
BRIEF COMMUNICATIONS

Cardinality of the Continuum of Closed Superclasses of Some Minimal Classes in the Partially Ordered Set \mathcal{L}_2^3

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Abstract—It is proved that the set of closed classes containing some minimal classes in the partially ordered set \mathcal{L}_2^3 of closed classes in the three-valued logic that can be homomorphically mapped onto the two-valued logic has the cardinality of continuum.

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All necessary definitions can be found in the introduction to [1]. It was also proved in [1] that the partially ordered set (further, p.o.s.) \mathcal{L}_2^3 of all closed classes of the three-valued logic \mathfrak{P}_3 that can be mapped to the two-valued logic \mathfrak{P}_2 contains only a finite number of minimal elements each of which has the basis composed of one function of two variables. All minimal classes in the p.o.s. \mathcal{L}_2^3 were described in [2]. There were 15 of them.

The countability of the number of closed superclasses of some minimal classes in the partially ordered set \mathcal{L}_2^3 was proved in [3].

In this paper we prove the following

Theorem. *The set of closed classes containing any of three minimal classes has the cardinality of continuum. One of these three classes is generated by the following function:*

$$\psi(x_1, x_2) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix}.$$

The other two classes are obtained by isomorphic mappings of this class generated by permutations of the cyclic group \mathbf{A}_3 of the third order generated by the permutation $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$.

Note that the matrix $\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix}$ defines the function $\varphi(x, y)$ by the relation $\varphi(x, y) = a_{xy}$.

Proof. It was proved in [4] that the set of closed classes of the three-valued logic containing the function of maximum for the partial order $\{(0,2), (1,2)\}$ has the cardinality of continuum. The structure of inclusion of these classes was also described in that paper. It is easy to check that the function $\psi(x_1, x_2)$ is obtained by superposition from this function and vice versa.

REFERENCES

1. A. V. Makarov, "The Homomorphisms of Functional Systems of Multi-Valued Logics," in: *Mathematical Problems of Cybernetics*, Issue 4, Ed. by S. V. Yablonskii (Nauka, Moscow, 1992), pp. 5–29.
2. A. V. Makarov, "The Description of all Minimal Classes in the Partly Ordered Set \mathcal{L}_2^3 of Closed Classes of the Three-Valued Logic that can be Homomorphically Mapped onto the Two-Valued Logic," *Vestn. Mosk. Univ., Matem. Mekhan.*, No. 1, 65 (2015) [*Moscow Univ. Math. Bull.* **70** (1), 48 (2015)].
3. A. V. Makarov and V. V. Makarov, "Countability of the Set of Closed Overclasses of Some Minimal Classes in the Partly Ordered Set \mathcal{L}_2^3 of All Closed Classes of Three-Valued Logic that Can be Mapped Homomorphically onto Two-Valued Logic," *Vestn. Mosk. Univ., Matem. Mekhan.*, No. 1, 62 (2017).
4. G. V. Bokov, "Lattice of Closed Classes of Three-Valued Logic Containing The Function of Maximum for a Nonlinear Partial Order," in: *Proc. XII Int. Workshop "Discrete Mathematics and its Applications" named after Acad. O. B. Lupanov* (Moscow State Univ., Moscow, 2016), pp. 187–190.

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