ANNEX

Al. Physical Quantities and Units

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>USUAL SYMBOL</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>$C$</td>
<td>Farad (F)</td>
</tr>
<tr>
<td>Charge</td>
<td>$Q$</td>
<td>Coulomb (C)</td>
</tr>
<tr>
<td>Conductance</td>
<td>$G$</td>
<td>Siemens ($S$) = $\Omega^{-1}$</td>
</tr>
<tr>
<td>Conductivity</td>
<td>$\sigma$</td>
<td>$S \text{ cm}^{-1} = \Omega^{-1} \text{ cm}^{-1}$</td>
</tr>
<tr>
<td>Current</td>
<td>$I$</td>
<td>Ampere (A)</td>
</tr>
<tr>
<td>Current density</td>
<td>$J$</td>
<td>$A \text{ cm}^{-2}$</td>
</tr>
<tr>
<td>Distance</td>
<td>$d, l, w, x, y, z$</td>
<td>Centimeter (cm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 micrometer ($\mu$m) = $10^{-4}$ cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 nanometer (nm) = $10^{-7}$ cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 angström (Å) = $10^{-8}$ cm</td>
</tr>
<tr>
<td>Electric field</td>
<td>$\mathcal{E} = -\nabla \Phi$</td>
<td>$V \text{ cm}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{E} = -d\Phi/dx \ (1 \text{ dimension})$</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>$E$</td>
<td>Joule (J)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Electron-volt ($1 \text{ eV} = 1.6\times10^{-19}$ J)</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>Potential</td>
<td>$V, \Phi$</td>
<td>Volt (V)</td>
</tr>
<tr>
<td>Resistance</td>
<td>$R, r$</td>
<td>Ohm ($\Omega$)</td>
</tr>
<tr>
<td>Resistivity</td>
<td>$\rho$</td>
<td>$\Omega \text{ cm}$</td>
</tr>
<tr>
<td>Temperature</td>
<td>$T$</td>
<td>Kelvin (K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0^\circ\text{C} = 273.15$ K</td>
</tr>
<tr>
<td>Time</td>
<td>$t$</td>
<td>Second (s)</td>
</tr>
</tbody>
</table>
## Annex

