Notes and References

Introduction

1. The work of Farquharson (1957) and Moulin (1981) should also be mentioned here as important forerunners of the Moore and Repullo approach.

1 Economies with Public Goods

1. The usual convention in general equilibrium is just the opposite, that is, that private good outputs are positive and private good inputs are negative. Notice that this convention does not apply to public goods since they are outputs and positive (public goods used as inputs could be considered at the cost of some complications).

2 Resource Allocation Mechanism

1. An alternative interpretation of Nash equilibrium is that it is the limit of a dynamic process of adjustment in which the players (endowed with an incomplete knowledge of the game) vary their strategies by means of some adjustment rule which is more or less rational. Such an interpretation (which coincides with Cournot's presentation of his ideas on market equilibrium) is without a doubt more attractive than the one where complete information is assumed. However, unfortunately it is not easy to formalize if we assume that the players are minimally rational.

3 Dominant Strategies and Direct Mechanism

1. The Gibbard–Satterthwaite theorem does not assume that the social choice function must select Pareto efficient or individually rational allocations, but that its range must be of cardinality greater than 2.
2. Notice that the actual price paid by the highest bidder is the same in the three kind of auctions (English, Dutch and Vickrey's). Thus it is not clear what the practical advantage of the latter is. Moreover, the Vickrey auction has untruthful equilibria, i.e. the second bidder is indifferent among any announcement about her reservation price. This may be the reason of why, in practice, if a Vickrey auction is performed, objects are sometimes sold at very low prices.

4 Implementation in Nash Equilibrium (I): General Results

1. In Section 4.4 we will consider the case where the feasible set is variable.
2. As we noticed in the previous chapter, if the dimension of both the domain of economies and the allocation space were finite, it would be pos-

sible to replace the announcement of a profile of preferences and an allocation for the announcement of two natural numbers.

3. This assumption can be relaxed to $A_i(s_{-i})$ being star-shaped with center in $(w_i, 0)$, that is if $(x_i, y) \in A_i(s_{-i})$, then any $(x_i', y') = \alpha_i(x_i, y) + (1 - \alpha_i)(w_i, 0)$ for some $\alpha_i \in (0, 1)$ also belongs to $A_i(s_{-i})$.
4. In fact in Hurwicz's version such an assumption is made which allows the differentibility to be relaxed to continuity and strict quasiconcavity to quasiconcavity. Furthermore, Hurwicz assumes that ϕ is implementable.
5. The paper of Glazer and Ma in *Games and Economic Behavior*, 1989 and the survey of John Moore (1991) popularized the story among implementation theorist. The latter credits the idea to A. Rubinstein.

5 Implementation in Nash Equilibrium (II): Applications

1. If the space of economies includes economies with several Lindahl equilibria, no social welfare function can be implemented in Nash equilibrium since the Lindahl correspondence is contained in any Nash implementable social choice correspondence (see exercise 27 in the previous chapter).
2. The author has to confess that he has changed his mind on this matter over the years and that four years ago he was closer to the critics of Maskin-type mechanisms that he is now. See e.g. the introduction of Corchón and Wilkie (1990). I should add that when considering matters of strategy domination, I find the above criticisms much more compelling (see Jackson 1992)).
3. An analogy might be useful here. Only 1% of genetic information in human cells is actually used (see R. Dawkins, *The Blind Watchmaker*, p. 116, (Norton, New York). The other information may be there to be used just in case some unforeseeable event happens.
4. An Appendix to Chapter 6 studies the implementation of the Ratio and the Lindahl correspondences in strict Nash equilibrium by means of the same mechanisms explained in this chapter.
5. The Walker mechanism belongs to a class of mechanisms called 'Tweed Ring'. The story behind this name is that Tweed was the New York Mayor at the end of the previous century. The famous cartoonist Thomas Nest pictured him and his friends in a circle. Faced with the question 'Who stole the people's money?' each of them pointed their finger at the person next to him. (See the entry corresponding on Caricature and Cartoon in the *Encyclopaedia Britannica*.)
6. An alternative definition of monotonicity using MT( ) is the following. ϕ is monotonic if for all preference profiles u and u' and for all $a \in \phi(u)$, if $\forall\ i \in I$, $u_i' \in MT(a, u_i)$, then $a \in \phi(u')$.

6 Refining Nash Implementation

1. In Appendix I we study the problem of the control of the externalities. In Appendix II we look for sufficient conditions in economic environments of double implementation in strict Nash and Nash equilibria. The former is the strongest refinement of Nash equilibrium.

2. The purpose of this part of the mechanism is to ascertain if agent 1 is of type ϑ_1 or ϕ_1. Since her choice is final there is no point in misrepresenting her preferences.
3. The interpretation of this part of the mechanism is that since agent 2 falsely accused agent 1 of being of type ϕ_1, the false accusation yields her a penalty of Δt. If the accusation was warranted the confident gets Δt, enough to compensate her for any loss.
4. Abreu and Sen do not assume ε to be close to zero in their original proof.
5. This is the other important reason for A to be defined as a set of lotteries.
6. This asymmetry is specially worrying if the principal is just a surrogate of the agents.
7. In this respect additional important points are that the message space used by Abreu and Matsushima may be of a very large dimensionality, that it is not clear how to deal with environments containing 'classical' exchange economies (i.e. those with perfectly divisible goods) and that agents are assumed to have von Neumann-Morgerstern utility functions. Given the amount of criticism that these utility functions have had bear this latter assumption appears to be too strong (see e.g. Peter C. Fishburn *Non Linear Preferences and Utility Theory*, Johns Hopkins University Press, Baltimore, 1988).

7 Bayesian Implementation

1. This only means that each player has preferences defined over the outcomes of the game. It does not imply that her preferences are selfish or that they are representable by a complete and transitive preordering. In fact many results of game theory (including implementation) can be proved without these assumptions. However, they are widely used because they simplify considerably the analysis.
2. However it has been shown by Ledyard that by choosing posterior probabilities adequately and without our assumption on islands, a mechanism is Bayesian incentive compatible if and only if for any agent, to tell the truth about her own characteristic is a dominant strategy (see Exercise 7.4).

Index

Note: The phrase 'Implementation in Nash (or Bayesian) equilibria' has been abbreviated in subheadings to 'Nash (or Bayesian) implementation'.