Appendix A

A Minimax Theorem for Zero-Sum Games

This appendix presents a general minimax theorem of the authors (Alpern and Gal, 1988), which establishes that every search game of the type considered in Book I has a value. Recalling equation (2.3), this means that if we consider all mixed search strategies s and all mixed hider strategies h, we have

$$\inf_s \sup_h c(s, h) = \sup_h \inf_s c(s, h) = v \text{ (value of game)}, \qquad \text{(A.1)}$$

where the cost function c is the expected capture time. Furthermore the inf on the left is a min, which means there is an optimal mixed strategy for the searcher. (The hider may have only ε-optimal strategies.) The original and most direct approach to this result is given in Gal (1980, app. 1); the result given here is an extension Gal's result and applies to a wider class of zero-sum games. Although it is customary in the literature to have the first player be the maximizer, we follow the approach taken in the main text, where the first player (searcher) is the minimizer.

We will show that a result of the type (A.1) holds for a wide class of zero-sum two-person normal-form games which includes the search games considered in Book I. A zero-sum two person normal form game is characterized by a cost function c (sometimes called a payoff function), which Player 1 (the searcher, in our games) wants to minimize and Player 2 (the hider, in our games) wants to maximize. The cost function c is initially given in terms of the pure strategies of the two players, $c : \mathcal{S} \times \mathcal{H} \to R$. Here $\mathcal{S}$ is the set of pure strategies for the minimizing Player 1 and $\mathcal{H}$ is the set of pure strategies of the maximizing Player 2. We use the letters $\mathcal{S}$ and $\mathcal{H}$ only to make the identification with search games clear, although this model applies to any game, and the strategy sets and payoff may be very different for other games.

Keeping this framework in mind, we first consider games in which the number of pure strategies is finite for each player; say, $\mathcal{S} = S_1, \ldots, S_m$ and $\mathcal{H} = H_1, \ldots, H_n$. (This would be the case, for example, for the search game with immobile hider on a tree, where H_j denote the leaves of the tree and the S_i denote the ways of searching all the leaves in a given order.) For notational simplicity, we denote the payoffs corresponding to pairs of pure strategies in the matrix notation

$$C_{ij} = c(S_i, H_j).$$

We may view the *i, j*-th entry C_{ij} of *the* $m \times n$ game matrix C as the amount the first player (minimizer) pays the second player (maximizer). It may happen that the matrix C will have a saddle point, i.e., an element $C_{\hat{\imath}\hat{\jmath}}$ such that

$$\max_{1 \le i \le m} C_{i\hat{\jmath}} = C_{\hat{\imath}\hat{\jmath}} = \min_{1 \le j \le n} C_{\hat{\imath}j}.$$

In this case, the game would be in a state of equilibrium if the first player chooses his $\hat{\imath}$-th pure strategy and the second player chooses his $\hat{\jmath}$-th pure strategy. The preceding strategies would be optimal, and thus this game could be solved using only pure strategies. Usually, however, such a saddle point does not exist because even in the simplest games (e.g., matching pennies, where $C = \left(\begin{smallmatrix} -1 & +1 \\ +1 & -1 \end{smallmatrix}\right)$, a player is at a disadvantage if he always uses the same pure strategy. The fact that usually there exists no optimal strategy in the set of pure strategies has led to the idea of using mixed strategies. Each player, instead of selecting a specific pure strategy, may choose an element from the set of his pure strategies according to a predetermined set of probabilities. Mixed strategies for the first and the second players will be denoted by $s = (x_1, x_2, \ldots, x_m)$ and $h = (y_1, y_2, \ldots, y_n)$, respectively, where $x_i, i = 1, \ldots, m$ is the probability that the first player will choose his *i*-th pure strategy S_i, and $y_j, j = 1 \ldots, n$ is the probability that the second player will choose his *j*-th pure strategy H_j.