### A2. Physical Constants

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>MEANING</th>
<th>VALUE</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_g$ (GaAs)</td>
<td>GaAs bandgap energy</td>
<td>1.42</td>
<td>eV</td>
</tr>
<tr>
<td>$E_g$ (Ge)</td>
<td>Ge bandgap energy</td>
<td>0.67</td>
<td>eV</td>
</tr>
<tr>
<td>$E_g$ (Si)</td>
<td>Si bandgap energy</td>
<td>1.124</td>
<td>eV</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of vacuum</td>
<td>8.854×10^{-14}</td>
<td>F cm^{-1}</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>Planck constant</td>
<td>6.63×10^{-34}</td>
<td>J s</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>Reduced Planck constant</td>
<td>$\hbar/2\pi$</td>
<td>J s</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann constant</td>
<td>1.3805×10^{-23}</td>
<td>J K^{-1}</td>
</tr>
<tr>
<td>$\kappa$ (GaAs)</td>
<td>Dielectric constant of GaAs</td>
<td>13.1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\kappa$ (Ge)</td>
<td>Dielectric constant of Ge</td>
<td>16</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\kappa$ (Si)</td>
<td>Dielectric constant of Si</td>
<td>11.7</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\kappa$ (SiO₂)</td>
<td>Dielectric constant of SiO₂</td>
<td>3.9</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$kT/q$</td>
<td>Thermal voltage (at T=300K)</td>
<td>0.02586</td>
<td>V</td>
</tr>
<tr>
<td>$L$ (GaAs)</td>
<td>Lattice parameter (GaAs)</td>
<td>5.6533×10^{-8}</td>
<td>cm</td>
</tr>
<tr>
<td>$L$ (Ge)</td>
<td>Lattice parameter (Ge)</td>
<td>5.64613×10^{-8}</td>
<td>cm</td>
</tr>
<tr>
<td>$L$ (Si)</td>
<td>Lattice parameter (Si)</td>
<td>5.43095×10^{-8}</td>
<td>cm</td>
</tr>
<tr>
<td>$\mu_n$ (GaAs)</td>
<td>Electron mobility (intrinsic GaAs)</td>
<td>8800</td>
<td>cm² V⁻¹ s⁻¹</td>
</tr>
<tr>
<td>$\mu_n$ (Ge)</td>
<td>Electron mobility (intrinsic Ge)</td>
<td>3900</td>
<td>cm² V⁻¹ s⁻¹</td>
</tr>
<tr>
<td>$\mu_n$ (Si)</td>
<td>Electron mobility (intrinsic Si)</td>
<td>1417</td>
<td>cm² V⁻¹ s⁻¹</td>
</tr>
<tr>
<td>$m_0$</td>
<td>Free electron mass</td>
<td>9.11×10⁻³¹</td>
<td>kg</td>
</tr>
<tr>
<td>$\mu_p$ (GaAs)</td>
<td>Hole mobility (intrinsic GaAs)</td>
<td>400</td>
<td>cm² V⁻¹ s⁻¹</td>
</tr>
<tr>
<td>$\mu_p$ (Ge)</td>
<td>Hole mobility (intrinsic Ge)</td>
<td>1900</td>
<td>cm² V⁻¹ s⁻¹</td>
</tr>
<tr>
<td>$\mu_p$ (Si)</td>
<td>Hole mobility (intrinsic Si)</td>
<td>471</td>
<td>cm² V⁻¹ s⁻¹</td>
</tr>
<tr>
<td>$N_c$ (GaAs)</td>
<td>Effective density of states in cond. band (GaAs)</td>
<td>4.7×10¹⁷</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$N_c$ (Ge)</td>
<td>Effective density of states in cond. band (Ge)</td>
<td>1.04×10¹⁹</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$N_c$ (Si)</td>
<td>Effective density of states in cond. band (Si)</td>
<td>2.8×10¹⁹</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$N_v$ (GaAs)</td>
<td>Effective density of states in valence band (GaAs)</td>
<td>7×10¹⁸</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$N_v$ (Ge)</td>
<td>Effective density of states in valence band (Ge)</td>
<td>6×10¹⁸</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$N_v$ (Si)</td>
<td>Effective density of states in valence band (Si)</td>
<td>1.04×10¹⁹</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$n_i$ (GaAs)</td>
<td>Intrinsic carrier concentration (GaAs)</td>
<td>1.1×10⁷</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$n_i$ (Ge)</td>
<td>Intrinsic carrier concentration (Ge)</td>
<td>2.5×10¹²</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$n_i$ (Si)</td>
<td>Intrinsic carrier concentration (Si)</td>
<td>1.45×10¹⁰</td>
<td>cm⁻³</td>
</tr>
<tr>
<td>$q$</td>
<td>Electron charge (absolute value)</td>
<td>1.6×10⁻¹⁹</td>
<td>C</td>
</tr>
</tbody>
</table>

All values are given for $T = 300K$. 
A3. Concepts of Quantum Mechanics

In this Annex the Reader is reminded of some concepts from quantum mechanics that will be used in this book.

1) A particle can be fully described by a function, called wave function. The wave function is noted $\Psi(x,y,z,t)$ and it contains all measurable information about the particle.

2) To each dynamic variable corresponds a quantum-mechanic operator:
   - To the position $x$ corresponds the operator $\hat{x} \equiv x$ (A3.1)
   - To momentum $p_x$ corresponds the operator $p_x \equiv \frac{\hbar}{j} \frac{\partial}{\partial x}$ (A3.2)
   - To the total energy $E$ corresponds the operator $\hat{E} \equiv -\frac{\hbar}{j} \frac{\partial}{\partial t}$ (A3.3)
   - To the potential energy $V(x,y,z)$ corresponds the operator $\hat{V} \equiv V(x,y,z)$ (A3.4)
   where $j = \sqrt{-1}$ and where $\hbar = h/2\pi$, $h$ being Planck's constant.

