Quan *et al. Fixed Point Theory and Applications* 2012, **2012**:16
http://www.fixedpointtheoryandapplications.com/content/2012/1/16

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH | Open Access

Strong and weak convergence of an implicit iterative process for pseudocontractive semigroups in Banach space

Jing Quan[1*], Shih-sen Chang[2] and Min Liu[1]

* Correspondence: quanjingcq@163.com
[1]Department of Mathematics, Yibin University, Yibin, Sichuan 644000, China
Full list of author information is available at the end of the article

Abstract
The purpose of this article is to study the strong and weak convergence of implicit iterative sequence to a common fixed point for pseudocontractive semigroups in Banach spaces. The results presented in this article extend and improve the corresponding results of many authors.

1 Introduction and preliminaries

Throughout this article we assume that E is a real Banach space with norm $||\cdot||$, E^* is the dual space of E; $\langle\cdot,\cdot\rangle$ is the duality pairing between E and E^*; C is a nonempty closed convex subset of E; $\mathbb{N}$ denotes the natural number set; $\Re^+$ is the set of nonnegative real numbers; The mapping $J: E \to 2^{E^*}$ defined by

$$J(x) = \left\{ f^* \in E^* : \left\langle x, f^* \right\rangle = \|x\|^2; \left\| f^* \right\| = \|x\|, \quad x \in E \right\} \tag{1}$$

is called *the normalized duality mapping*. We denote a single valued normalized duality mapping by j.

Let T: $C \to C$ be a nonlinear mapping; $F(T)$ denotes the set of fixed points of mapping T, i.e., $F(T) := \{x \in C, x = Tx\}$. We use "$\to$" to stand for strong convergence and "$\rightharpoonup$" for weak convergence. For a given sequence $\{x_n\} \subset C$, let $\omega_w(x_n)$ denote the weak ω-limit set.

Recall that T is said to be *pseudocontractive* if for all $x, y \in C$, there exists $j(x - y) \in J(x - y)$ such that

$$\left\langle Tx - Ty, j(x - y) \right\rangle \le \left\| x - y \right\|^2; \tag{2}$$

T is said to be *strongly pseudocontr active* if there exists a constant $\alpha \in (0,1)$, such that for any $x, y \in C$, there exists $j(x - y) \in J(x - y)$

$$\left\langle Tx - Ty, j(x - y) \le \alpha \left\| x - y \right\|^2 \right. . \tag{3}$$

In recent years, many authors have focused on the studies about the existence and convergence of fixed points for the class of pseudocontractions. Especially in 1974, Deimling [1] proved the following existence theorem of fixed point for a continuous and strong pseudocontraction in a nonempty closed convex subset of Banach spaces.

© 2012 Quan et al; licensee Springer. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/2.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Theorem D. Let E be a Banach space, C be a nonempty closed convex subset of E and T: $C \to C$ be a continuous and strong pseudocontraction. Then T has a unique fixed point in C.

Recently, the problems of convergence of an implicit iterative algorithm to a common fixed point for a family of nonexpansive mappings or pseudocontractive mappings have been considered by several authors, see [2-5]. In 2001, Xu and Ori [2] firstly introduced an implicit iterative $x_n = \alpha_n x_{n-1} + (1 - \alpha_n)T_n x_n$, $n \in \mathbb{N}$, $x_0 \in C$ for a finite family of nonexpansive mappings $\{T_i\}_{i=1}^{N}$ and proved some weak convergence theorems to a common fixed point for a finite family of nonexpansive mappings in a Hilbert space. In 2004, Osilike [3] improved the results of Xu and Ori [2] from nonexpansive mappings to strict pseudocontractions in the framework of Hilbert spaces. In 2006, Chen et al. [4] extended the results of Osilike [3] to more general Banach spaces.

On the other hand, the convergence problems of semi-groups have been considered by many authors recently. Suzuki [6] considered the strong convergence to common fixed points of nonexpansive semigroups in Hilbert spaces. Xu [7] gave strong convergence theorem for contraction semigroups in Banach spaces. Chang et al. [8] proved the strong convergence theorem for nonexpansive semi-groups in Banach space. He also studied the weak convergence problems of the implicit iteration process for Lipschitzian pseudocontractive semi-groups in the general Banach spaces [9]. The pseudocontractive semi-groups is defined as follows.