When the first player plays a mixed strategy *s* and the second player a mixed strategy *h*, the expected cost is given by the function

$$c(s, h) = \sum_i \sum_j C_{ij} x_i y_j$$

The fundamental theorem of two-person zero-sum finite games is due to Von Neumann (see Von Neumann and Morgenstern, 1953). It states that

$$\min_s \max_h c(s, h) = \max_h \min_s c(s, h).$$

This minimax value of *c* is called the *value* of the game and denoted by *v*. An equivalent result is that there exists a pair of mixed strategies $\bar{s}$ and $\bar{h}$ such that

$$\min_s c(s, \bar{h}) = c(\bar{s}, \bar{h})(= v) = \max_h c(\bar{s}, h).$$

Thus, *c* has a saddle point, or in other words, if the first player chooses the mixed strategy $\bar{s}$ and the second player uses the mixed strategy $\bar{h}$ then each of them can guarantee an expected payoff of *v*. Thus, $(\bar{s}, \bar{h})$ is a pair of optimal strategies and the game has a solution in mixed strategies, (e.g., in the matching pennies game, $\bar{s} = (\frac{1}{2}, \frac{1}{2})$, $\bar{h} = (\frac{1}{2}, \frac{1}{2})$, and $v = 0$).

The situation is more complicated if the game has an infinite number of pure strategies. In this case, the mixed strategies are probability measures on the set of pure strategies. Such a game is said to have a value v if, for any positive ε, the first player has a (mixed) strategy s_ε that limits him to an expected loss of at most $(1+\varepsilon)v$ and the second player has a (mixed) strategy h_ε that guarantees him an expected payoff of at least $(1-\varepsilon)v$. If one of the players has an infinite number of pure strategies while the other player has only a finite number of pure strategies, then the game has a value. However, if both players have an infinite number of pure strategies, then the existence of a value is not assured. (For details see Luce and Raiffa, 1957, app. 7.)

In the search games considered in the main text, both players have an infinite number of pure strategies. Nevertheless, Gal (1980, app. 1) has proved that any search game has a value. Using the more general formulation of Alpern and Gal (1988) the minimax theorem can be stated as follows.

Theorem A.1 *Let X be a compact Hausdorff space and (Y, A) a measurable space. Let $f : X \times Y$ be a measurable function that is bounded below and lower semicontinuous on X for all fixed y in Y. Let M be any convex set of probability measures (mixed strategies) on (Y, A) and B(X) the regular probability measures on X. Then*

$$\min_{\beta\in B(X)} \sup_{\gamma\in M} \iint f(x,y)\,d\beta\,d\gamma = \sup_{\gamma\in M} \min_{\beta\in B(X)} \iint f(x,y)\,d\beta\,d\gamma.$$

For our search trajectories we use the topology of uniform convergence for any finite interval. Since any $S \in \mathcal{S}$ is Lipshitz (with constant 1) it follows from the Ascoli theorem that $\mathcal{S}$ is compact. Under that topology $\mathcal{S}$ is also Hausdorff (two distinct trajectories always have disjoint neighborhoods). Since the capture time $C(S, H)$ can only "jump" down, it easily follows that it is *lower semicontinuous* $(C(\lim) \le \lim C(\cdot))$ in each of its arguments (see Gal, 1980). Thus, we can use the general minimax theorem A.1 and obtain

$$\min_s \sup_h \int C(S,H)\,d(s\times h) = \sup_h \min_s \int C(S,H)\,d(s\times h)$$

so that the search games considered in our book always have a value and an optimal search strategy. Note that the lower semicontinuity implies that $C(S, H)$ is Borel measurable, in both arguments. Thus, the above integral

$$\int C(S,H)\,d(s\times h)(\equiv c(s,h))$$

is well defined.

Appendix B

Theory of Alternating Search

In some cases, particularly for the undirected circle and line, the asymmetric rendezvous problem can be reduced to a problem in which two searchers act as a team to locate a stationary hidden object. The object is placed in one of two disjoint regions, with a searcher in each. The searchers can only move one at a time (hence the term "alternating"), and each has a maximum speed of 2. To make the space of search strategies closed, we also allow the limiting case in which the searchers can move simultaneously with a combined speed of 2.

In this appendix we review the relevant results of Alpern and Howard (2000) on the theory of *alternating search at two locations.* We will need only the special case of this theory in which each search region is a ray (a copy of $[0, \infty)$) and each searcher starts at the end (labeled 0). We will also assume that the object is equally likely to be on either ray.