3) The wave function also gives the probability of finding the particle in a given region of space. If the wave function is real (i.e., not complex) the probability of finding the particle between positions $a$ and $b$ in one dimension ($x$) is given by:
   \[
   \text{probability} = \int_{a}^{b} \Psi^* \Psi \, dx \quad (= \int_{a}^{b} \Psi^2 \, dx \text{ if } \Psi \text{ is a real function})
   \]
   For all space in one dimension the particle must be somewhere between $x = -\infty$ and $x = +\infty$ and therefore, we obtain the normalization condition:
   \[
   \int_{-\infty}^{+\infty} \Psi^* \Psi \, dx = 1 \quad (\int_{-\infty}^{+\infty} \Psi^2 \, dx = 1 \text{ if } \Psi \text{ is a real function}) \quad (A3.5)
   \]
   Consider the total energy of a particle in a classical Newtonian physics approach. If the particle has a momentum $p$ and a potential energy $V$, its total energy is given by:
   \[
   E = \frac{p^2}{2m} + V \quad (A3.6)
   \]
Annex

Note that $p = p(x,y,z)$, $p^2 = p_x^2 + p_y^2 + p_z^2$ and $V = V(x,y,z)$.

Applying these concepts to an electron having a mass $m$ for the one-dimensional case one obtains Table A.1:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Classical mechanics</th>
<th>Quantum mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>$p = mv$</td>
<td>$\frac{\hbar}{j} \frac{d}{dx}$</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>$\frac{p^2}{2m}$</td>
<td>$\frac{1}{2m} \frac{\hbar}{j} \frac{d}{dx} \left( \frac{\hbar}{j} \frac{d}{dx} \right) = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$</td>
</tr>
<tr>
<td>Potential energy</td>
<td>$V$</td>
<td>$\hat{V}$</td>
</tr>
<tr>
<td>Total energy</td>
<td>$E = \frac{p^2}{2m} + V$</td>
<td>$-\frac{\hbar}{j} \frac{\partial}{\partial t}$</td>
</tr>
<tr>
<td>Mass</td>
<td>$m = \frac{1}{d^2E/dp^2}$</td>
<td>$m = \frac{\hbar^2}{d^2E/dk^2}$</td>
</tr>
<tr>
<td>Velocity,</td>
<td>$v = \frac{dE}{dp}$</td>
<td>$V_k = \frac{1}{\hbar} \frac{dE}{dk}$</td>
</tr>
<tr>
<td>group velocity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this Table, $k$ is a wave vector or a wave number that corresponds to the momentum of the particle.

The Schrödinger equation is basically the quantum mechanical equivalent of classical mechanics $E = \frac{p^2}{2m} + V$. For the one-dimensional case the quantum mechanical equivalent of total energy is:

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi}{\partial x^2} + V(x,t)\Psi = -\frac{\hbar}{j} \frac{\partial \Psi}{\partial t} \tag{A3.7}$$

and, in three dimensions:

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + V(x,y,z,t)\Psi = -\frac{\hbar}{j} \frac{\partial \Psi}{\partial t} \tag{A3.8}$$

where $\nabla^2$ is the Laplacian operator defined by:

$$\nabla^2 \Psi(x,y,z,t) = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}$$
If the potential energy function is time independent \( \frac{\partial V}{\partial t} = 0 \) one is able to construct a solution to the Schrödinger equation through the technique of separation of variables where the wave function is written as the product of a time-independent term, \( \psi(x,y,z) \) and a space-independent term, \( T(t) \), such that \( \Psi(x,y,z,t) = \psi(x,y,z) \ T(t) \). The introduction of these terms into (A3.8) yields:

\[
T(t) \left( -\frac{\hbar^2}{2m} \nabla^2 \psi(x,y,z) \right) + V(x,y,z) \ \psi(x,y,z) \ T(t) = \psi(x,y,z) \left( -\frac{\hbar}{j} \frac{\partial T(t)}{\partial t} \right)
\]

or

\[
\frac{1}{\psi(x,y,z)} \left( -\frac{\hbar^2}{2m} \nabla^2 \psi(x,y,z) + V(x,y,z) \ \psi(x,y,z) \right) = \frac{1}{T(t)} \left( -\frac{\hbar}{j} \frac{\partial T(t)}{\partial t} \right) \quad (A3.9)
\]