Definition 1.1 *(1) One-parameter family* $\mathbf{T}: = \{T(t): t \geq 0\}$ *of mappings from C into itself is said to be a pseudo-contraction semigroup on C, if the following conditions are satisfied:*

(a). $T(0)x = x$ *for each* $x \in C$*;*
(b). $T(t + s)x = T(s)T(t)$ *for any* $t, s \in \Re^+$ *and* $x \in C$*;*
(c). For any $x \in C$*, the mapping* $t \to T(t)x$ *is continuous;*
(d). For all $x, y \in C$*, there exists* $j(x - y) \in J(x - y)$ *such that*

$$\left\langle T(t)x - T(t)y, j(x - y) \le \left\| x - y \right\|^2, \quad \text{for any } t > 0. \right. \tag{4}$$

(2) A pseudo-contraction semigroup of mappings from C into itself is said to be a Lipschitzian if the condition (a)-(d) and following condition (f) are satisfied.

(f) there exists a bounded measurable function L: $[0, \infty) \to [0, \infty)$ *such that for any* x*,* $y \in C$*,*

$$\left\| T(t)x - T(t)y \right\| \le L(t) \left\| x - y \right\|$$

for any $t > 0$*. In the sequel, we denote it by*

$$L = \sup_{t \ge 0} L(t) < \infty \tag{5}$$

Cho et al. [10] considered viscosity approximations with continuous strong pseudocontractions for a pseudocontraction semigroup and prove the following theorem.

Theorem Cho. Let E be a real uniformly convex Banach space with a uniformly Gâteaux differentiable norm, and C be a nonempty closed convex subset of E. Let $T(t)$: $t \geq 0$ be a strongly continuous L-Lipschitz semigroup of pseudocontractions on C such that $\Omega \neq \emptyset$, where Ω is the set of common fixed points of semi-group $T(t)$. Let f: $C \to$

C be a fixed bounded, continuous and strong pseudocontraction with the coefficient α in (0,1), let α_n and t_n be sequences of real numbers satisfying $\alpha_n \in (0, 1)$, $t_n > 0$, and $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \frac{\alpha_n}{t_n} = 0$; Let $\{x_n\}$ be a sequence generated in the following manner:

$$x_n = (1 - \alpha_n) f(x_n) + \alpha_n T(t_n) x_n, \quad \forall n \geq 1. \tag{6}$$

Assume that $LIM||T(t)x_n - T(t)x^*|| \leq ||x_n - x^*||$, $\forall x^* \in K$, $t \geq 0$, where $K := \{x^* \in C: \Phi(x^*) = \min_{x\in C} \Phi(x)\}$ with $\Phi(x) = LIM||x_n - x||^2$, $\forall x \in C$. Then x_n converges strongly to $x^* \in \Omega$ which solves the following variational inequality: $\langle (I - f)x^*, j(x^* - x)\rangle \leq 0$, $\forall x \in \Omega$.

Qin and Cho [11] established the theorems of weak convergence of an implicit iterative algorithm with errors for strongly continuous semigroups of Lipschitz pseudocontractions in the framework of real Banach spaces.

Theorem Q. Let E be a reflexive Banach space which satisfies Opial's condition and K a nonempty closed convex subset of E. Let $\mathcal{T} := \{T(t) : t \geq 0\}$ be a strongly continuous semigroup of Lipschitz pseudocontractions from K into itself with $\mathfrak{F} := \bigcap_{t\geq 0} F(T(t)) \neq \emptyset$; Assume that $sup_{t\geq 0}\{L(t)\} < \infty$, where $L(t)$ is the Lipschitz constant of the mapping $T(t)$. Let $\{x_n\}$ be a sequence generated by the following iterative process:

$$x_0 \in K; x_n = \alpha_n x_{n-1} + \beta_n T(t_n) x_n + \gamma_n u_n; \quad \forall n \geq 1; \tag{7}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ are sequences in (0,1), $\{t_n\}$ is a sequence in $(0, \infty)$ and $\{u_n\}$ is a bounded sequence in K. Assume that the following conditions are satisfied:

(*a*) $\alpha_n + \beta_n + \gamma_n = 1$;
(*b*) $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \frac{\alpha_n + \gamma_n}{t_n} = 0$.