Let $F_i(t), i = 1, 2,$ denote the probability that the object is placed in the interval [0, *t]* on ray *i,* given that it is somewhere on ray *i*. Thus F_1 and F_2 are probability distributions. The alternation of the two searchers may be described by a rule α that determines when each of the searchers is moving (at speed 2). In this interpretation, we may let $\alpha(t)$ denote twice the total time up to t that Searcher 1 has been moving. In this case, Searcher 1 will have covered the interval $[0, \alpha(t)]$ on ray 1, while Searcher 2 will have covered the interval $[0, 2t - \alpha(t)]$ on ray 2. Hence the object will have been found by time t with probability

$$F_\alpha(t) = \tfrac{1}{2}[F_1(\alpha(t)) + F_2(2t - \alpha(t))].$$

The expected time to find the object will be $\int_0^\infty t \, dF_\alpha(t)$, and the least expected time is denoted by

$$v(F_1, F_2) = \min_\alpha \int_0^\infty t \, dF_\alpha(t).$$

An α for which the minimum is attained is called an *optimal alternation rule.* In order to justify the *existence* of the minimum, we need to consider a wider class of alternation rules α. The rule described above, with intervals of alternating motion of the searchers, could be described by a continuous piecewise linear function $\alpha : [0, \infty) \to [0, \infty)$, which has intervals of slope 2 (when Searcher 1 moves) and intervals of slope 0 (when Searcher 2 moves). As these intervals get smaller and smaller, the motions of the two searchers become, in the limit, simultaneous motions. This is easily formalized as follows. We define an alternation rule α to be any increasing function with maximum slope (Lipshitz constant) 2, with $\alpha(0) = 0$. The set of all such alternation rules $\mathcal{A}$ is compact with respect to the topology of uniform convergence on compact intervals. Since the integral $\int_0^\infty t \; dF_\alpha(t)$ is lower semicontinuous in α with respect to this topology, the existence of the minimum is established. An alternation rule α has a derivative almost everywhere. We interpret the position of searcher 1 to be $\alpha(t)$ and that of searcher 2 to be $2t - \alpha(t)$. Their speeds are $\alpha'(t)$ and $2 - \alpha'(t)$, which sum to 2.

In this presentation of the problem, the distributions F_1 and F_2 are given. This will be the case when the two regions are each rays, and the starting points are the ends of the rays. However, in general (e.g., alternating search on two circles) there may be *many* ways of searching a given region (say region 1), and each way will determining a *different* distribution F_1. We note that in the general case it is not necessarily true that $v(F_1, F_2)$ is minimized by taking two distributions F_1^* and F_2^* that individually minimize the expected time to find the object if it is certain to be in that region. However, we can say the following.

Lemma B.1 *Suppose that $\hat{F}_1$ dominates F_1 in the sense that it is at least as large for all t. Then for any distribution F_2, we have*

$$v(\hat{F}_1, F_2) \leq v(F_1, F_2).$$

This inequality holds simply because we can write the moment $\int_0^\infty t \, dF_\alpha(t)$ as $\int_0^\infty (1 - F_\alpha(t))\, dt$, and the latter will not increase when F_1 is replaced by $\hat{F}_1$. The importance of this observation is that if there is a strategy (like the Columbus strategy in Chapter 14) or a family of search strategies for a given region that dominates any strategy, we may assume that a strategy of this type is used.

We now list some results obtained in Alpern and Howard (2000) on optimal alternation rules α corresponding to various assumptions on the distributions F_1 and F_2.

Theorem B.2 *Suppose an optimal alternation rule searches in the two rays alternately in consecutive time intervals. Then the interval on line i for which F_i has the higher average density is searched first.*

Theorem B.3 *If one of the distributions F_i is convex on an interval, then there is an optimal alternation rule for which this interval (on line i) is traversed at maximum speed without interruption.*

Theorem B.4 *If one of the distributions F_i is constant on an interval (which has density zero and cannot contain the object), then* **any** *optimal alternation rule traverses this interval (on line i) at maximum speed without interruption.*

For the next result we assume that $F_1 = F_2$ and we denote by $\bar{F}$ the *concavification* of F, that is, the smallest concave function satisfying $\bar{F}(x) \geq F(x)$ for all x. The following presents a complete characterization of the optimal solution in terms of the F_i and their concavifications.