The left-hand term of this equation depends only on space, while the right-hand term depends only on time, which indicates that the separation of \( \Psi \) into the product of \( \psi \) and \( T \) was successful. We can now solve the Schrödinger equation for the variables \( \psi \) and \( T \) separately, and with this solution find \( \Psi = \psi T \). Equation A3.9 makes sense only if both terms are equal to a constant which we shall call \( E \), therefore, we can write:

\[
E \ T(t) = -\frac{\hbar}{j} \frac{\partial T(t)}{\partial t} \quad \Rightarrow \quad T(t) = \exp \left( -\frac{iEt}{\hbar} \right) \quad (A3.10)
\]

and therefore:

\[
\Psi(x,y,z,t) = \psi(x,y,z) \ \exp \left( -\frac{iEt}{\hbar} \right) \quad (A3.11)
\]

Introducing Expression A3.11 into A3.8 one obtains the time-independent Schrödinger equation:

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi(x,y,z) + [V(x,y,z) - E] \ \psi(x,y,z) = 0 \quad (A3.12)
\]

where \( E \) is the (constant) energy of the particle, where the energy of the particle is given by:

\[
-\frac{\hbar}{j} \frac{\partial \psi(x,y,z,t)}{\partial t} = \psi(x,y,z) \left( -\frac{\hbar}{j} \frac{\partial T(t)}{\partial t} \right) = \psi(x,y,z) \ E \ T(t) = E \ \Psi(x,y,z,t)
\]
A4. Crystallography – Reciprocal Space

Most semiconductors are crystalline materials. Elemental semiconductor atoms such as silicon or germanium belong to column IV of the periodic table and have four electrons on their outer shell. In a crystal these atoms form four covalent bonds with neighboring atoms in order to complete their outer shell. Each atom is thus in the center of a tetrahedron, the corners of which are occupied by other similar atoms (Figure A.1).

![Silicon atom forming covalent bonds to other silicon atoms.](image)

The atoms in a crystal form a pattern that is repeated in the three directions of space with perfect regularity. That pattern is called the "unit cell". Silicon and germanium have the diamond lattice structure. This structure can be viewed as two interweaving face-centered lattices. In this case the unit cell is a cube (Figure A.2). The length of each cube side is called the "lattice parameter", which is equal to 5.43 and 5.64 Å in silicon and germanium, respectively.

In the unit cell presented in Figure A.2 atoms labeled "1" are completely enclosed in the unit cell. Atoms at the center of each of the six sides of the cell and labeled "1/2" belong half to the unit cell and half to an adjacent cell. Atoms located at the corners of the cube and labeled "1/8" have one-eighth of their volume included in the unit cell and contribute to seven other cells. Therefore, the unit cell contains $4 \times 1 + 6 \times 1/2 + 8 \times 1/8 = 8$ atoms. Semiconductors formed using elements from columns III and V of the periodic table, such as gallium arsenide (GaAs), have the zincblende crystal structure. The GaAs lattice cell can be viewed as two interpenetrating face-centered lattices, one containing gallium atoms, and the other containing arsenic atoms. It is also represented by Figure A.2 where atoms labeled "1" are gallium and atoms labeled "1/2" and "1/8" are arsenic (and vice-versa). The lattice parameter of GaAs is 5.65 Å.
The most basic property of a crystal is that the same pattern of atoms is repeated over and over again in the three directions of space. The position of any cell in the crystal is given by a vector $\mathbf{l}$ defined by:

$$
\mathbf{l} = m\mathbf{a} + n\mathbf{b} + p\mathbf{c}
$$

where $m$, $n$ and $p$ are integer numbers, and $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the vectors of the lattice parameters of the unit crystal cell (Figure A.3). In most semiconductors the cell is cubic and $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$ have the same length.