Then the sequence $\{x_n\}$ generated in (7) converges weakly to a common fixed point of the semigroup $\mathcal{T} := \{T(t) : t \geq 0\}$;

Agarwal et al. [12] studied strongly continuous semigroups of Lipschitz pseudocontractions and proved the strong convergence theorems of fixed points in an arbitrary Banach space based on an implicit iterative algorithm.

Theorem A. Let E be an arbitrary Banach space and K a nonempty closed convex subset of E. Let $\mathcal{T} := \{T(t) : t \geq 0\}$ be a strongly continuous semigroup of Lipschitz pseudocontractions from K into itself with $\mathfrak{F} := \bigcap_{t\geq 0} F(T(t)) \neq \emptyset$. Assume that $\sup_{t\geq 0}\{L(t)\} < \infty$, where $L(t)$ is the Lipschitz constant of the mapping $T(t)$. Let $\{x_n\}$ be a sequence in

$$x_0 \in K; x_n = \alpha_n x_{n-1} + \beta_n T(t_n) x_n + \gamma_n u_n; \quad \forall n \geq 1, \tag{8}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ are sequences in (0,1) such that $\alpha_n + \beta_n + \gamma_n = 1$, $\{t_n\}$ is a sequence in $(0, \infty)$ and $\{u_n\}$ is a bounded sequence in K. Assume that $\lim_{n\to\infty} \frac{\gamma_n}{\alpha_n + \gamma_n} < \infty$, $\lim_{n\to\infty} \frac{\gamma_n}{\alpha_n + \gamma_n} < \infty$ and there is a nondecreasing function f: $(0, \infty) \to (0, \infty)$ with $f(0) = 0$ and $f(t) > 0$ for all $t \in (0, \infty)$ such that, for all $x \in C$, $\sup\{\|x - T(t)x\| : t \geq 0\} \geq f(\mathrm{dist}(x, \mathfrak{F}))$. Then the sequence $\{x_n\}$ converges strongly to a common fixed point of the semigroup $\mathcal{T} := \{T(t) : t \geq 0\}$.

The purpose of this article is to prove the strong and weak convergence of implicit iterative process

$$x_n = (1-\alpha_n)x_{n-1} + \alpha_n T(t_n)x_n, \quad n \in \mathbb{N}, \quad x_0 \in C \tag{9}$$

for a pseudocontraction semigroup $\mathbf{T}: = \{T(t): t \geq 0\}$ in the framework of Banach spaces, which improves and extends the corresponding results of many author's. We need the following Lemma.

Lemma 1.1 [9]*Let E be a real reflexive Banach space with Opial condition. Let C be a nonempty closed convex subset of E and $T: C \to C$ be a continuous pseudocontractive mapping. Then $I - T$ is demiclosed at zero, i.e., for any sequence $\{x_n\} \subset E$, if $x_n \rightharpoonup y$ and $||(I - T)x_n|| \to 0$, then $(I - T)y = 0$.*

2 Main results

Theorem 2.1 *Let E be a real Banach space and C be a nonempty compact convex subset of E. Let $\mathbf{T}: = \{T(t): t \geq 0\}: C \to C$ be a Lipschitian and pseudocontraction semigroup defined by Definition 1.1 with a bounded measurable function $L: [0, \infty) \to [0, \infty)$. Suppose $F(\mathbf{T}) := \bigcap_{t\geq 0} F(T(t)) \neq \emptyset$. Let α_n and t_n be sequences of real numbers satisfying $t_n > 0$, $\alpha_n \in [a, 1) \subset (0, 1)$ and $\lim_{n\to\infty} \alpha_n = 1$. Then the sequence $\{x_n\}$ defined by (9) converges strongly to a common fixed point $x^* \in F(\mathbf{T})$ in C.*

Proof. We divide the proof into five steps.