Theorem B.5 *Suppose the object has a common conditional distribution F on both rays.*

1. *If $\bar{F}(x) > F(x)$ for all $a < x < b$, $\bar{F}(a) = F(a)$, $\bar{F}(b) = F(b)$, then there is an optimal alternation rule α which satisfies either $\alpha' = 2$ on $(a, (a+b)/2)$ and $\alpha' = 0$ on $((a+b)/2, b)$, or the reverse. That is, the intervals $[a, b]$ on the two rays are searched consecutively at speed 2.*
2. *If F is concave then the constant alternation rule $\alpha' = 1$, which searches both rays in parallel, is optimal. If it is strictly concave, then this strategy is uniquely optimal.*

This work may be thought of as an extension of the Gittens Index to continuous time and general distribution. See Gittens (1989).

B.1 Arbitrary Regions

The analysis given above for alternating search on two rays can be useful in solving the more general problem of searching two arbitrary regions Q_1 and Q_2. In the general problem, there is a searcher starting at a point $q_i \in Q_i, i = 1, 2$. The two searchers move subject to a maximum combined speed of 2. If searcher i moves with maximum speed all the time, there are many possible cumulative distribution functions F_i of capture time given that the hidden object lies in Q_i. The minimum time required for the two searchers to find the object is given by

$$\min_{F_1, F_2} v(F_1, F_2).$$

Appendix C

Rendezvous-Evasion Problems

The situations described in Books I and II are either search-evasion or rendezvous search. The novel element that was introduced in the article (Alpern and Gal, 2002) on which this appendix is based, is an uncertainty regarding the motives of the lost agent: he may be a mobile or immobile hider (evader) as in Book I, or he may be a cooperating rendezvouser who shares the same aim as the searcher, as in Book II. We assume that the probability p of cooperation is known to the searcher and to the agent. In any given search context (search space and player motions) G, we obtain a continuous family $\Gamma_G(p)$ of search problems, $0 \leq p \leq 1$, where $\Gamma_G(0)$ is a search game with mobile or immobile hider and $\Gamma_G(1)$ is an asymmetric rendezvous problem. We obtain a unique continuous value function $V_G(p)$ that gives the least expected value of T for the given (cooperation) probability p that the target wants to be found. As we shall show, this uncertainty regarding *a priori* agent motives affects both the paths (strategies) chosen by the searcher and the paths chosen by the (cooperating or evading) agent. One might say that here we introduce the game theoretic notion of incomplete information into the theory of search. Our formalization of the cooperative portion of the problem will be that of asymmetric rendezvous search, where the searcher and agent may agree on the roles (paths) they will take in the event that the agent gets lost and wants to be found. For example a mother (the searcher) may tell her child (the agent) what to do in this event, knowing, however, that this instruction may be disregarded if the child does not want to be found. The child will follow these (rendezvous) instructions if he wants to be found and may use the knowledge of these instructions in deciding on an evasion strategy if he does not.

This uncertainty as to the agent's aims is common, for example, when a teenager is reported missing to the police by the parents. In such cases the police usually ascribe a probability ($1 - p$ in our notation) that the teenage is not lost or abducted but rather a runaway who does not want to be found. Parents usually complain that the police overestimate this probability, and indeed this is a subject of some controversy, as in the following passage from Clancy (1999):

> Bannister had gone to a local police station to make a report in person . . . From a police detective . . . he'd heard "Look, its only been a few

> weeks . . . She's probably alive and healthy somewhere, and ninety nine out of a hundred of these cases turn out to be a girl who just wanted to spread her wings [an evader]." Not his Mary, Bannister had replied.

Another example of such incomplete information search is found in the novel *Hunt for Red October* (Clancy, 1995) where a Russian submarine of that name becomes lost to Soviet command. The difficulties faced by the Russian search effort are exactly those formalized in this article, as they are uncertain whether the sub is indeed lost (their first assumption) or is defecting to the West (as they gradually come to believe). The current SETI (Search for Extraterrestrial Intelligence) Project is based on an implicit assumption regarding not only the existence of EI but of a sufficiently large value *of* p. In the SETI context, T would be the first time when transmitter and listener are on the same frequency and the message is understood to be nonrandom. Search and Rescue operations (seeking lost hikers, for example) also make judgements about p in determining where to look first.