**Figure A.2:** Atoms in the unit cell of silicon (diamond lattice structure).

**Figure A.3:** Unit cell of a cubic crystal lattice.
One can define three new vectors:

\[ a^* = 2\pi \frac{b \times c}{a \cdot b \times c} , \quad b^* = 2\pi \frac{c \times a}{a \cdot b \times c} , \quad c^* = 2\pi \frac{a \times b}{a \cdot b \times c} \]  

(A4.2)

Vectors \( a^* \), \( b^* \) and \( c^* \) belong to what is called the "reciprocal lattice". While vectors \( a \), \( b \) and \( c \) belong to real space and are measured in meters or centimeters, vectors \( a^* \), \( b^* \) and \( c^* \) belong to a space where the measurement unit is \( \text{meter}^{-1} \) or \( \text{centimeter}^{-1} \), which is called the "reciprocal space". Note that \( a \cdot a^* = b \cdot b^* = c \cdot c^* = 2\pi \) and \( a \cdot b^* = a \cdot c^* = b \cdot c^* = 0 \); \( a^* \) is thus parallel to \( a \) and perpendicular to \( b \) and \( c \), if there is such a thing as being parallel or perpendicular to a vector belonging to another space.

Figure A.4 represents vectors \( a^* \), \( b^* \) and \( c^* \). They are perpendicular to crystal planes (100), (010) and (001), respectively. Vectors perpendicular to planes (110) and (111) are represented as well. Any vector \( k \) in the reciprocal space obeys the following equation:

\[ k = fa^* + gb^* + hc^* \]  

(A4.3)

where \( f \), \( g \) and \( h \) are integer numbers.

**Figure A.4**: Main crystal planes of a semiconductor having a cubic lattice. Vectors \( a^* \), \( b^* \) and \( c^* \) belong to the reciprocal space and are represented here in the real-space unit cells for a visualization purpose only.
Problems

Problem A4.1:
a: Calculate the number of atoms in a cubic centimeter of silicon and germanium.

b: Calculate the number of atoms per square centimeter at the surface of an (100)-oriented silicon sample.

Problem A4.2:
Using Matlab place silicon atoms in the silicon unit cell in order to produce a 3D plot similar to Figure A.2. View it from different directions: random, (100), (110) and (111). The lattice parameter is 5.43 Å. Use commands [sx,sy,sz]=sphere(20) and surf1(sx,sy,sz) to draw the atoms. Use command line([X1 X2],[Y1 Y2],[Z1 Z2]) to plot the bonds between the atoms.

Problem A4.3:
Using Matlab place silicon atoms in 3x3x3=27 silicon unit cells in order to produce a 3D plot of the lattice. View it from different directions: random, (100), (110) and (111). The lattice parameter is 5.43 Å. Use commands [sx,sy,sz]=sphere(20) and surf1(sx,sy,sz) to draw the atoms. Use command line([XI X2],[Y1 Y2],[Z1 Z2]) to plot the bonds between the atoms.
A5. Getting Started with Matlab

◊ Matlab contains a powerful and user-friendly HELP function. For example:

```
help help
help graphics
help * or help +
```

will display a general help message, help on graphic functions, and help on operations such as multiplication and addition, respectively.

◊ Matlab is based on matrix operations. The following commands:

```
1  a = 1
2  b = a + a
```

will of course produce b=2 as a result, but internally both a and b are treated as 1 x 1 matrices, such that a = \[1\] and b = \[2\].

