interfaces* **16**:, 1–19.
- Klingman, D., Phillips, N., Steiger, D., and Young, W. (1987). "The Successful Deployment of Management Science throughout Citgo Petroleum Corporation," *Interfaces* **17**:, 4–25.
- Klotz, E.S. (1988). "Dynamic Pricing Criteria in Linear Programming," Technical Report SOL 88-15, Department of Operations Research, Stanford University, Stanford, CA.
- Kohler, D.A. (1973). "Translation of a Report by Fourier on His Work on Linear Inequalities," *OPSEARCH* **10**, 38–42.
- Kojima, M., Megiddo, N., and Mizuno, S. (1993). "A Primal-Dual Infeasible Interior Point Algorithm for Linear Programming," *Mathematical Programming* **61**, 263–280.
- Kojima, M., Mizuno, S., and Yoshise, A. (1989a). "A Primal-Dual Interior Point Method for Linear Programming," in N. Megiddo (ed.), *Progress in Mathematical Programming: Interior Point and Related Methods* Springer-Verlag, Berlin and New York, 29–47.
- Kojima, M., Mizuno, S., and Yoshise, A. (1989b). "A Polynomial-Time Algorithm for a Class of Linear Complementarity Problems," *Mathematical Programming* **44**, 1–26.
- Koopmans, T.C. (1947). "Optimum Utilization of the Transportation System," *Proceedings of the International Statistical Conference*, Washington, DC. Volume 5 was reprinted as a supplement to *Econometrica*, **17**, 1949.
- Koopmans, T.C. (ed.), (1951). *Activity Analysis of Production and Allocation*, John-Wiley and Sons, New York.

- Koopmans, T.C. and Reiter, S. (1951). "A Model of Transportation," in T.C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, July–October 1949, Cowles Commission Monograph 13, Proceedings of Linear Programming Conference, June 20–24, 1949, John Wiley and Sons, New York, 33–97.
- Kortanek, K.O. and Zhu, J. (1988). "New Purification Algorithms for Linear Programming," *Naval Research Logistics Quarterly*, **35**, 571–583.
- Knuth, D.E. (1973a). *Fundamental Algorithms, The Art of Computer Programming, Vol. 1*, Addison-Wesley Publishing Company, Reading, Massachusetts; originally published in 1968.
- Knuth, D.E. (1973b). *Sorting and Searching, The Art of Computer Programming, Vol. 3*, Addison-Wesley Publishing Company, Reading, Massachusetts.
- Knuth, D.E. (1981). *Seminumerical Algorithms, The Art of Computer Programming, Vol. 2*, Addison-Wesley Publishing Company, Reading, Massachusetts; originally published in 1969.
- Kranich, E. (1991). "Interior Point Methods for Mathematical Programming: A Bibliography Discussion Paper 171, Institute of Economy and Operations Research, Fern Universität Hagen, P.O. Box 940, D-5800 Hagen 1, West Germany.
- Krishna, A.S. (1989). Unpublished Manuscript.
- Krishna, A.S. (1993). "Enhanced Algorithms for Stochastic Programming," Technical Report SOL 93-8, Department of Operations Research, Stanford University, Stanford, CA.
- Kruskal, J.B., Jr. (1956). "On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem," *Proceedings of the American Mathematical Society* **7**, 48–50.
- Kuhn, H.W. (1955). "The Hungarian Method for the Assignment Problem," *Naval Research Logistics Quarterly*, **2**, 1 and 2, 83–97.
- Kuhn, H.W. (1956). "Solvability and Consistency for Linear Equations and Inequalities," *The American Mathematical Monthly* **63**, 217–232.
- Kuhn, H.W. and Tucker, A.W. (1950). "Nonlinear Programming," in J. Neyman (ed.), *Proceedings 2nd Berkeley Symposium on Mathematical Statistics and Probability, 1950*, University of California Press, Berkeley, California, 481–492.
- Kuhn, H.W. and Tucker, A.W. (1958). "John von Neumann's Work in the Theory of Games and Mathematical Economics," *Bulletin of the American Mathematical Society* **64**, 100–122.
- Kusy, M.I. and Ziemba, W.T. (1986). "A Bank Asset and Liability Management Model," *Operations Research* **34**, 356–378.

L

- Laderman, J. (1947).
- Lansdowne, Z.F. (1979). "Survey of Research on Model Simplification," Technical Report SOL 79-26, Department of Operations Research, Stanford University, Stanford, CA.
- Lasdon, L.S. (1970). *Optimization Theory for Large Systems*, Macmillan, London.

- Lawler, E.L. (1976). *Combinatorial Optimization: Networks and Matroids*, Holt, Rinehart and Winston, New York.
- Lawson, C.L. and Hanson, R.J. (1974). *Solving Least Squares Problems*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Lawson, C.L., Hanson R.J., Kincaird, D.R., and Krogh, F.T. (1979a). "Basic Linear Algebra Subprograms for Fortran Usage," *ACM Transactions on Mathematical Software* **5**, 308–323.
- Lawson, C.L., Hanson R.J., Kincaird, D.R., and Krogh, F.T. (1979b). "Algorithm 539, Basic Linear Algebra Subprograms for Fortran Usage," *ACM Transactions on Mathematical Software* **5**, 324–325.
- Lee, S.M. (1972). *Goal Programming for Decision Analysis*, Auerbach, Philadelphia.
- Leichner, S.A., Dantzig, G.B., and Davis, J.W. (1993). "A Strictly Improving Linear Programming Phase I Algorithm," *Annals of Operations Research* **14**, 409–430.
- Lemke, C.E. (1954). "The Dual Method of Solving the Linear Programming Problem," *Naval Research Logistics Quarterly*, **1**, 36–47.
- Lenstra, J.K., Rinnooy Kan, A.H.G., and Schrijver, A. (eds.) (1991). *History of Mathematical Programming*, Elsevier Science Publishing Company, New York.
- Levin, A.Y. (1965). "On an Algorithm for the Minimization of Convex Functions," *Soviet Mathematics Doklady* **6**, 286–290.
- Lewis, R.E. (1955). "Top Management Looks at Linear Programming and Inventory Management," *Proceedings of the Linear Programming and Inventory Management Seminar*, Methods Engineering Council, Pittsburgh, B-1 to B-8.
- Litty, C.J. (1994). "Optimal Lease Structuring at GE Capital," *Interfaces* **24**:3, 34–45.
- Liu, J. (1985). "Modification of the Minimum-Degree Algorithm by Multiple Elimination," *ACM Transactions on Mathematical Software* **11**, 141–153.
- Luenberger, D.G. (1989). *Introduction to Linear and Non-Linear Programming*, Addison-Wesley Publishing Company, Reading, Massachusetts.
- Lustig, I.J. (1987). "An Analysis of an Available Set of Linear Programming Test Problems," Technical Report SOL 87-11, Department of Operations Research, Stanford University, Stanford, CA.
- Lustig, I.J. (1991). "Feasibility Issues in an Interior Point Method for Linear Programming," *Mathematical Programming* **49**, 145–162.
- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1990). "The Primal-Dual Interior Point Method on the Cray Supercomputer, in T.F. Coleman and Y. Li (eds.), *Large-Scale Numerical Optimization*, papers from the workshop held at Cornell University, Ithaca, New York, October 1989, vol. 46 of SIAM Proceedings in Applied Mathematics, Society of Industrial and Applied Mathematics, Philadelphia, PA, 70–80.
- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1991a). "Computational Experience with a Primal-Dual Interior Point for Linear Programming," *Linear Algebra and Its Applications* **152**, 191–222.
- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1991b). "Interior Method vs Simplex Method: Beyond NETLIB," *Mathematical Programming Society COAL Newsletter* **19**, 41–44.

- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1992a). "Computational Experience with a Globally Convergent Primal-Dual Predictor-Corrector Algorithm for Linear Programming," Technical Report SOR 92-10, School of Engineering and Applied Science, Department of Civil Engineering and Operations Research, Princeton University, Princeton, New Jersey.
- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1992b). "The Interaction of Algorithms and Architectures for Interior Point Methods, in P.M. Pardalos (ed.), *Advances in Optimization and Parallel Computing, North-Holland*, the Netherlands, 190–205.
- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1992c). "On Implementing Mehrotra's Predictor-Corrector Interior Point Method for Linear Programming," *SIAM Journal on Optimization* **2**, 435–449.
- Lustig, I.J., Marsten, R.E., and Shanno, D.F. (1994). "Interior Point Methods for Linear Programming: Computational State of the Art," *ORSA Journal on Computing* **6**, 1–14.
- Lustig, I.J., Mulvey, J.M., and Carpenter, T.J. (1991). "Formulating Two-Stage Stochastic Programs for Interior Point Methods," *Operations Research* **39**, 757–770; previously published in 1990 as Technical Report SOR 89-16, School of Engineering and Applied Science, Department of Civil Engineering and Operations Research, Princeton University, Princeton, New Jersey.

M

- Madansky, A. (1959). "Bounds on the Expectation of a Convex Function of a Multivariate Random Variable," *Annals of Mathematical Statistics* **30**, 743–746.
- Makuch, W.M., Dodge, J.L., Ecker, J.G., Granfors, D.C., and Hahn, G.J. (1992). "Managing Consumer Credit Delinquency in the US Economy: A Multi-billion Dollar Management Science Application," *Interfaces* **22**:1, 90–109.
- Malhotra, V.M., Kumar, M.P., and Maheshwari, S.N. (1978). "An $O(|V|^3)$ Algorithm for Finding Maximum Flows in Networks," *Information Processing Letters*, **7**, 277–278.
- Manley, B.R. and Threadgill, J.A. (1991). "LP Used for Valuation and Planning of New Zealand Plantation Forests," *Interfaces* **21**:6, 66–79.
- Manne, A.S. (1956). *Scheduling of Petroleum Refinery Operations*, Harvard University Press, Cambridge, Massachusetts.
- Manne, A.S. (1974). "Waiting for the Breeder," in *Review of Economic Studies Symposium*, 47–65.
- Marcus, M. (1960). "Some Properties of Doubly Stochastic Matrices," *The American Mathematical Monthly* **67**, 215–221.
- Markowitz, H.M. (1952). "Portfolio Selection," *The Journal of Finance* **7**, 77–91.
- Markowitz, H.M. (1957). "The Elimination Form of the Inverse and its Applications to Linear Programming," *Management Science* **3**, 255–269.
- Markowitz, H.M. (1959). *Portfolio Selection: Efficient Diversification of Investments*, John Wiley and Sons, New York.