(*I*). The sequence $\{x_n\}$ defined by $x_n = (1 - \alpha_n)x_{n-1} + \alpha_n T(t_n)x_n$, $n \in \mathbb{N}$, $x_0 \in C$ is well defined.

In fact for all $n \in \mathbb{N}$, we define a mapping S_n as follows:

$$S_n x = (1-\alpha_n)x_{n-1} + \alpha_n T(t_n)x, \quad n \in, \quad \forall x \in C. \tag{10}$$

Then we have

$$\langle S_n x - S_n y, j(x-y)\rangle = \alpha_n \langle T(t_n)x - T(t_n)y, \quad j(x-y)\rangle \leq \alpha_n \|x-y\|^2. \tag{11}$$

So S_n is strongly pseudo-contraction, thus from Theorem *D*, there exists a point x_n such that $x_n = (1 - \alpha_n)x_{n-1} + \alpha_n T(t_n)x_n$, that is the sequence $\{x_n\}$ defined by $x_n = (1 - \alpha_n)x_{n-1} + \alpha_n T(t_n)x_n$, $n \in \mathbb{N}$, $x_0 \in C$ is well defined.

(*II*). Since the common fixed-point set $F(\mathbf{T})$ is nonempty let $p \in F(\mathbf{T})$. For each $p \in F(\mathbf{T})$, we prove that $\lim_{n\to\infty} ||x_n - p||$ exists.

In fact

$$\begin{aligned} \|x_n - p\|^2 &= \langle x_n - p, \quad j(x_n - p)\rangle \\ &= \langle (1-\alpha_n)(x_{n-1} - p) + \alpha_n(T(t_n)x_n - p), \quad j(x-p)\rangle \\ &\leq (1-\alpha_n)\|x_{n-1} - p\|\,\|x_n - p\| + \alpha_n \|x_n - p\|^2. \end{aligned} \tag{12}$$

So we get $||x_n - p|| \leq (1 - \alpha_n)||x_{n-1} - p|| + \alpha_n||x_n - p||$, that is

$$\|x_n - p\| \leq \|x_{n-1} - p\|.$$

This implies that the limit $\lim_{n\to\infty} ||x_n - p||$ exists.

(*III*). We prove $\lim_{n\to\infty} ||T(t_n)x_n - x_n|| = 0$.

The sequence $\{||x_n - p||_{n\in\mathbb{N}}\}$ is bounded since $\lim_{n\to\infty} ||x_n - p||$ exists, so the sequence $\{x_n\}$ is bounded. Since

$$\begin{aligned}\left\|T(t_n)x_n\right\| &= \left\|\frac{x_n-(1-\alpha_n)x_{n-1}}{\alpha_n}\right\| \\ &\le \frac{\|x_n\|}{\alpha_n}+\frac{(1-\alpha_n)\|x_{n-1}\|}{\alpha_n} \\ &\le \frac{\|x_n\|}{a}+\frac{(1-\alpha_n)\|x_{n-1}\|}{a},\end{aligned} \tag{13}$$

This shows that $\{T(t_n)x_n\}$ is bounded. In view of

$$\left\|x_n-T(t_n)x_n\right\| = \left\|(1-\alpha_n)(x_{n-1}-T(t_n)x_n)\right\| = \|1-\alpha_n\|\cdot\left\|x_{n-1}-T(t_n)x_n\right\|$$

and condition $\lim_{n\to\infty}\alpha_n = 1$, we have

$$\lim_{n\to\infty}\left\|T(t_n)x_n-x_n\right\| = 0. \tag{14}$$

(IV). Now we prove that for all $t > 0$, $\lim_{n\to\infty} ||T(t)x_n - x_n|| = 0$.
Since pseudocontraction semigroup $\mathbf{T}$: = $\{T(t) : t \ge 0\}$ is Lipschitian, for any $k \in \mathbb{N}$,

$$\begin{aligned}&\left\|T((k+1)t_n)x_n-T(kt_n)x_n\right\| \\ &= \left\|T(kt_n)T(t_n)x_n-T(kt_n)x_n\right\| \\ &\le L(kt_n)\left\|T(t_n)x_n-x_n\right\| \\ &\le L\left\|T(t_n)x_n-x_n\right\|.\end{aligned} \tag{15}$$