◊ Characters preceded by a percent sign (%) are treated as comments. Here is an example of commands:

```
1  clear % Clears all variables
2  A=[1 2;3 4] % Build a 2x2 matrix
3  B=A/A % Divide the A by itself
4  C=A*A % Multiply A itself
5  D=A .*A % Multiply the elements of A by themselves
6  E=A ./A % Divide the elements of A by themselves
7  a=1:2:12 % Generate a vector
8  b=a' % Transpose it
```

The resulting matrices and vectors are:

```
A = [1  2; 3  4]  B = [1  0; 0  1]  C = [7 10; 15 22]  D = [1  0; 9 16]  E = [1  1]

a = [1 3 5 7 9 11]  b =
```

Note the important difference between "*" and ".*" or "/" and "./"!
Using Matlab graphic results can be produced very easily. Here are some examples:

Plot sin(x) and cos(x)

1. clear %Clear all variables
2. X=0:0.1:2*pi; % x varies from 0 to $2\pi$ in steps of 0.1
3. SINE=sin(X);COSINE=cos(X);
4. plot(X,SINE,'-r',X,COSINE,'--b');
5. title('Sine and Cosine functions')

Note that x, sin(x) and cos(x) are vectors. There is no need for FOR or DO loops!

Plot a spiral

1. clear; clf % Clear all variables; clear figure
2. R=0:0.1:5*pi; % R varies from 0 to $2\pi$ in steps of 0.1
3. SINE=sin(R);COSINE=COS(R);
4. plot(SINE.*R,COSINE.*R,'-b')
5. axis square
6. title('Spiral')
Plot a two-dimensional "Mexican hat"

clear; clf;  % Clear all variables; clear figure

t=50;  % number of mesh points in each direction
A=zeros(t);  % build a 50x50 matrix array
for i=1:t;
    for j=1:t;
        r=sqrt(((i-t/2)/2)^2+((j-t/2)/2)^2);
        A(i,j)=sin(r)/r;
    end
A(t/2, t/2)=1; % center point of matrix is equal to 1
surf1(A)  % Plot the 2D graph
shading interp;
colormap(pink);
title ('"Mexican hat function" ')
Matlab can be used to conveniently solve many matrix problems. Here is a simple example. Consider the circuit below. We need to find the value of currents $I_1$ and $I_2$, as well as voltage $V_1$.

\[
\begin{align*}
100\,\Omega \, I_1 + 50\,\Omega \, (I_1 + I_2) &= 10 \, V \\
100\,\Omega \, I_2 + 50\,\Omega \, (I_1 + I_2) &= 10 \, V \\
100\,\Omega \, I_2 - V_1 &= 0 \, V
\end{align*}
\]

or, in a matrix form:

\[
\begin{bmatrix}
150 & 50 & 0 \\
50 & 150 & 0 \\
0 & 100 & -1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
V_1
\end{bmatrix}
= 
\begin{bmatrix}
10 \\
10 \\
0
\end{bmatrix}
\]

Using this simple program:

```matlab
A=[150 50 0;50 150 0;0 100 -1];
B=[10 10 0]';
IV=A\B
```

The solution is $IV = \begin{bmatrix} I_1 \\ I_2 \\ V_1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ 5.0 \end{bmatrix}$ from which we infer $I_1 = I_2 = 500$ mA and $V_1 = 5$ V.
Here are some Matlab functions that can be useful to solve some Problems from this Book:

**Concatenation and iterative equation solving:**

If \( A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \) then writing \( B = [A A A] \) yields:

\[
B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}
\]

The following example solves the equation \( x=\cos(x) \) iteratively and uses concatenation to plot the values of \( x \) at each iteration:

```matlab
clear
test=1;x=0;graph=[];
while test>le-4
    x2=cos(x);
test=abs(x2-x);
    graph=[graph x];
    x=x2;
end
('the solution is')
x
plot(graph)
xlabel('Iteration number');ylabel('X value');
```

![Graph of iteration number vs. X value](image)
If one tries to solve $x = 2\cos(x)$ using the iterative method described above, convergence will not be reached. Convergence can be improved by introducing a relaxation factor, $\alpha$, used during each evaluation of a new $x$ value. The value of $\alpha$ ranges between 0 and 1.

Instead of writing $x_2 = \cos(x)$
one can write $x_2 = x \cdot (\alpha - 1) + \alpha \cdot \cos(x)$

such that $x_2$ is some average value between the old $x$ value and the newly calculated value for $x$.