- Maros, I.G. (1986). "A General Phase-I Method in Linear Programming," *European Journal of Operations Research* **23**, 64–77.
- Marshall, K.T. and Suurballe, J.W. (1969). "A Note on Cycling in the Simplex Method," *Naval Research Logistics Quarterly*, **16**, 121–137.
- Marsten, R.E. (1981). "The Design of the XMP Linear Programming Library," *ACM Transactions on Mathematical Software* **7**, 481–497.
- Marsten, R.E., Saltzman, M.J., Shanno, D.F., Ballinton, J.F., and Pierce, G.S. (1989). "Implementation of a Dual Affine Interior Point Algorithm for Linear Programming," *ORSA Journal on Computing* **1**, 287–297.
- Mascarenhas, W.F. (1993). "The Affine Scaling Algorithm Fails for $\lambda = 0.999$," Technical Report, Universidade Estadual de Campinas, Campinas S. P., Brazil.
- Massé, P. and Gibrat, R. (1957). "Applications of Linear Programming to Investments in the Electric Power Industry," *Management Science* **3**, 149–166.
- Maynard, H.B. (1955). "Putting New Management Tools to Work," *Proceedings of the Linear Programming and Inventory Management Seminar*, Methods Engineering Council, Pittsburgh.
- McCarthy, C. and Strang, G. (1973). "Optimal Conditioning of Matrices," *SIAM Journal on Numerical Analysis* **10**, 370–388.
- McCormick, S.T. (1983). "Optimal Approximation of Sparse Hessians and its Equivalence to a Graph Coloring Problem," *Mathematical Programming* **26**, 153–171.
- McDiarmid, C. (1990). "On the Improvement per Iteration in Karmarkar's Algorithm for Linear Programming," *Mathematical Programming* **46**, 299–320.
- McGowan, C.L. and Kelly, J.R. (1975). *Top-Down Structured Programming Techniques*, Petrocelli/Charter, New York.
- McKeeman, W.M. (1962). "Crout with Equilibration and Iteration," *Communications of the Association for Computing Machinery* **5**, 553–555.
- McShane, K.A., Monma, C.L., and Shanno, D.F. (1989). "An Implementation of a Primal-Dual Interior Point Method for Linear Programming," *ORSA Journal on Computing* **1**, 70–83.
- Megiddo, N. (1986). "Introduction: New Approaches to Linear Programming," special issue of *Algorithmica* **1**, 387–394, Springer-Verlag, Berlin and New York.
- Megiddo, N. (1988). "Pathways to the Optimal Set in Linear Programming," in N. Megiddo (ed.), *Progress in Mathematical Programming: Interior Point and Related Methods* Springer-Verlag, Berlin and New York, 131–158.
- Megiddo, N. (1991). "On Finding Primal- and Dual-Optimal Bases," *ORSA Journal on Computing* **3**, 63–65.
- Mehrotra, S. (1992a). "On the Implementation of a Primal-Dual Interior Point Method," *SIAM Journal on Optimization* **2**, 575–601.
- Mehrotra, S. (1992b). "Implementation of Affine Scaling Methods: Approximate Solutions of Systems of Linear Equations Using Preconditioned Conjugate Gradient Methods," *ORSA Journal on Computing* **4**, 103–118.
- Mehrotra, S. (1993). "Quadratic Convergence in a Primal-Dual Method," *Mathematics of Operations Research* **18**, 741–751.

- Meijerink, J. and van der Vorst, H.A. (1981). "Guidelines for the Usage of Incomplete Decomposition in Solving Sets of Linear Equations as They Occur in Practical Problems," *Journal of Computational Physics* **44**, 134–155.
- Millham, C.B. (1976). "Fast Feasibility Methods for Linear Programming," *OPSEARCH* **13**, 198–204.
- Minkowski, H. (1896). *Geometrie der Zahlen*, B.G. Teubner, Leipzig and Berlin, 1910. First printing, 1896; also reprinted by Chelsea Publishing Company, New York, 1953.
- Mitchell, J.E. and Borchers, B. (1992). "A Primal-Dual Interior Point Cutting Plane Method for the Linear Ordering Problem," *COAL Bulletin* **21**, 13–18.
- Mitchell, J.E. and Todd, M.J. (1992). "Solving Combinatorial Optimization Problems Using Karmarkar's Algorithm," *Mathematical Programming* **56**, 245–284.
- Mizuno, S. (1992). "Polynomiality of the Kojima-Megiddo-Mizuno Infeasible Interior Point Algorithm for Linear Programming," Technical Report 1006, School of Operations Research and Industrial College of Engineering, Cornell University, Ithaca, New York 14853.
- Mizuno, S., Kojima, M., and Todd, M. (1995). "Infeasible-Interior-Point-Primal-Dual Potential-Reduction Algorithms for Linear Programming," *SIAM Journal on Optimization* **5**, 52–67.
- Monma, C.L. and Morton, A.J. (1987). "Computational Experiments with a Dual Affine Variant of Karmarkar's Method for Linear Programming," *Operations Research Letters* **6**, 261–267.
- Monteiro, R.C. and Adler, I. (1989a). "Interior Path Following Primal-Dual Algorithms—Part I: Linear Programming," *Mathematical Programming* **44**, 27–42.
- Monteiro, R.C. and Adler, I. (1989b). "Interior Path Following Primal-Dual Algorithms—Part II: Convex Quadratic Programming," *Mathematical Programming* **44**, 43–66.
- Monteiro, R.C., Tsuchiya, T., and Wang, Y. (1993). "A Simplified Global Convergence Proof of the Affine Scaling Algorithm," *Annals of Operations Research* **47**, 443–482.
- Mood, A.M., Graybill, F.A., and Boes, D.C. (1974). *Introduction to the Theory of Statistics*, McGraw-Hill, New York.
- Mooney, J.W. (1975). "Organized Program Maintenance," *Datamation* **21**, 63–64.
- Moore, E.F. (1959). "The Shortest Path Through a Maze," in *Proceedings of the International Symposium on the Theory of Switching*, Harvard University Press, Cambridge, Massachusetts, 285–292.
- More, J.J. and Sorenson, D.C. (1981). "Computing a Trust Region step," Report ANL-81-83, Argonne National Laboratory, Argonne, Illinois.
- Morin, G. (1955). "More Effective Production Planning with Linear Programming," Paper F from *Proceedings of the Linear Programming and Inventory Management Seminar*, Methods Engineering Council, Pittsburgh.
- Morris, W.T. (1967). "On the Art of Modeling," *Management Science* **13**, 707–717.
- Morton, D.P. (1993). "Algorithmic Advances in Multi-Stage Stochastic Programming," Ph.D. thesis, Department of Operations Research, Stanford University, Stanford, CA.

- Morton, D.P. (1995). "Stochastic Network Interdiction," working paper presented at a Workshop in Stochastic Programming at University of California, Davis, March, 1995.
- Motzkin, T.S. (1936). "Beiträge zur Theorie der linearen Ungleichungen," doctoral thesis, University of Zurich.
- Muir, T. (1960). *Determinants*, Dover Publications, New York.
- Mulvey, J.M. (1978). "Pivot Strategies for Primal-Simplex Network Codes," *Journal of the Association for Computing Machinery* **25**, 266–270.
- Munksgaard, N. (1980). "Solving Sparse Symmetric Sets of Linear Equations by Pre-conditioned Conjugate Gradient," *ACM Transactions on Mathematical Software* **6**, 206–219.
- Murray, W. (1976). "Constrained Optimization" in L.C.W. Dixon (ed.), *Optimization in Action*, Academic Press, London and New York, 217–251.
- Murray, W. and Wright, M.H. (1978). *Projected Lagrangian Methods Based on Trajectories of Penalty and Barrier Functions*, Technical Report SOL 78-23, Department of Operations Research, Stanford University, Stanford, CA.
- Murtagh, B.A. and Saunders, M.A. (1978). "Large-Scale Linearly Constrained Optimization," *Mathematical Programming* **14**, 41–72.
- Murtagh, B.A. and Saunders, M.A. (1995). "MINOS 5.4 User's Guide," Technical Report SOL 83-20R, Department of Operations Research, Stanford University, Stanford, CA.
- Murty, K.G. (1980). "Computational Complexity of Parametric Linear Programming," *Mathematical Programming* **19**, 213–219.
- Murty, K.G. (1983). *Linear Programming*, John Wiley and Sons, New York.
- Musalem, J. (1980). "Converting Linear Models to Network Models," Ph.D. thesis, University of California, Los Angeles, California.
- Myers, D.C. and Shih, W. (1988). "A Constraint Selection Technique for a Class of Linear Programs," *Operations Research Letters* **7**, 191–194.

N

- Nazareth, J.L. (1984). "Numerical Behavior of LP Algorithms Based Upon the Decomposition Principle," *Linear Algebra and Its Applications* **57**, 181–189.
- Nazareth, J.L. (1986). "Implementation Aids for the Optimization Algorithms that Solve Sequences of Linear Programs," *ACM Transactions on Mathematical Software* **12**, 307–323.
- Nazareth, J.L. (1987). *Computer Solution of Linear Programs*, Monographs on Numerical Analysis, Oxford University Press, Oxford and New York.
- Nering, E.D. and Tucker, A.W. (1993). *Linear Programs and Related Problems*, Academic Press, London and New York.
- Ng, E. (1988). "On the Solution of Sparse Linear Least-Squares Problems," Presentation at Stanford, Mathematical Science Section, Oak Ridge National Laboratory, Oak Ridge, Tennessee.
- Nishiya, T. (1983). "A Basis Factorization Method for Multi-Stage Linear Programming with an Application to Optimal Operation of an Energy Plant," Draft Report.