Because $\lim_{n\to\infty} ||T(t_n)x_n - x_n|| = 0$, so for any $k \in \mathbb{N}$,

$$\lim_{n\to\infty}\left\|T((k+1)t_n)x_n-T(kt_n)x_n\right\| = 0. \tag{16}$$

Since

$$\begin{aligned}&\left\|T(t)x_n-T\left(\left[\frac{t}{t_n}\right]t_n\right)x_n\right\| \\ &= \left\|T\left(\left[\frac{t}{t_n}\right]t_n\right)T\left(t-\left[\frac{t}{t_n}\right]t_n\right)x_n-T\left(\left[\frac{t}{t_n}\right]t_n\right)x_n\right\| \\ &\le L\left\|T\left(t-\left[\frac{t}{t_n}\right]t_n\right)x_n-x_n\right\|\end{aligned} \tag{17}$$

and $T(\cdot)$ is continuous, we have

$$\lim_{n\to\infty}\left\|T\left(\left[\frac{t}{t_n}\right]t_n\right)x_n-T(t)x_n\right\| = 0. \tag{18}$$

So from

$$\begin{aligned}&\left\|x_n-T(t)x_n\right\| \\ &\le \sum_{k=0}^{\left[\frac{t}{t_n}\right]-1}\left\|T((k+1)t_n)x_n-T(kt_n)x_n\right\|+\left\|T\left(\left[\frac{t}{t_n}\right]t_n\right)x_n-T(t)x_n\right\|,\end{aligned} \tag{19}$$

and $\lim_{n\to\infty} ||T((k+1)t_n)x_n - T(kt_n)x_n|| = 0$ as well as $\lim_{n\to\infty}\left\|T\left(\left[\frac{t}{t_n}\right]t_n\right)x_n-T(t)x_n\right\| = 0$, we can get

$$\lim_{n\to\infty} \|T(t)x_n - x_n\| = 0. \tag{20}$$

(V). We prove $\{x_n\}$ converges strongly to an element of $F(\mathbf{T})$.

Since C is a compact convex subset of E, we know there exists a subsequence $\{x_{n_j}\} \subset \{x_n\}$, such that $x_{n_j} \to x \in C$. So we have $\lim_{j\to\infty} \|T(t)x_{n_j} - x_{n_j}\| = 0$ from $\lim_{n\to\infty} ||T(t)x_n - x_n|| = 0$, and

$$\|x - T(t)x\| = \lim_{j\to\infty} \|T(t)x_{n_j} - x_{n_j}\| = 0. \tag{21}$$

This manifests that $x \in F(\mathbf{T})$. Because for any $p \in F(\mathbf{T})$, $\lim_{n\to\infty} ||x_n - p||$ exists, and $\lim_{n\to\infty} \|x_n - x\| = \lim_{j\to\infty} \|x_{n_j} - x\| = 0$, we have that $\{x_n\}$ converges strongly to an element of $F(\mathbf{T})$. This completes the proof of Theorem 2.1.

Theorem 2.2 *Let E be a reflexive Banach space satisfying the Opial condition and C be a nonempty closed convex subset of E. Let $\mathbf{T}$: = $\{T(t): t \geq 0\}$: $C \to C$ be a Lipschitian and pseudocontraction semigroup defined by Definition* 1.1 *with a bounded measurable function L: $[0, \infty) \to [0, \infty)$. Suppose $F(\mathbf{T}) := \bigcap_{t\geq 0} F(T(t)) \neq \emptyset$. Let α_n and t_n be sequences of real numbers satisfying $t_n > 0$, $\alpha_n \in [a, 1) \subset (0,1)$ and $\lim_{n\to\infty} \alpha_n = 1$. Then the sequence $\{x_n\}$ defined by $x_n = (1 - \alpha_n)x_{n-1} + \alpha_n T(t_n)x_n$, $x_0 \in C$, $n \in \mathbb{N}$, converges weakly to a common fixed point $x^* \in F(T)$ in C.*