It would be natural for the searcher to behave in the following way: If p is relatively high, then he assumes a cooperative agent and hence he first goes to the agreed meeting point, switching into a search-evasion mode if the agent is not found at this location. If p is small, then the searcher assumes an evading agent right from the start and acts accordingly. This situation is characterized by a threshold value for p that separates between the above two strategy types. For some cases this natural strategy is indeed optimal. For example:

Immobile agent on a tree This can represent the following search problem. A mother drives a teenager son to a roadside drop-off point O. From there he will hike in a large park. If it rains, he will go to one of a group of huts (none of which are *at* O), where he will wait, while the mother drives back to O and, covered with proper raingear, begins a search. The Mother tells the child which hut he should go to if it rains, knowing full well that if he is enjoying himself at that time he will disregard her instructions. The huts and the drop-off point O are all connected by a network of paths that forms a tree.

In this model the search domain will be a tree Q with a distinguished node called O where the searcher must start his search. (In keeping with the search and rendezvous literature, we will refer to the agents as "he" even when they adopt the "wait for mummy" strategy, or when the motivating example includes females.) Each edge of Q has a certain length, and the sum of these is called the length of Q, denoted μ-(Q)- For simplicity we will normalize the length of the given tree so that $\mu = 1$. The agent in this problem is immobile; he simply picks a node of the tree other than O and stays there. The searcher moves along the tree at unit speed until he reaches the node chosen by the agent, aiming to minimize the expected time $\hat{T}$ to reach this node. With probability p, the agent is a cooperator (or rendezvouser) who also wishes to minimize $\hat{T}$; and with probability $1 - p$ the agent is an evader who wishes to aximize $\hat{T}$.Simple domination arguments are sufficient to show that a cooperator will always chose a node adjacent to the starting node O, while the evader will always chose a terminal node of Q. We will denote by $v(Q,\ O, p)$ the minimal expected time for this specific problem.

Denote by $h^*(q, o)$ an optimal hiding distribution for a tree q with root o (see Section 3.3). We have the following theorem.

Theorem C.1 *Let Q be a unit-length tree with searcher starting node O. Let X be any node at minimum distance* d_X *to O and let Z be any node that determines a subtree* Q_Z *of maximum total length* $\mu(Q_Z) = w_Z$. *Denote* $Q \backslash Q_Z$ *by Q–z. Then the value v(Q, O, p) of the rendezvous-evasion problem on (Q, O) with cooperation probability p is given by*

$$v(Q, O, p) = \begin{cases} 1 - pw_Z, & \text{if } p \le \dfrac{d_X}{1 - w_Z}, \\ 1 - p + d_X, & \text{if } p \ge \dfrac{d_X}{1 - w_Z}. \end{cases}$$

For $p \le d_X/(1 - w_Z)$, *Z is an optimal rendezvous strategy for the cooperator, a traversal of a minimal tour of Q from O (equiprobably in either direction) is optimal for the searcher, and an optimal strategy for the evader is to adopt h*(Q–z, O) with probability* $\mu(Q_{-Z})/(1 - p)$ *and h*(Qz, O) with the complementary probability. For* $p \ge d_X/(1 - w_Z)$, *X is an optimal rendezvous strategy for the cooperator, a move to X followed by a minimal tour of Q from X (equiprobably in either direction) is optimal of the searcher, and the strategy h*(Q, X) is optimal for the evader.*

However, this situation, of the above (p) threshold type strategy being optimal, does not always hold.