O

- Oettli, W., and Prager, W. (1964). "Compatibility of Approximate Solutions of Linear Equations With Given Error Bounds For Coefficients and Right-Hand Sides," *Numerische Mathematik* **6**, 405–409.
- O'Leary, D.P. (1980). "Estimating Matrix Condition Numbers," *SIAM Journal on Scientific and Statistical Computing* **1**, 205–209.
- Orchard-Hays, W. (1954). *A Composite Simplex Algorithm—II, RM-1275*, The RAND Corporation, May.
- Orchard-Hays, W. (1956). "Evolution of Computer Codes for Linear Programming," The RAND Corporation, Paper P-810, (March 14), 22–24.
- Orchard-Hays, W. (1968). *Advanced Linear-Programming Computing Techniques*, McGraw-Hill, New York.
- Orchard-Hays, W. (1978). "History of Mathematical Programming Systems," in H.J. Greenberg (ed.), *Design and Implementation of Optimization Software*, Sijthoff and Noordhoff, Alphen aan den Rijn, 1–26.
- Orchard-Hays, W., Cutler, L., and Judd, H. (1956). "Manual for the RAND IBM Code for Linear Programming on the 704," The RAND Corporation, Paper P-842, (May 16), 24–26.
- Orden, A. (1956). "The Transshipment Problem," *Management Science* **2**, 276–285.
- Orden, A. (1993). "LP from the 40s to the 90s," *Interfaces* **23:5**, 2–12.
- Orlin, J.B. and Ahuja, R.K. (1992). "New Scaling Algorithms for the Assignment and Minimum Cycle Mean Problems," *Mathematical Programming* **54**, 41–56.
- Ortega, J.M. (1973). *Numerical Analysis: A Second Course*, Academic Press, London and New York.
- Ortega, J.M. (1988). "The *ijk* Forms of Factorization Methods I: Vector Computers," *Parallel Computing* **7**, 135–147.
- Ortega, J.M. and Rheinboldt, W.C. (1970). *Iterative Solutions of Nonlinear Equations in Several Variables*, Academic Press, London and New York.
- Osterby, O. and Zlatev Z. (1983). "Direct Methods for Sparse Matrices," *Lecture Notes in Computer Science* **157**, Springer-Verlag, Berlin and New York.

P

- Pan, V. (1984). "How Can We Speed Up Matrix Multiplication," *SIAM Review* **26**, 393–416.
- Papadimitriou, C.H. and Steiglitz, K. (1982). *Combinatorial Optimization, Algorithms and Complexity*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Pape, U. (1974). "Implementation and Efficiency of Moore Algorithms for the Shortest Root Problem," *Mathematical Programming* **7**, 212–222.
- Parlett, B.N., and Wang, Y. (1975). "The Influence of the Compiler on the Cost of Mathematical Software—in Particular on the Cost of Triangular Factorization," *ACM Transactions on Mathematical Software* **1**, 35–46.

- Paull, A. E. and Walter, J.R. (1955). "The Trim Problem: An Application of Linear Programming to the Manufacture of Newsprint Paper," *Econometrica*, **23**, 336 (abstract).
- Pereira, M.V. and Pinto, L.M.V.G. (1989). "Stochastic Dual Dynamic Programming," Technical Note, DEE-PUC/RJ-Catholic University of Rio de Janeiro, Caixa Postal 38063 Gávea, Rio de Janeiro, RJ CEP, Brazil.
- Pereira, M.V., Pinto, L.M.V.G., Oliveira, G.C., and Cunha, S.H.F. (1989). "A Technique for Solving LP-Problems with Stochastic Right-Hand Sides," CEPEL, Centro del Pesquisas de Energia Electria, Rio de Janeiro, Brazil.
- Perold, A. F. (1984). "Large Scale Portfolio Optimizations," *Management Science* **30**, 1143-1160.
- Peters, G. and Wilkinson, J.H. (1970). "The least-squares problem and pseudo-inverses," *Computer Journal* **13**, 309-316.
- Pissanetzky, Sergio (1984). *Sparse Matrix Technology*, Academic Press, London and New York.
- Powell, M.J.D. (1970). A Hybrid Method for Nonlinear Equations, in P. Rabinowitz (ed.), *Numerical Methods for Nonlinear Algebraic Equations*, Academic Press, London and New York, 29-55.
- Powell, M.J.D. (1971). "On the Convergence of the Variable Metric Algorithm," *Journal of the Institute of Mathematics and Its Applications* **7**, 21-36.
- Powell, M.J.D. (1976). "Some Global Convergence Properties of a Variable Metric Algorithm Without Exact Line Searches," in R.C. Cottle and C.E. Lemke (eds.), *SIAM-AMS proceedings, Volume IX, Mathematical Programming*, American Mathematical Society, Providence, Rhode Island, 53-72.
- Powell, M.J.D. (1991). "On the Number of Iterations of Karmarkar's Algorithm for Linear Programming," Technical Report DAMTP 1991/NA23, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK.
- Powell, M.J.D. and Toint, Ph.L. (1979). "On the Estimation of Sparse Hessian Matrices," *SIAM Journal on Numerical Analysis* **16**, 1060-1073.
- Prékopa, A. (1990). "Sharp Bounds on Probabilities Using Linear Programming," *Operations Research* **38**, 227-239.

R

- Reid, J.K. (1971). "A Note on the Stability of Gaussian Elimination," *Journal of the Institute of Mathematics and Its Applications* **8**, 374-375.
- Reid, J.K. (1976). "Fortran Subroutines for Handling Sparse Linear Programming Bases," *Report AERE R8269*, Atomic Energy Research Establishment, Harwell, England.
- Reid, J.K. (1982). "A Sparsity-Exploiting Variant of the Bartels-Golub Decomposition for Linear Programming Bases," *Mathematical Programming* **24**, 55-69.
- Reinfeld, N.V., and Vogel, W.R. (1958). *Mathematical Programming*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

- Renegar, J. (1988). "A Polynomial-Time Algorithm, Based on Newton's Method, for Linear Programming," *Mathematical Programming* **40**, 59–93.
- Rice, J.R. (1966a). "Experiments with Gram-Schmidt orthogonalization," *Mathematics of Computation* **20**, 325–328.
- Rice, J.R. (1966b). "A Theory of Condition," *SIAM Journal on Numerical Analysis* **3**, 287–310.
- Rice, J.R. (1976a). "Parallel Algorithm for Adaptive Quadrature III: Program Correctness," *ACM Transactions on Mathematical Software* **2**, 1–30.
- Rice, J.R. (1976b). "The Algorithm Selection Problem," in M. Rubicoff and M. Yovits (eds.), *Advances in Computers*, Vol. 15, Academic Press, London and New York, 65–118.
- Rice, J.R. (1985). *Matrix Computations and Mathematical Software*, McGraw-Hill, San Francisco, California.
- Rigby, B., Lasdon, L.L., and Waren, A.D. (1995). "The Evolution of Texaco's Blending Systems from OMEGA to StarBlend," *Interfaces* **25:5**, 64–83.
- Robertson, H.H. (1977). "The Accuracy of Error Estimates for Systems of Linear Algebraic Equations," *Journal of the Institute of Mathematics and Its Applications* **20**, 409–414.
- Rockafellar, R.T. (1970). *Convex Analysis*, Princeton University Press, Princeton, New Jersey.
- Rockafellar, R.T. (1984). *Network Flows and Monotropic Optimization*, Wiley, New York.
- Rockafellar, R.T. and Wets, R.J. (1989). "Scenario and Policy Aggregation in Optimization under Uncertainty," *Mathematics of Operations Research* **16**, 119–147.
- Rose, D.J. and Willoughby, R.A. (eds.) (1972). *Sparse Matrices and Their Applications*, Plenum Press, New York, 67–76.
- Rosen, S. (1969). "Electronic Computers: A Historical Survey," *ACM Computing Surveys* **1**, 7–36.
- Rosen, J.B. (1960). "The Gradient Projection Method for Nonlinear Programming, Part I—Linear Constraints," *SIAM Journal of Applied Mathematics* **9**, 181–217.
- Rosen, J.B. (1964). "Primal Partition Programming for Block Diagonal Matrices," *Numerische Mathematik* **6**, 250–260.
- Roush, W.B., Stock, R.H., Cravener, T.L., and D'Alfonso, T.H. (1994). "Using Chance-Constrained Programming for Animal Feed Formulation at Agway," *Interfaces* **24:2**, 53–58.
- Roy, J. and Crainic, T.G. (1992). "Improving Freight Routing with a Tactical Planning Model," *Interfaces* **22:3**, 31–44.
- Russel, E.J. (1969). "Extensions of Dantzig's Algorithm for Finding an Initial Near-Optimal Basis for the Transportation Problem," *Operations Research* **17**, 187–191.
- Ruszczynski, A. (1986). "A Regularized Decomposition Method for Minimizing a Sum of Polyhedral Functions," *Mathematical Programming* **35**, 309–333.
- Ryan, D.M., and Osborne, M.R. (1988). "On the Solution of Highly Degenerate Linear Programs," *Mathematical Programming* **41**, 385–392.
- Ryder, B.G. (1974). "The PFORT Verifier," *Software Practice and Experience* **4**, 359–378.

S

- Saigal, R. (1992). "A Simple Proof of the Primal Affine Scaling Method," Technical Report 92-60, Department of Industrial Engineering and Operations Research, University of Michigan, Ann Arbor, Michigan, to appear in *Annals of Operations Research*.
- Saigal, R. (1993a). "A Three Step Quadratically Convergent Implementation of the Primal Affine Scaling Method," Technical Report 93-9, Department of Industrial Engineering and Operations Research, University of Michigan, Ann Arbor, Michigan.
- Saigal, R. (1993b). "The Primal Power Affine Scaling Method," Technical Report 93-21, Department of Industrial Engineering and Operations Research, University of Michigan, Ann Arbor, Michigan, to appear in *Annals of Operations Research*.
- Sakarovitch, M. and Saigal, R. (1967). "An Extension of Generalized Upper Bounding Techniques for Structured Linear Programs," *SIAM Journal of Applied Mathematics* **15**, 4, 906–914.
- Salveson, M.E. (1953). "A Computational Technique for the Fabrication Scheduling Problem," Management Sciences Research Project, University of California, Los Angeles, California.
- Samelson, H. (1974). *An Introduction to Linear Algebra*, John Wiley and Sons, New York.
- Sargent, R.W.H., and Westerberg, A.W. (1964). "'Speed-up' in Chemical Engineering Design," *Transactions of the Institute of Chemical Engineers*, **42**, 190–197.
- Saunders, M.A. (1972). "Large-Scale Linear Programming Using the Cholesky Factorization," Report STAN-CS-72-252, Department of Computer Science, Stanford University, Stanford, CA.
- Saunders, M.A. (1976a). "The Complexity of LU Updating in the Simplex Method," in R. S. Anderssen and R. P. Brent (eds.), *The Complexity of Computational Problem Solving*, University Press, Queensland, 214–230.
- Saunders, M.A. (1976b). "A Fast, Stable Implementation of the Simplex Method Using Bartels-Golub Updating," in J.R. Bunch and D.J. Rose (eds.), *Sparse Matrix Computations*, Academic Press, London and New York, 213–226.
- Saunders, M.A. (1979). "Sparse Least Squares by Conjugate Gradients: A Comparison of Preconditioning Methods," in J.F. Gentlemen (ed.), *Computer Science and Statistics: 12th Annual Symposium on the Interface*, University of Waterloo, Waterloo, Ontario, Canada, 15–20.
- Saunders, M.A. (1980). "Large-scale Linear Programming," Notes for the tutorial conference *Practical Optimization*, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, CA.
- Schönauer, W. (1987). *Scientific Computing on Vector Computers*, North-Holland, Amsterdam, the Netherlands.
- Schrage, L. (1975). "Implicit Representation of Variable Upper Bounds in Linear Programming," *Mathematical Programming Study* **4**, 118–132.
- Schrage, L. (1981). "Some Comments on Hidden Structure in Linear Programs," in H. Greenberg and J. Maybee (eds.), *Computer Assisted Analysis and Model Simplification*, Academic Press, London and New York, 389–395.