The program below uses the values 0.2, 0.4, 0.6 and 0.8 for $\alpha$. Convergence is obtained for the lower $\alpha$ values, but not for $\alpha = 0.8$. Not using a relaxation factor is equivalent to writing $\alpha = 1$, for which there is no convergence.

```matlab
1 clear;clf
2 graph2=[]
3 for alpha=0.2:0.2:.8
4     x=0;graph1=[];x=0;
5     for counter=1:12
6         x2=2*cos(x);
7         test=abs(x2-x);
8         graph1=[graph1 x];
9         x=x*(1-alpha)+alpha*x2;
10    end
11    graph2=[graph2 graph1];
12 end
13 plot(graph2,'-k')
14 xlabel('Iteration number');ylabel('X value');
```

17. Annex

Relaxation factor:
Diagonal matrices: The following program

```matlab
clear
t=6;
A=diag(ones(1,t),0)
B=diag(ones(1,t-1),1)
C=diag(ones(1,t-1),-1)
A=-2*A+B+C
A(1,1)=1;A(1,2)=0;A(t,t)=1;A(t,t-1)=0
```

yields:

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
1 & -2 & 1 & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & 0 & 0 \\
0 & 0 & 1 & -2 & 1 & 0 \\
0 & 0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

A similar matrix is used in problems based on a numerical (finite-differences) simulation technique.

Numerical integration and differentiation:

The following program integrates and differentiates \( y = x^2 \):

```matlab
dx=0.01;
x=-5:dx:5;
y=x.^2;
integral=sum(y)*dx %Definite integral (from x=-5 to x=5)
integral_curve=cumsum(y)*dx;%Integral curve
% derivative=diff(y)./diff(x);
% Since the differentiation of an n-element
% vector produces an (n-1)-element vector we add
% a dummy "Not a Number"(NaN) at the end of the
% derivative vector, such that it has the same
% length as the x-vector:
derivative=[derivative NaN];
plot(x,y,'-b',x,integral_curve,'--r',x,derivative,'--k')
text(-4,80,'BLUE: y=x^2')
text(-4,70,'RED: integral of y')
text(-4,60,'BLACK: dy(x)/dx')
```
Note 1: On some computers some versions of Matlab may give you frustrating problems if you use uppercase letters in file names. So, it is good practice to use file names such as "test.m" instead of "Test.m", for example. The Problems in this Book were designed using the Student Edition of Matlab, version 5.0 for Macintosh, and version 5.3 for PC.

Note 2: Some people may find the font size in Matlab plots too small for easy reading. Plot properties such as font size and line width can be modified using the following commands:

```matlab
set(0,'defaultaxesfontsize',14) sets the axes font size to 14
set(0,'defaulttextfontsize',14) sets the text font size to 14
set(0,'defaultlinelinewidth',14) sets the plot linewidth to 2
set(0,'defaultaxeslinewidth',14) sets the axes linewidth to 2
set(0,'defaultaxesfontname','Arial') sets the axes font name to Arial
set(0,'defaulttextfontname','Arial') sets the text font name to Arial
```
### A6. Greek alphabet

<table>
<thead>
<tr>
<th>LETTER</th>
<th>LOWERCASE</th>
<th>UPPERCASE</th>
</tr>
</thead>
<tbody>
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A7. Basic Differential Equations

In the examples below, A and B are given constants, and \( C_n (n=0,1,2,3,4) \) are integration constants. Integration constants can be numerically determined by applying boundary conditions to the general solution of the equation.

\[ \frac{dF(x)}{dx} + Ax + B = 0 \]

To solve:

\[ dF(x) = -(Ax + B) \, dx \quad \Rightarrow \quad \int dF(x) = - \int (Ax + B) \, dx \]

which yields the general solution: \( F(x) = -\frac{A}{2} x^2 - Bx + C_1 \)

\[ \frac{dF(x)}{dx} + A \, F(x) + B = 0 \]

To solve:

\[ dF(x) = -(A \, F(x) + B) \, dx \]

or:

\[ \frac{dF(x)}{AF(x) + B} = -dx \quad \Rightarrow \quad \frac{A \, dF(x)}{AF(x) + B} = -A \, dx \]