Rendezvous-evasion in two cells The searcher and agent are initially placed at time $t = 0$ into distinct cells, which we (and the players), respectively, call 1 and 2. At each time $t = 1, 2, 3, \ldots$, searcher and agent locate in cell x_t and y_t, and meet at the first time when they are in the same cell. We may assume (by relabeling in each period) that the cooperator strategy is (2, 2, …). For p close to 1, the optimal searcher strategy is (2, *b, b,* ...) and the optimal evader strategy is (1, *b, b,* ...), where *b* denote the Bernoulli strategy of picking either location equiprobably and independently of previous choices. For p near 0 the optimal strategy for both searcher and evader is *(b, b, b, ...)*. These strategy pairs give respective expected meeting times of $p(1) + (1 - p)(1 + 2) = 3 - 2p$ and 2, which give the same time 2 when $p = 1/2$. However in this intermediate case $(p = 1/2)$ both searcher strategies are dominated by the mixed strategy 0.9(2, *b, b,* …) + 0.1(1, 2, *b, b,* ...). Since a best response of the evader is (1, *b, b,* ...), this mixed searcher strategy ensures that the expected meeting time is no more than $(0.9)(3 - 2p) + (0.1)(2p + (1 - p)) = 1.95 < 2$, for $p = 1/2$. This analysis shows that for intermediate values of p the optimal searcher strategy is not optimal for either extreme case ($p = 0$ or 1). The determination of optimal strategies for all p, and the corresponding optimal expected meeting time $v(p)$, seems an interesting problem.

Alpern and Gal (2002) also analyze mobile agent on the line (the agent is assumed to have a smaller speed than the searcher) and suggest additional other interesting problems for further research:

Rendezvous-evasion in *n* cells The problem is the same as for two cells, except that for $n > 2$ the two players can never achieve a common labeling of the cells. For $p = 1$ the cooperator stays still while the searcher picks a random permutation of the remaining cells. For $p = 0$ both searcher and evader should move randomly. What is the solution for general p? See Anderson and Weber (1990) for a discussion of a related problem.

Mobile rendezvous-evasion on the circle The searcher and agent are placed randomly (uniform probability density) on a circle of unit circumference and can both move with unit speed. For the search game $p = 0$ it has been shown Alpern (1974), and Zelikin (1972), that the optimal search strategy is "cohatu." This is short for "coin half tour": at times $i/2$, $i = 0, 1, 2, \ldots$, go half way around the circle equiprobably in either direction. All evader responses give the same expected meeting time of $v(0) = 3/4$. For the rendezvous problem $p = 1$, posed in Alpern (1995), the optimal strategy is for the searcher to move clockwise while the cooperator moves counterclockwise (for a proof see Section 14.2), with expected meeting time $v(1) = 1/4$. For p near to 1 the searcher should use "clockwise-cohatu": this means clockwise for $t \leq 1/2$ (by which time he will have met the cooperator) and then use cohatu. Any evader strategy that goes clockwise for $t \leq 1/2$ is an optimal response. Thus this strategy has an expected meeting time of

$$p\left(\frac{1}{4}\right) + (1-p)\left(\frac{1}{2} + \frac{3}{4}\right) = \frac{5-4p}{4}.$$

The two strategies, cohatu and clockwise-cohatu, give the same expected time of 3/4 when $p = 1/2$. However, neither is optimal in this case ($p = 1/2$), since both are dominated by a mixture of 0.9 (clockwise-cohatu) and 0.1 (counterclockwise–clockwise-cohatu). The latter strategy goes in the indicated directions for times (0, 1/2) and (1/2, 1) and then in random directions. Against this mixture, an optimal evader must go clockwise for the first half time unit. Thus, if clockwise-cohatu is used, then the expected meeting time is the same (3/4) as for either of the two searcher strategies already calculated. However, in the case that counterclockwise–clockwise-cohatu is used, the searcher does better: The cooperator will be found in the same expected time $1/2 + v(1) = 3/4$ (no improvement), but the evader will be found in the smaller expected time 1/4. This analysis is similar to that for two cells (there are two directions on the circle), and as in that problem the solution for intermediate values of p would be interesting. This analysis assumes that the searcher and agent have a common notion of direction around the circle. The problem is also well defined (but distinct) without this assumption.

Immobile rendezvous-evasion on networks To what extent does the dichotomy of search strategies found for trees apply to other networks?

Two-sided ambiguity of aims In the analysis given in the article, only the aims of the agent are uncertain. How should the problem be analyzed if the agent has a probability p_A of wanting to minimize T (otherwise he wants to maximize) and the searcher has a similar probability p_S?

We believe that the introduction into search theory of models with uncertain target motives can present many interesting new problems into the area that may stimulate further research.

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