- Scott, D.M. (1985). "A Dynamic Programming Approach to Time-Staged Convex Programs," Technical Report SOL 85-3, Department of Operations Research, Stanford University, Stanford, CA.
- Sethi, A.P. and Thompson, G.L. (1984). "The Pivot and Probe Algorithm for Solving a Linear Program," *Mathematical Programming* **29**, 219–233.
- Shapiro, J.F. (1979). *Mathematical Programming: Structures and Algorithms*, John Wiley and Sons, New York.
- Shapiro, R.D. (1984). *Optimization Models for Planning and Allocation: Text and Cases in Mathematical Programming*, Wiley, New York.
- Shanno, D.F. and Bagchi, A. (1990). "A Unified View of Interior Point Methods for Linear Programming," *Annals of Operations Research* **22**, 55–70.
- Sharpe, W.F. (1971). "A Linear Programming Approximation for the General Portfolio Selection Problem," *Journal of Financial Quantitative Analysis* **6**, 1263–1275.
- Shefi, A. (1969). "Reduction of Linear Inequality Constraints and Determination of All Feasible Extreme Points," Ph.D. thesis, Department of Engineering-Economic Systems, Stanford University, Stanford, CA.
- Sherman, A.H. (1978). "Algorithms for Sparse Gaussian Elimination with Partial Pivoting," *ACM Transactions on Mathematical Software* **4**, 330–338.
- Sherman, D.H. and Ladino, G. (1995). "Managing Bank Productivity Using Data Envelopment Analysis (DEA)," *Interfaces* **25:2**, 60–73.
- Sherman, J. and Morrison, W.J. (1949). "Adjustment of an Inverse of Matrix Corresponding to Changes in Elements of a Given Column or a Given Row of the Original Matrix," *Annals of Mathematical Statistics* **20**, 621.
- Shor, N.Z. (1970a). "Utilization of the Operation of Space Dilatation in the Minimization of Convex Functions," *Kibernetika* **6**, 6–11; English translation in *Cybernetics* **6**, 7–15.
- Shor, N.Z. (1970b). "Convergence Rate of the Gradient Descent Method with Dilatation of the Space," *Kibernetika* **6**, 80–85; English translation in *Cybernetics* **6**, 102–108.
- Shor, N.Z. (1971a). "A Minimization Method Using the Operation of Extension of the Space in the Direction of the Difference of Two Successive Gradients," *Kibernetika* **7**, 51–59; English translation in *Cybernetics* **7**, 450–459.
- Shor, N.Z. (1971b). "Certain Questions of Convergence of Generalized Gradient Descent," *Kibernetika* **7**, 82–84; English translation in *Cybernetics* **7**, 1033–1036.
- Shor, N.Z. (1972a). "Solution of the Minimax Problems by the Method of Generalized Gradient Descent with Dilatation of the Space," *Kibernetika* **8**, 82–88; English translation in *Cybernetics* **8**, 88–94.
- Shor, N.Z. (1972b). "A Class of Almost-Differentiable Functions and a Minimization Method for Functions in this Class," *Kibernetika* **8**, 65–70; English translation in *Cybernetics* **8**, 599–606.
- Shor, N.Z. (1975). "Convergence of a Gradient Method with Space Dilatation in the Direction of the Difference Between Two Successive Gradients," *Kibernetika* **11**, 48–53; English translation in *Cybernetics* **11**, 564–570.

- Shor, N.Z. (1977a). "Cut-off Method with Space Extension in Convex Programming Problems," *Kibernetika* **13**, 94–95; English translation in *Cybernetics* **13**, 94–96.
- Shor, N.Z. (1977b). "New Development Trends in Nondifferentiable Optimization," *Kibernetika* **13**, 87–91; English translation in *Cybernetics* **13**, 881–886.
- Simonard, M. (1966). *Linear Programming*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey; English translation by W. S. Jewell.
- Sinha, G.P., Chandrasekaran, B.S., Mitter, N., Dutta, G., Singh, S.B., Choudhry, A.R., and Roy, P.N. (1995). "Strategic and Operational Management with Optimization at Tata Steel," *Interfaces* **25**:1, 6–19.
- Skeel, R.D. (1979). "Scaling for Numerical Stability in Gaussian Elimination," *Journal of the Association for Computing Machinery* **26**, 494–526.
- Skeel, R.D. (1980). "Iterative Refinement Implies Numerical Stability for Gaussian Elimination," *Mathematics of Computation* **35**, 817–832.
- Sleator, D.K. (1980). "An $O(nm \log n)$ Algorithm for Maximum Network Flows," Ph.D. thesis, Department of Computer Science, Stanford University, Stanford, CA.
- Smale, S. (1976). "A Convergent Process of Price Adjustment and Global Newton Methods," *Journal of Mathematical Economics* **3**, 107–120.
- Smale, S. (1982). "On the Average Speed of the Simplex Method of Linear Programming," Department of Mathematics, University of California, Berkeley.
- Smale, S. (1983). "The Problem of the Average Speed of the Simplex Method," in A. Bachem, M. Grötschel, and B. Korte (eds.), *Mathematical Programming: The State of the Art*, Springer-Verlag, Berlin and New York, , 530–539.
- Smith, B.T., Boyle, J.M., Dongarra, J.J., Garbow, B.S., Ikebe, Y., Klema, V.C., and Moler, C.B. (1976). *Matrix Systems Routines—EISPACK Guide, Lecture Notes in Computer Science* **6**, Springer-Verlag, Berlin and New York.
- Solow, D. (1984). *Linear Programming: An Introduction to Finite Improvement Algorithms*, North-Holland, Amsterdam, the Netherlands.
- Sonnevend, G. (1986). "An 'Analytic Center' for Polyhedrons and New Classes of Global Algorithms for Linear (smooth, convex) Programming, in A. Prékopa, J. Szelezsan, and B. Strazicky (eds.), *System Modelling and Optimization: Proceedings of the 12th IFIP Conference Held in Budapest, Hungary, September, 1985, Volume 84 of Lecture Notes in Control and Information Science*, Springer-Verlag, Berlin, West Germany, 866–876.
- Sorensen, D. (1980). "Newton's Method with a Model Trust Region Modification," Report ANL-80-106, Argonne National Laboratory, Argonne, Illinois.
- Sorensen, D. (1985). "Analysis of Pairwise Pivoting in Gaussian Elimination," *IEEE Transactions on Computers*, **C-34**, 274–278.
- Sponk, J. (1981). *Interactive Multiple Goal Programming: Applications to Financial Management*, Martinus Nijhoff, Boston, Massachusetts.
- Steinitz, E. (1913). *Journal für die Reine and Angewandte Mathematik* **143**, 128–275.
- Steuer, R.E. (1985). *Multiple Criteria Optimization: Theory, Computation, and Application*, Wiley, New York

- Stiemke, E. (1915). "Über positive Lösungen homogener linearer Gleichungen," *Mathematical Annals* **76**, 340–342.
- Stigler, G.J. (1945). "The Cost of Subsistence," *Journal of Farm Economics* **27**, 303–314.
- Starfield, A.M., Smith, K. A., and Bleloch, A. L. (1990). *How to Model It—Problem Solving for the Computer Age*, McGraw-Hill, New York.
- Stewart, G.W. (1973). *Introduction to Matrix Computations*, Academic Press, London and New York.
- Strang, G. (1986). *Introduction to Applied Mathematics*, Wellesley-Cambridge Press, Massachusetts.
- Strang, G. (1988). *Linear Algebra*, Academic Press, London and New York.
- Strassen, V. (1969). "Gaussian Elimination is Not Optimal," *Numerische Mathematik* **13**, 354–356.
- Stone, H.S. (1972). *Introduction to Computer Organization and Data Structures*, McGraw-Hill, San Francisco, California.

T

- Tarjan, R.E. (1972). "Depth-First Search and Linear Graph Algorithms," *SIAM Journal on Computing* **1**, 146–160.
- Thapa, M.N. (1976). "Operations Research Techniques in Iron and Steel Making," B. Tech. Thesis, Indian Institute of Technology, Bombay, India.
- Thapa, M.N. (1983). "Optimization of Unconstrained Functions with Sparse Hessian Matrices—Quasi-Newton Methods," *Mathematical Programming* **25**, 158–182.
- Thapa, M.N. (1984a). "Optimal Leveraged Leasing Within LEA2," Final Report for ATEL, Inc.
- Thapa, M.N. (1984b). "Optimization of Unconstrained Functions with Sparse Hessian Matrices—Newton-Type Methods," *Mathematical Programming* **29**, 156–186.
- Thapa, M.N. (1991). "A Gasoline/Diesel Retail Distribution Model," Final Report for United Refining Company.
- Thapa, M.N. (1992). "A Multi-time Period Asphalt Distribution Model," Final Report for United Refining Company.
- Thie, P.R. (1988). *An Introduction to Linear Programming and Game Theory*, John Wiley and Sons, New York.
- Tilanus, C.B., DeGans, O.B., and Lenstra, J.K. (1986). *Quantitative Methods in Management: Case Studies of Failures and Successes*, Wiley, New York.
- Todd, M.J. (1980). "The Monotonic Hirsch Conjecture is False for Dimension at Least 4," *Mathematics of Operations Research* **5**, 599–601.
- Todd, M.J. (1982). "An Implementation of the Simplex Method for Linear Programming Problems with Variable Upper Bounds," *Mathematical Programming* **23**, 23–49.
- Todd, M.J. (1983). "Large Scale Linear Programming: Geometry, Working Bases and Factorizations," *Mathematical Programming* **26**, 1–20.