Noting that \( d(AF(x) + B) = A \, dF(x) \) and using a change of variables where \( AF(x) + B = y \) we can write:

\[ \frac{d(y)}{y} = -A \, dx \quad \Rightarrow \quad \int \frac{d(y)}{y} = -A \int dx \]

The integration results in: \( \ln(y) = \ln(AF(x) + B) = -Ax + C_0 \)

Therefore, the general solution is:

\[ F(x) = \frac{\exp(-Ax + C_0) - B}{A} \]

or, noting \( C_1 = \frac{\exp(C_0)}{A} \):

\[ F(x) = C_1 \exp(-Ax) - \frac{B}{A} \]
To solve:
\[ \frac{d^2 F(x)}{dx^2} = A \]

we integrate a first time to find:
\[ \frac{dF(x)}{dx} = Ax + C_1 \]

and then integrate a second time to obtain the general solution:
\[ F(x) = \frac{A}{2} x^2 + C_1 x + C_2 \]

To solve:
\[ \frac{d^2 F(x)}{dx^2} = A F(x) \text{ with } A > 0 \]

we must find a function that is equal to its second derivative, multiplied by a positive constant. The only function satisfying this condition is the exponential function, since:
\[
\frac{d^2(C_1 \exp(Bx))}{dx^2} = \frac{d}{dx} \frac{d(C_1 \exp(Bx))}{dx} = C_1 B \frac{d(\exp(Bx))}{dx} = C_1 B^2 \exp(Bx)
\]

and
\[
\frac{d^2(C_2 \exp(-Bx))}{dx^2} = \frac{d}{dx} \frac{d(C_2 \exp(-Bx))}{dx} = -C_2 B \frac{d(\exp(-Bx))}{dx} = C_2 B^2 \exp(-Bx)
\]

Comparing the initial differential equation and the possible solutions, we find that \( A = B^2 \). Therefore, the general solution is:
\[ F(x) = C_1 \exp(\sqrt{A} \ x) + C_2 \exp(-\sqrt{A} \ x) \]

Since \( \sinh(y) = \frac{\exp(y) - \exp(-y)}{2} \) and \( \cosh(y) = \frac{\exp(y) + \exp(-y)}{2} \), we can also write:
\[ F(x) = C_3 \sinh(\sqrt{A} \ x) + C_4 \cosh(\sqrt{A} \ x) \]
To solve: \( \frac{d^2 F(x)}{dx^2} = -A \ F(x) \) with \( A > 0 \)

we must find a function that is equal to its second derivative, multiplied by a negative constant. The only functions satisfying this condition are the sine and cosine functions since:

\[
\frac{d^2(C_1 \sin(Bx))}{dx^2} = \frac{d}{dx} \frac{d(C_1 \sin(Bx))}{dx} = C_1 B \frac{d(\cos(Bx))}{dx} = -C_1 B^2 \sin(Bx)
\]

and

\[
\frac{d^2(C_2 \cos(Bx))}{dx^2} = \frac{d}{dx} \frac{d(C_2 \cos(Bx))}{dx} = -C_2 B \frac{d(\sin(Cx))}{dx} = -C_2 B^2 \cos(Cx)
\]

Comparing the initial differential equation and the possible solutions, we find that \( A = B^2 \). Therefore, the general solution is:

\[
F(x) = C_1 \sin(\sqrt{A} \ x) + C_2 \cos(\sqrt{A} \ x)
\]

Using \( \cos(y) = \frac{e^{iy} + e^{-iy}}{2} \) and \( \sin(y) = \frac{e^{iy} - e^{-iy}}{2j} \) we can write:

\[
F(x) = \frac{C_1}{2} \left( \exp(j \sqrt{A} x) + \exp(-j \sqrt{A} x) \right) - \frac{jc_1}{2} \left( \exp(j \sqrt{A} x) - \exp(-j \sqrt{A} x) \right)
\]

or:

\[
F(x) = C_3 \exp(j \sqrt{A} \ x) + C_4 \exp(-j \sqrt{A} \ x)
\]
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