- Todd, M.J. (1989). "Recent Developments and New Directions in Linear Programming," in M. Iri and K. Tanabe (eds.), *Mathematical Programming: Recent Developments and Applications*, Kluwer Academic Press, Dordrecht, the Netherlands, 109–157.
- Todd, M.J. (1990a). "The Effects of Degeneracy, Null, and Unbounded Reduction Algorithm for Linear Programming," Technical Report No. 902, School of Operations Research and Industrial College of Engineering, Cornell University, Ithaca, New York 14853.
- Todd, M.J. (1990b). "Combining Phase I and Phase II in a Potential Reduction Algorithm for Linear Programming," Technical Report No. 902, School of Operations Research and Industrial College of Engineering, Cornell University, Ithaca, New York 14853.
- Todd, M.J. (1994a). "Theory and Practice of Interior-Point Methods," *ORSA Journal on Computing* **6**, 28–31.
- Todd, M.J. (1994b). "A Lower Bound on the Number of Iterations of Primal-Dual Interior Point Methods for Linear Programming," Technical Report 1050, School of Operations Research and Industrial College of Engineering, Cornell University, Ithaca, New York 14853.
- Todd, M.J. and Burrell, B.P. (1986). "An Extension of Karmarkar's Algorithm for Linear Programming Using Dual Variables," special issue of *Algorithmica* **1**, Springer-Verlag, Berlin and New York, 409–424.
- Toint, Ph.L. (1977). "On Sparse and Symmetric Matrix Updating Subject to a Linear Equation," *Mathematics of Computation* **32**, 839–851.
- Toint, Ph.L. (1979). "On the Superlinear Convergence of an Algorithm for Solving a Sparse Minimization Problem," *SIAM Journal on Numerical Analysis* **16**.
- Tomlin, J.A. (1972a). "Pivoting for Size and Sparsity in Linear Programming Inversion Routines," *Journal of the Institute of Mathematics and Its Applications* **10**, 289–295.
- Tomlin, J.A. (1972b). "Modifying Triangular Factors of the Basis in the Simplex Method," in D.J. Rose and R.A. Willoughby (eds.), *Sparse Matrices and their Applications*, Plenum Press, New York, .
- Tomlin, J.A. (1975a). "LPM1 User's Guide," Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, CA.
- Tomlin, J.A. (1975b). "On Scaling Linear Programming Problems," *Mathematical Programming Study* **4**, 146–166.
- Tomlin, J.A. (1975c). "An Accuracy Test for Updating Triangular Factors," *Mathematical Programming Study* **4**, 142–145.
- Tomlin, J.A. (1987). "An Experimental Approach to Karmarkar's Projective Method for Linear Programming," *Mathematical Programming Study* **31**, 175–191.
- Tomlin, J.A. and Welch, J.S. (1983a). "Formal Optimization of some Reduced Linear Programming Problems", *Mathematical Programming* **27**, 232–240.
- Tomlin, J.A. and Welch, J.S. (1983b). "A Pathological Case in the Reduction of Linear Programs," *Operations Research Letters* **2**, 53–57.
- Tomlin, J.A. and Welch, J.S. (1985). "Integration of a Primal Simplex Network Algorithm with a Large-Scale Mathematical Programming System," *ACM Transactions on Mathematical Programming* **11**, 1–11.

- Tomlin, J.A. and Welch, J.S. (1986). "Finding Duplicate Rows in a Linear Programming Model," *Operations Research Letters* **5**, 7–11.
- Tompkins, C.B. (1955). "Projection Methods in Calculation," in H.A. Antosiewicz (ed.), *Proceedings of the Second Symposium in Linear Programming*, Vol. 2, National Bureau of Standards and Directorate of Management Analysis, DCS/Comptroller, USAF, Washington, DC, 425–488. 1036–1045.
- Tompkins, C.B. (1957). "Some Methods of Computational Attack on Programming Problems, Other than the Simplex Method," *Naval Research Logistics Quarterly*, **4**, 95–96.
- Tsuchiya, T. and Monteiro, R.D.C. (1996). "Superlinear Convergence of the Affine Scaling Algorithm," *Mathematical Programming* **75**, 77–110.
- Tsuchiya, T. and Muramatsu, M. (1995). "Global Convergence of the Long-Step Affine Scaling Algorithm for Degenerate Linear Programming Problems," *SIAM Journal on Optimization* **5**, 525–551; previously published as a Research Memorandum 423, The Institute of Statistical Mathematics, 4-6-7 Minami-Azubu, Minato-ku, Tokyo 106, Japan, 1992.
- Tucker, A.W. (1955). "Linear Inequalities and Convex Polyhedral Sets," in H.A. Antosiewicz (ed.), *Proceedings of the Second Symposium in Linear Programming*, Vol. 2, National Bureau of Standards and Directorate of Management Analysis, DCS/Comptroller, USAF, Washington, DC, 569–602.
- Tucker, A.W. (1956). "Dual Systems of Homogeneous Linear Relations," in H.W. Kuhn and A.W. Tucker (eds.), *Linear Inequalities and Related Systems*, Annals of Mathematics Study No. 38, Princeton University Press, Princeton, New Jersey, 3–18.

U

- Ubhaya, V.A. (1974a). "Isotone Optimization, I," *Journal of Approximation Theory* **12**, 146–159.
- Ubhaya, V.A. (1974b). "Isotone Optimization, II," *Journal of Approximation Theory* **12**, 315–331.

V

- Vajda, S. (1961). *Mathematical Programming*, Addison-Wesley Publishing Company, Reading, Massachusetts.
- Vanderbei, R.J. (1991). "Splitting Dense Columns in Sparse Linear Systems," *Linear Algebra and Its Applications* **152**, 107–117.
- Vanderbei, R.J. (1992). "LOQO User's Manual," School of Engineering and Applied Science, Department of Civil Engineering and Operations Research, Princeton University, Princeton, New Jersey, 92–95.
- Vanderbei, R.J. (1994). "Interior Point Methods: Algorithms and Formulations," *ORSA Journal on Computing* **6**, 32–34.

- Vanderbei, R.J. (1995). "Symmetric Quasi-Definite Matrices," *SIAM Journal on Optimization* **5**, 100-113; previously published in 1991 in School of Engineering and Applied Science, Department of Civil Engineering and Operations Research, Princeton University, Princeton, New Jersey 91-100.
- Vanderbei, R.J. and Carpenter, T.J. (1993). "Symmetric Indefinite Systems for Interior Point Methods," *Mathematical Programming* **58**, 1-32.
- Vanderbei, R.J. and Lagarias, J.C. (1988). "I.I. Dikin's Convergence Result for the Affine-Scaling Algorithm," in J.C. Lagarias and M.J. Todd (eds.), *Mathematical Developments Arising from Linear Programming*, Contemporary Mathematics, 109-119.
- Vanderbei, R.J., Meketon, M.S., and Freedman, B.A. (1986). "A Modification of Karmarkar's Linear Programming Algorithm," *Algorithmica* **1**, 395-407.
- Van Loan, C. (1985). "How Near is a Stable Matrix to an Unstable Matrix?" *Contemporary Mathematics* **47**, 465-477.
- Van Slyke, R.M. and Wets, R.J. (1969). "L-Shaped Linear Programs with Applications to Optimal Control and Stochastic Programming," *SIAM Journal of Applied Mathematics* **17**, 638-663.
- Van Slyke, R.M. and Wets, R.J. (1966). "Programming Under Uncertainty and Stochastic Optimal," *SIAM Journal on Control and Optimization* **4**, 179-193.
- Van Tassel, D. (1974). *Program Style, Design, Efficiency, Debugging and Testing*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- van de Panne, C. and Whinston, A. (1969). "The Symmetric Foundation of the Simplex Method for Quadratic Programming," *Econometrica*, **37**, 507-527.
- Varga, R.S. (1962). *Matrix Iterative Analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Vavasis, S.A. (1993). "Stable Finite Elements for Problems with Wild Coefficients," Report TR 93-1364, Department of Computer Science, Cornell University, Ithaca, New York.
- Vavasis, S.A. (1994). "Stable Numerical Algorithms for Equilibrium Systems," *SIAM Journal on Matrix Analysis and Applications* **15:4**, 1108-1131.
- Vertinsky, I., Brown, S., Schreier, H., Thompson, W.A., and van Kooten, G.C. (1994). "A Hierarchical-GIS Based Decision Model for Forest Management: The Systems Approach," *Interfaces* **24:4**, 38-53.
- Ville, J.A. (1938). "Sur la théorie général des jeux où intervient l'habileté des joueurs," *Applications aux jeux de hasard* by Emil Borel and Jean Ville, Tome 4, Fascicule 2, in *Traité du Calculus des Probabilités et de ses Applications*, by Emile Borel, 105-113.
- Vliet, A.V., Boender, C.G., and Rinnooy Kan, A.H.G. (1991). "Interactive Optimization of Bulk Sugar Deliveries," *Interfaces* **22:3**, 4-14.
- von Neumann, J. (1928). "Zur Theorie de Geselleschaftsspiele," *Mathematical Annals* **100**, 295-320; translated by Sonya Bargmann in H.W. Kuhn and A.W. Tucker (eds.), *Contributions to the Theory of Games*, Vol. IV, Annals of Mathematics Study No. 40, Princeton University Press, Princeton, New Jersey, 13-42.
- von Neumann, J. (1937). "Über ein Ökonomisches Gleichungssystem und ein Verallgemeinerung des Brouwerschen Fixpunktsatzes," *Ergebnisse eines Mathematischen Kolloquiums*, No. 8; translated in *The Review of Economic studies* **13**, 1-9.

- von Neumann, J. (1947). "On a Maximization Problem,," (manuscript), Institute for Advanced Study, Princeton, New Jersey.
- von Neumann, J. (1948a). "A Numerical Method for the Determination of the Value and Best Strategies of a Zero-Sum Two-Person Game," (manuscript), Institute for Advanced Study, Princeton, New Jersey.
- von Neumann, J. (1948b). Private Communication on an Interior Method.
- von Neumann, J. (1953). "A Certain Zero-Sum Two-Person Game Equivalent to the Optimal Assignment Problem," in H.W. Kuhn and A.W. Tucker (eds.), *Contributions to the Theory of Games*, Vol. 2, Annals of Mathematics Study No. 28, Princeton University Press, Princeton, New Jersey, 12–15.
- von Neumann, J. and Goldstine, H.H. (1947). "Numerical Inverting of Matrices of High Order," *Bulletin of the American Mathematical Society* **53**, 1021–1089.
- von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, New Jersey.

W

- Wegner, P. (1968). *Programming Languages, Information Structures and Machine Organization*, McGraw-Hill, New York.
- Wets, R.J. (1984). "Programming Under Uncertainty: The Equivalent Convex Program," *SIAM Journal of Applied Mathematics* **14**, 89–105.
- Whiting, P.D. and Hillier, J.A. (1960). "A Method for Finding the Shortest Route Through a Road Network," *Operations Research Quarterly* **11**, 37–40.
- Wilkinson, J.H. (1961). "Error Analysis of Direct Methods of Matrix Inversion," *Journal of the Association for Computing Machinery* **8**, 281–330.
- Wilkinson, J.H. (1963). *Rounding Errors in Algebraic Processes*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Wilkinson, J.H. (1965). *The Algebraic Eigenvalue Problem*, Oxford University Press, Oxford and New York.
- Wilkinson, J.H. (1971). "Modern Error Analysis," *SIAM Review* **13**, 548–568.
- Wilkinson, J.H. and Reinsch, C. (eds.), (1971). *Handbook for Automatic Computation, Vol. 2, Linear Algebra*, Springer-Verlag, Berlin and New York.
- Williams, H.P. (1985). *Model Building in Operations Research*, Wiley, New York.
- Wittrock, R.J. (1983). "Advances in a Nested Decomposition Algorithm for Solving Staircase Linear Programs," Technical Report SOL 83-2, Department of Operations Research, Stanford University, Stanford, CA.
- Wolfe, P. (1960). "Accelerating the Cutting Plane Method for Nonlinear Programming," *Journal of the Society for Industrial and Applied Mathematics*, **9**, 481–488.
- Wolfe, P. (1962). "The Reduced-Gradient Method," unpublished manuscript, RAND Corporation.
- Wolfe, P. (1963). "A Technique for Resolving Degeneracy in Linear Programming," *SIAM Journal of Applied Mathematics* **11**, 205–211.

- Wolfe, P. (1965). "The Composite Simplex Algorithm," *SIAM Review* **7**, 42–55.
- Wolfe, P. (1967). "Methods for Nonlinear Programming," in J. Abadie (ed.), *Nonlinear Programming*, North-Holland, Amsterdam, the Netherlands, 67–86.
- Wolfe, P. and Cutler, L. (1963). "Experiments in Linear Programming," in R.L. Graves and P. Wolfe (eds.), *Recent Advances in Mathematical Programming*, McGraw-Hill, New York, 177–200.
- Wright, M.H. (1976). *Numerical Methods for Nonlinearly Constrained Optimization*, Ph.D. thesis, Department of Computer Science, Stanford University, Stanford, CA.
- Wright, M.H. (1992). "Interior Methods for Constrained Optimization," in A. Iserles (ed.), *Acta Numerica*, Cambridge University Press, New York, New York, 341–407.
- Wright, M.H. and Glassman, S.C. (1978). "FORTRAN Subroutines to Solve the Linear Least Squares Problem and Compute the Complete Orthogonal Factorization," Technical Report SOL 78-8, Department of Operations Research, Stanford University, Stanford, CA.
- Wright, S.J. (1993). "A Superlinear Infeasible-Interior-Point Algorithm for Monotone Nonlinear Complementarity Problems, Technical Report MCS-P344-1292, Mathematical and Computer Science Division, Argonne National Laboratory, Argonne, Illinois.
- Wright, W. (1980). "Automatic Identification of Network Rows in Large-Scale Optimization Models," M.S. thesis, Naval Postgraduate School, Monterey, CA.

Y

- Yamada, T. and Kitahara, T. (1985). "Qualitative Properties of Systems of Linear Constraints," *Journal of the Operations Research Society of Japan* **28**, 331–343.
- Yamashita, H. (1986). "A Polynomially and Quadratically Convergent Method for Linear Programming," working paper, Mathematical Systems Institute, Inc., Tokyo, Japan.
- Ye, Y. (1987). "Eliminating Columns in the Simplex Method for Linear Programming," Technical Report SOL 87-14, Department of Operations Research, Stanford University, Stanford, CA.
- Ye, Y. (1990). "A 'Build-Down' Scheme for Linear Programming," *Mathematical Programming* **46**, 61–72.
- Ye, Y. (1997). *Interior Point Algorithms: Theory and Analysis*, John Wiley and Sons, New York.
- Ye, Y., Güler, O., Tapia, R.A., and Zhang, Y. (1993). "A Quadratically Convergent $O(\sqrt{n}L)$ -Iteration Algorithm for Linear Programming," *Mathematical Programming* **59**, 151–162.
- Ye, Y., Todd, M.J., and Mizuno, S. (1994). "An $O(\sqrt{n}L)$ -Iteration Homogeneous and Self-Dual Linear Programming Algorithm," *Mathematics of Operations Research* **19**, 53–67.
- Ye, Y. and Tse, E. (1989). "An Extension of Karmarkar's Projective Algorithm for Convex Quadratic Programming," *Mathematical Programming* **44**, 157–180.
- Yohe, J.M. (1974). "An Overview of Programming Practices," *ACM Computing Surveys* **6**, 221–245.
- Young, D.M. (1971). *Iterative Solution for Large Linear Systems*, Academic Press, London and New York.

Z

- Zadeh, N. (1973). "A Bad Network Problem for the Simplex Method and Other Minimum Cost Flow Algorithms," *Mathematical Programming* **5**, 255–266.
- Zaki, H. (1990). "A Comparison of Two Algorithms for the Assignment Problem," Technical Report ORL-90-002, Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL.
- Zangwill, W.I. (1969). *Nonlinear Programming: A Unified Approach*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Ziemba, W.J. (1970). "Computational Algorithms for Convex Stochastic Programs with Simple Recourse," *Operations Research* **18**, 273–294.
- Ziemba, W.J. (1974). "Stochastic Programs with Simple Recourse," in P.L. Hammond and H. Zoutendijk (eds.), *Mathematical Programming: Theory and Practice*, North-Holland, Amsterdam, the Netherlands.
- Zikan K. and Cottle R.W. (1987). "The Box Method for Linear Programming: Part I—Basic Theory," Technical Report SOL 87-6, Department of Operations Research, Stanford University, Stanford, CA.
- Zhang, Y. (1994). "On the Convergence of a Class of Infeasible Interior-Point Methods for the Horizontal Linear-Complementarity Problem," *SIAM Journal on Optimization* **4**, 208–227.
- Zhang, Y., Tapia, R.A., and Dennis, J.E. (1992). "On the Superlinear and Quadratic Convergence of Primal-Dual Interior Point Linear Programming Algorithms," *SIAM Journal on Optimization* **2**, 304–324.
- Zhang, Y. and Zhang, D. (1995). "On Polynomiality of the Mehrotra-Type Predictor-Corrector Interior Point Algorithms," *Mathematical Programming* **68**, 303–318.
- Zlatev, Z. (1980). "On Some Pivotal Strategies in Gaussian Elimination by Sparse Technique," *SIAM Journal on Numerical Analysis* **17**, 12–30.
- Zlatev, Z., Wasniewski, J., and Schaumburg, K. (1981). "Y12M—Solution of Large and Sparse Systems of Linear Equations," *Lecture Notes in Computer Science* **121**, Springer-Verlag, Berlin and New York.

Index

- ϵ -Optimal Polynomial Time, 70
- 2-Stage Stochastic LP
 - algorithm, 341
 - deterministic case, 335
 - estimating lower bound z_{LB} , 352
 - estimating upper bound z_{UB} , 351
 - Full Formulation, 337
 - geometric description, 338
 - theory behind the algorithm, 348
- Abadie, J., 59
- Abrahamson, P., 361
- Adjacent, 18
- Adjusted costs, 287, 302, 307
- Adler, I., 120, 142, 143, 316
- Affine combination, 1, 2
- Affine hull, 2
- Affine transformation, 133
- Ahuja, R., 243, 244
- Ali, A., 244
- Alternatives
 - theorems, 43
- Analytic center, 141
- Andersen, E., 146
- Anstreicher, K., 120, 121
- anstreicher, K., 141
- Anti-cycling, 149
- Arc, 231
 - saturated, 240
- Argmin, xxv
- Associated network, 235
- Augmenting path algorithm, 243
- Avriel, M., 31, 146, 361
- Axelsson, O., 144
- Bagchi, A., 140
- Bahn, O., 121
- Balanski, M., 166
- Ballinton, J., 142
- Barnes, E., 120, 141
- Barrier function methods, 128
 - primal logarithmic, 131
 - primal-dual, 134
- Basis norm, 197
- Basis triangularity, 210
- Bastian, M., 264
- Bazaraa, M., 38, 244
- Beale's Example, 150
- Beale, E., 150, 166, 201, 316, 332, 360
- Beale, M., 324
- Bellman, R., 244
- Ben-Tal, A., 361
- Benders decomposition, 299
 - dual of Dantzig-Wolfe decomposition, 299
 - using, 309
- Benders Full Master, 300, 301
- Benders, J., 316, 361
- Benichou, M., 166
- Bennet, 264
- Berge, C., 31
- Berry-Esséen, 361
- Bertsekas, D., 245
- Bertsekas, D., 243
- Bipartite graph, 245
- Birge, J., 332, 360, 361, 363
- Bisschop, J., 316
- Bixby, R., 146, 200, 201
- Björck, A., 202
- Bland's Rule, 158–160

- Bland, R., 166
Block-angular, 266
Block-angular system, 306
Block-triangular, 267
Blossom algorithm, 201
Borchers, B., 143
Borgwardt, K., 31
Bradley, G., 244
Bradley, S., 316
Breadth-first unblocked search, 237
Brown, G., 244, 264
Brumelle, S., 363
Burrel, B., 121
- Canonical form, 176
Carpenter, T., 143–145
Center, 141
Center-of-Gravity Problem, 72
Central Limit Theorem, 372
Charnes, A., 146, 166, 169, 264
Cherkassky, B., 244
Chi-square distribution, 373
Choi, I., 142
Cholesky factorization, 140
 partial, 144
Chvátal, V., 227, 244
Classical transportation problem, *see*
 transportation problem
Column-restricted primal, 191–196
Complementary pair, 56
Complementary slackness, 49–50
Cone, 9
 convex polyhedral, 10
 simplicial, 10
Confidence interval, 376
 one sided, 377
Confidence level, 377
Confidence limit, 377
Conjugate gradient method, 144
Conservation of flows, 232
Convex combination, 1, 2
Convex hull, 2, 35
Convex polygon, 9
Convex polyhedral cone, 10
Convex polyhedron, 9
Convex programming, 120
Convex regions
 general, 6
 two-dimensional, 3
Convex set, 4
 closed, 5
Convexity
 feasible solutions, 9
 half-space, 8
 hyperplane, 8
Convexity and linear inequality
 systems, 1
Convexity constraint, 2
Correlation coefficient, 369
Correspondence of primal and dual
 bases, 174
Covariance, 369
Cover, 245
CPM, 231, 244
Criss-Cross Method, 188
Cross-over to a basic solution, 137
Cubic fit, 115
Cunha, S., 332, 360
Cunningham, W., 227, 244
Cut, 239
 feasibility, 302
 in a network, *see* Maximal flow
 optimality, 302
 separate, 239
 value, 240
Cut value, 241
Cutler, L., 201
Cuts, 302
- D-W, 266
D-W Decomposition Principle, 280
Dantzig's Inductive Method, 154
Dantzig, G. B., 32–37, 58, 59, 70, 71,
 119, 120, 138, 146, 154, 166,
 169, 200–202, 243, 244, 264,
 315–317, 324, 332, 333, 360,
 361
Dantzig, T., 59
Dantzig-Wolfe Decomposition
 Principle, 266, 280

-
- Davis, J., 202
 Davis, P., 361
 de Ghellinck, G., 121
 Decomposition
 nested, 316
 Benders, 299
 central planning, 313
 D-W, 280
 D-W Principle, 266, 284
 Dantzig-Wolfe, 280
 illustrated, 292
 initial solution, 291
 staircase, 308
 Decomposition of flow, 233
 Decopositon
 D-W Principle, 280
 Degeneracy, 149–171
 Bland's Rule, 158–160
 example, 16
 examples of cycling, 149
 Beale's example, 150
 Hoffman's example, 149
 Tucker's Example, 150
 Extra Column Rule, 160–164
 Inductive Method, 154
 lexicographic method, 169
 lexicographic rule, 168
 on resolving, 153
 order of, 164
 perturbation method, 167, 169
 random choice rule, 158, 167
 resolving, 149
 transportation problem, 216
 Wolfe's Rule, 156–158
 Degenerate dual, 175
 Degenerate pivots
 avoidance, 164–165
 Delayed Column Generation
 Procedure, 288
 Delayed Row Generation, 299
 Dempster, M., 316
 Denardo, E., 244
 Dennnis, J., 143
 density, 79
 Descent direction, 124
 Deák, I., 361
 Dial, R., 244
 Dijkstra, E., 244
 Dikin's algorithm, 87, 88
 assumptions, 89
 convergence, 89
 using t in S , 93
 ellipsoid strictly in interior, 91
 proof of convergence, 93
 valid steps, 90
 Dikin's method, 68, 84–100, 133
 dual solution, 85
 ellipsoid, 86
 ellipsoid subproblem, 88
 interior solution, 85
 partition of indices, 85
 primal solution, 85
 Dikin, I., 68, 84, 119, 120, 141, 142
 Dilworth's theorem, 249
 Dinic, E., 244
 Dual Simplex Method, 179
 correspondence with primal, 180
 Duality
 Additional Theorem, 47
 Duality Gap, 45
 Duality Theorem, 45, 59
 Von Neumann, 45
 Duality theorem, 43
 Duality theorems, 43
 Duff, I., 144
 Edge, 14, 18
 Edmonds, J., 243
 Edmondson, J., 166
 Egerváry, E., 201, 243
 El-Bakry, A., 143
 Elias, P., 243
 Ellipsoid, 86
 Elmaghaby, S., 245
 Entriken, R., 316, 361
 Erisman, A., 144
 Ermoliev, Y., 332, 360–362
 Event, 367
 Expected value, 368
 Extra Column Rule, 160–164

- Extremal problem, 285
Extreme homogeneous normalized solution, 280
Extreme point, 14, 15
 degenerate, 16
- Fanning out procedure, 237
Farkas's Lemma, 59
 for homogeneous systems, 52
Farkas, J., 59
Faulkner, J., 167
Feasibility cut, 302
Feinstein, A., 243
Feller, W., 332, 377
Ferguson, A., 324, 332, 360
Fiacco, A., 119, 141, 146
Fletcher, R., 166
Flow value, 235
Ford, L., 201, 243, 316
Forrest, J., 200, 201
Forsgren, A., 145
Fourer, R., 144, 145, 316
Fourier-Motzkin Elimination, 35
Fox, B., 244
Fraley, C., 121
Frauendorfer, K., 332, 360, 361
Fredman, M., 244
Freedman, B., 120, 141
Freund, R., 121, 141
Frisch, K., 119, 141
Fulkerson, D., 201, 243, 316
Full Master, 285
 Benders, 300
Full Master Program, 307
- Gabow, H., 244
Gaivoronski, A., 362
Gale, D., 59
Galil, Z., 244
Gallo, G., 244
Gamma function, 374
Gass, S., 166, 201
Gassner, B., 227
Gauthier, J., 166
Gay, D., 121, 142
General position, 14
Generalized Upper Bounds, 251
Generic Column, 273
Geoffrion, A., 361
Geometry of Vector Spaces, 1
George, A., 144
Gill, P., 31, 140, 144–146, 166, 167
Glassey, C., 316
Global minimum, 14, 15
Glynn, P., 361
Goffin, J., 121
Goldberg, A., 244
Goldfarb, D., 200, 201
Goldman, A., 59, 316
Gomory, R., 166, 202
Gonzaga, C., 121, 140, 141
Gordan's theorem for homogeneous systems, 51
Gordan, P., 59
Gradient vector, 123
Graves, G., 244
Grcar, J., 144
Greenberg, H., 166
Gregory, J., 146
Grötschel, M., 120
GUB, 251
Güler, O., 143
- Hadley, G., 202
Half line, 5
Half-line, 6, 8
Half-plane, 8
Half-space, 7, 8
 closed, 8
 convexity, 8
Hall's theorem, 249
Hall, L., 141, 146
Hall, P., 361
Hammersly, J., 361
Handscomb, D., 361
Hanson, R., 202
Harris, P., 166
Hax, A., 316
Helgason, R., 244
Hentges, J., 166

-
- Hertog, D., 141
 Hessian matrix, 123
 Hight, J., 332, 360
 Hillier, F., 244
 Hillier, J., 244
 Hirsch conjecture, 25, 31, 33
 Hirsch, W. M., 25, 31, 33, 316
 Ho, J., 316, 317
 Hoffman's Example, 149
 Hoffman, A., 119, 149, 316
 Hoffman, A., 166
 Holmes, D., 360
 homogeneous inequalities, 47
 Homogeneous system, 59
 Huang, C., 361
 Huard, P., 141
 Hughes, 316
 Hungarian Method, 201
 Hyperplane, 7
 - convexity, 8
 - lowest dimensional, 8
 - separating, 11
 - supporting, 339
- Iglehart, D., 361
 Importance sampling, 350, 356
 Independent
 - distribution, 369
 Independent random variables, 369
 Inductive Method, 154
 Infanger, G., 360, 361
 Infeasibility Theorem, 11, 44, 59
 Infeasible equation, 44
 Interior solution to basic solution, 137
 Interior-point
 - potential reduction, 67
 Interior-point methods, 123–147
 - affine, 67
 - Early, 67–122
 - path-following, 67
 - primal affine, 67
 - projective, 67
 - versus Simplex Method, 69
- Jarvis, J., 38, 244
 Jensen's inequality, 361
 Jewell, W., 316
 Johnson, D., 244
 Johnson, L., 227
 Jones, M., 144
 Judd, H., 201
 Judin, D., 120
- König-Evary, 249
 Kalan, J., 166
 Kall, P., 332, 360, 361
 Kallio, M., 316
 Karmarkar's algorithm, 114–115
 - versus von Neumann Algorithms, 118
 - computational comments, 116
 - converting an LP, 115
 - null space, 104
 Karmarkar's method, 68, 100–118
 - assumptions, 102
 - inverse mapping, 101
 - inverse projective transformation, 101
 - mapping \mathcal{T} , 101
 - projective algorithm, 101
 - projective transformation, 101
 - proof of convergence, 105
 - simplex, 102
 - center, 102
 - radius of circumscribed sphere, 102
 - radius of inscribed sphere, 102
 - standard form LP, 102
- Karmarkar, N., 68, 120, 140, 142
 Karp, R., 243
 Kaul, R., 264
 Kelley, 36
 Kennington, J., 244
 Khachian's method, 68
 Khachian, I., 119, 120
 Khachian, L., 68
 Kim, K., 144
 Kirchoff's Law, 232
 Klee, V., 31, 201

- Klinecicz, J., 227, 244
 Klotz, E., 31, 166
 Kojima, M., 142, 143
 Koopmans, T., 227, 228
 Kortanek, K., 146
 Kotiah, T., 166
 Krishna's Rule, 160–164
 Krishna, A., 166
 Kuhn, H., 59, 146, 201, 243, 315
 Kumar, M., 244
- Lagarias, J., 120
 Lagrange multipliers, 59
 Lall, H., 244
 Law of large numbers, 372
 Lawler, E., 243
 Lawson, C., 202
 Leichner, S., 202
 Lemke, C., 200, 264
 Levin, A., 120
 Lexico-positive, 169
 Lexicographic method, 169
 Lexicographic Rule, 168
 Lieberman, G., 244
 Line search, 115, 121
 Line segment, 5
 Linear combination, 1
 nonnegative, 2
 Linear least-squares problem, 127
 Liu, J., 144
 Local minimum, 15
 Loute, E., 316
 Louveaux, F., 332, 360, 363
 Lovász, L., 120
 Lower-blocked, 238
 Lustig, I., 31, 120, 141–146
- Madansky, A., 332, 360
 Magnanti, M., 316
 Magnanti, T., 243
 Maheshwari, S., 244
 Malhotra, V., 244
 Manne, A., 201, 316
 Mannos, M., 119
 Marginal Column, 208
 Marginal row, 208
 Markowitz, H., 144, 324, 332
 Marshall, K., 166
 Marsten, R., 120, 141–143, 145, 146
 Mascarenhas, W., 141
 Master Program, 285
 Benders, 301
 full, 285
 restricted, 286
 Matching, 245
 Matching problem, 201
 Matrix
 lower triangular, 210
 triangular, 210
 upper triangular, 210
 Max improvement per iteration, 176
 Maximal flow, 232–241
 cuts in a network, 239–241
 Maximal matching, 245
 McCormick, G., 119, 141, 146
 McDiarmid, C., 120
 McGill, J., 363
 McShane, K., 142
 Mean, 368
 Meeraus, A., 316
 Megiddo, N., 142, 143, 146
 Mehlhorn, K., 244
 Mehrotra, S., 142–145
 Meijerink, J., 144
 Meketon, M., 120, 141
 Merle, O., 121
 Min-cut = max-flow, 240
 Minimal cover, 245
 Minimum Cost Flow Problem, 242
 Minkowski, H., 36
 MINOS, 166
 Minty, G., 31, 201
 Mitchell, J., 121, 143
 Mizuno, S., 142–144
 Monma, C., 120, 142
 Monte Carlo methods, 355
 Monteiro, R., 141–143
 Moore, E., 244
 Morton, A., 120
 Morton, D., 246, 361

-
- Motzkin's Transposition Theorem, 54
Motzkin, T., 59
Multi-commodity flow, 245, 317
Mulvey, J., 143, 144, 244
Munksgaard, N., 144
Muramatsu, M., 142
Murray, W., 31, 140, 144–146, 166, 167
Murtagh, B., 166
Murty, K., 201, 227
- Naive sampling, 350
Nakayama, M., 361
Nazareth, L., 144, 166, 317
Neighbor, 18
Nemirovskii, A., 120
Nested decomposition, 309, 316
Network, 221
Network Simplex Method, 243
 cycling, 244
 efficiency, 244
 pathological example, 244
Newton direction, 125
Newton's Method, 123–127, 201
Nishiya, T., 316
Node, 231
Node-arc incidence matrix, 232
Nondegenerate dual, 176
Nonnegative linear combination, 2
nonzero coefficient density, 79
Norm
 ∞ , xxiv
 one, xxiv
 two, xxiv
Normal distribution, 370
NP, 69
Null space, 104
- Oliveira, G., 332, 360
Open interval, 376
Open set, 38
Optimality cut, 302
Orchard-Hays, W., 201
Orden, A., 166, 169, 219
Orlin, J., 243, 244
- Ortega, J., 140
Osborne, M., 167
- Pallottino, S., 244
Papadimitriou, C., 120, 201
Pape, U., 244
Parametric objective, 201
Parametric programming, 183
 worst-case behavior, 201
Pereira, M., 332, 360
Perold, A., 316
PERT, 231, 244
Perturbation method, 167, 169
Phase I Least-Squares, 197–200
 algorithm steps, 198–199
Pierce, G., 142
Pinto, L., 332, 360
Plassman, P., 144
Polygon, 9
Polyhedral set, 16
Polyhedron, 9
Polynomial Time
 ϵ -Optimal, 70
Polynomial time, 69
Ponceleón, D., 144, 145
Positive Matrix, 39
Positive semi-definite, 86
Positive-definite, 86
Potential function, 107
Powell, M., 121
Preconditioned conjugate gradient
 method, 144
Predictor-Corrector Method, 142, 143
Primal affine method, 133
 Dikin's method, 133
Primal logarithmic barrier method, 131
Primal-Dual Algorithm, 191
Primal-dual logarithmic barrier
 method, 134
Probability density, 368
 joint, 371
Probability distribution, 368
Probability theory, 367–377

- Projection matrix, 128, 133
 Projective algorithm, 101
 Projective transformation, 101
 Projective transformation inverse,
 101

 QR factorization, 133
 Quadratic, 86, 328

 Rabinowitz, P., 361
 Radzik, T., 244
 Random Choice Rule, 158, 167
 Random sample, 371
 Random variable, 368
 continuous, 370
 discrete, 368
 Ray, 6
 Rays, 5
 Reduced array, 211
 Redundant constraints, 16
 Reid, J., 144
 Reiter, S., 227, 228
 Renegar, J., 140, 141
 Resende, M., 120, 142
 Resolution Theorem, 281
 Restricted linear program, 155
 Restricted Master, 286
 Rheinboldt, W., 140
 Ribière, G., 166
 Rockafellar, R., 31, 166, 332, 360
 Roos, C., 141
 Rosen, J., 264
 Ruszczynski, A., 332, 360
 Ryan, D., 167

 Saaty, T., 166, 201
 Saigal, R., 141, 142, 264
 Sakarovitch, M., 264
 Saltzman, M., 142, 146
 Sample mean, 371
 Sample point, 367
 Sample space, 367
 discrete, 368
 Sample variance, 371
 unbiased, 372

 Sampling
 importance, 350
 naive, 350
 Saturated arc, 240
 Saunders, M., 31, 140, 144, 145, 166,
 167
 Scarf, H., 332
 Schrage, L., 264
 Schrijver, A., 120
 Scott, D., 361
 Self-Dual Parametric Algorithm, 188
 worst-case behavior, 201
 Sen, S., 332, 360
 Separable quadratic function, 83
 Separating hyperplane, 11
 Shanno, D., 120, 140–143, 145, 146
 Shannon, C., 243
 Sherali, H., 38, 244
 Shor, N., 68, 120
 Shortest path, 244
 Shortest route, 244
 Simplex, 102
 center, 102
 extreme point, 14
 radius of circumscribed sphere,
 102
 radius of inscribed sphere, 102
 vertex, 14
 Simplex Algorithm related to von
 Neumann Algorithm, 83
 Simplex defined, 13
 Simplex Interpretation
 Simplex Method, 24–30
 Simplex Method
 simplex interpretation, 24–30
 steepest descent, 20–24
 versus interior-point methods,
 69
 worst-case complexity, 201
 Simplicial cone, 10
 Sleator, D., 244
 Smale, S., 31, 201
 Small, R., 316
 Sokolowsky, D., 119
 Sonnevend, G., 141

-
- Staircase, 266, 308
 Standard deviation, 369
 Standard form miscellaneous
 theorems, 48
 Statistic, 371
 Steepest descent, 20–24
 along edges, 24
 Steepest direction, 70, 100,
 103, 104, 124
 Steiglitz, K., 120
 Steinberg, D., 166
 Steinitz, E., 37
 Stieglitz, K., 201
 Stiemke's theorem for homogeneous
 systems, 53
 Stiemke, E., 59
 Stone, J., 361
 Stoyan, D., 361
 Strict complementary slackness, 56
 Strong Duality Theorem, 45
 first known proof, 59
 first rigorous proof, 59
 originator, 59
 Student's t -distribution, 375
 Support, 339
 Supporting hyperplane, 11
 Suurballe, J., 166
 System of distinct representatives,
 249

 Tapia, R., 143
 Tarjan, R., 244
 Thapa, M., 144
 Theorems of the alternatives, 43, 50,
 59
 Thomen, D., 264
 Todd, M., 121, 143, 144, 264
 Tomlin, J., 121, 140
 Tompkins, C., 119
 Transportation problem, 207–229
 basis triangularity, 210
 bounded partial sums of
 variables, 225
 classical, 207–213
 compact representatoin, 208
 mathematical statement, 208
 properties, 208
 rank, 208
 degeneracy, 216
 Fundamental Theorem, 211
 improved solution, 214
 initial solution, 213
 integer solution property, 212
 simplex multipliers and reduced
 costs, 214
 Triangularity Rule, 213
 Transshipment problem, 219
 reduction to classical form, 222
 reduction to classical form by
 transshipment, 222
 Triangular matrix, 210
 Triangularity Rule, 213
 Tsuchiya, T., 141, 142
 Tucker's example, 150
 Tucker's strict complementary
 slackness theorem, 56
 Tucker, A., 31, 59, 146, 150, 166,
 169, 315, 316

 Ülkücü, A., 316
 Unbiased estimator, 371
 Unbiased sample variance, 372
 Unboundedness Theorem, 47
 Uncertain costs, 326
 Uncertain demand, 329
 convex costs, 331
 Unimodular, 212
 total, 212
 Upper-blocked, 238
 Using t in S , 93

 van de Panne, C., 202
 van der Vorst, H., 144
 Van Slyke, 332, 361
 Van Slyke, R., 264, 332, 360
 Vanderbei, R., 120, 141, 144–146
 Variable cost
 minimum expected cost, 326
 minimum variance, 327
 Variance, 368

- Variants of the Simplex Method, 173
Vavasis, S., 144
Vector, 1
Veiga, G., 120, 142
Vertex, 14, 15
Vial, J. P., 121
Vial, J.-Ph., 121
Ville's theorem of alternatives for
 matrices, 55
Ville, J., 59
von Neumann
 skew-symmetric, 43
von Neumann Algorithm, 73
 as a Variant of the Simplex
 Algorithm, 83–84
 Improving the Rate of
 convergence, 81
 steps, 81
von Neumann symmetric form, 43
von Neumann versus Karmarkar
 Algorithms, 118
von Neumann's method, 70–84
von Neumann, J., 58, 67, 70, 119, 145

Wallace, S., 332, 360, 361
Wang, Y., 142
Watteyne, P., 121
Weak Duality Theorem, 44
Weak law of large numbers
 (Chebyshev), 372
Wets, R., 332, 360, 361
Whinston, A., 202
Whiting, P., 244
Wiegman, N., 119
Willard, D., 244
Wittrock, R., 361
Wolfe's Generalized Program, 267
Wolfe's Rule, 156–158
Wolfe, P., 36, 146, 154, 166, 169, 267,
 315, 316
Wright, M., 31, 120, 140, 144, 146,
 166, 167
Wright, S., 143

Ye, Y., 120, 143, 144, 146
Ye, Yinyu, 40
Yoshise, A., 142, 143

Zadeh, N., 244
Zangwill, W., 146
Zhang, D., 143
Zhang, Y., 143
Zhu, J., 146
Ziemba, W., 361

Yamashita, H., 121