Proof. It can be proved as in Theorem 2.1, that for each $p \in F(T)$, the limit $\lim_{n\to\infty} ||x_n - p||$ exists and $\{T(t_n)x_n\}$ is bounded, for all $t > 0$, $\lim_{n\to\infty} ||T(t)x_n - x_n|| = 0$. Since E is reflexive, C is closed and convex, $\{x_n\}$ is bounded, there exist a subsequence $\{x_{n_j}\} \subset \{x_n\}$ such that $x_{n_j} \rightharpoonup x$. For any $t > 0$, we have $\lim_{n_j\to\infty} \|T(t)x_{n_j} - x_{n_j}\| = 0$. By Lemma 1.1, $x \in F(T(t))$, $\forall t > 0$. Since the space E satisfies Opial condition, we see that $\omega_w(x_n)$ is a singleton. This completes the proof.

Remark 2.1 *There is no other condition imposed on t_n in the Theorems* 2.1 *and* 2.2 *except that in the definition of pseudo-contraction semigroups. So our results improve corresponding results of many authors such as* [10-12], *of cause extend many results in* [4-8].

Acknowledgements
This work was supported by National Research Foundation of Yibin University (No.2011B07).

Author details
[1]Department of Mathematics, Yibin University, Yibin, Sichuan 644000, China [2]College of Statistics and Mathematics, Yunnan University of Finance and Economics, Kunming, Yunnan 650221, China

Authors' contributions
All the authors contributed equally to the writing of the present article. And they also read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 4 November 2011 Accepted: 15 February 2012 Published: 15 February 2012

References

1. Deimling, K: Zeros of accretive operators. Manuscripta Math. **13**, 365–374 (1974). doi:10.1007/BF01171148
2. Xu, HK, Ori, RG: An implicit iteration process for nonexpansive mappings. Numer Funct Anal Optim. **22**, 767–773 (2001). doi:10.1081/NFA-100105317
3. Osilike, MO: Implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps. J Math Anal Appl. **294**, 73–81 (2004). doi:10.1016/j.jmaa.2004.01.038
4. Chen, RD, Song, YS, Zhai, HY: Convergence theorems for implicit iteration press for a finite family of continuous pseudocontractive mappings. J Math Anal Appl. **314**, 701–709 (2006). doi:10.1016/j.jmaa.2005.04.018

5. Zhou, HY: Convergence theorems of common fixed points for a finite family of Lipschitzian pseudocontractions in Banach spaces. Nonlinear Anal. **68**, 2977–2983 (2008). doi:10.1016/j.na.2007.02.041
6. Suzuki, T: On strong convergence to common fixed points of nonexpansive semigroups in Hilbert spaces. Proc Am Math Soc. **131**, 2133–2136 (2003). doi:10.1090/S0002-9939-02-06844-2
7. Xu, HK: A strong convergence theorem for contraction semigruops in Banach spaces. Bull Aust Math Soc. **72**, 371–379 (2005). doi:10.1017/S000497270003519X
8. Chang, SS, Yang, L, Liu, JA: Strong convergence theorem for nonexpansive semigroups in Banach spaces. Appl Math Mech. **28**, 1287–1297 (2007). doi:10.1007/s10483-007-1002-x
9. Zhang, SS: Convergence theorem of common fixed points for Lipschitzian pseudo-contraction semigroups in Banach spaces. Appl Math Mech (English Edition). **30**(2):145–152 (2009). doi:10.1007/s10483-009-0202-y
10. Cho, SY, Kang, SM: Approximation of fixed points of pseudocontraction semigroups based on a viscosity iterative process. Appl Math Lett. **24**, 224–228 (2011). doi:10.1016/j.aml.2010.09.008
11. Qin, X, Cho, SY: Implicit iterative algorithms for treating strongly continuous semigroups of Lipschitz pseudocontractions. Appl Math Lett. **23**, 1252–1255 (2010). doi:10.1016/j.aml.2010.06.008
12. Agarwal, RP, Qin, X, Kang, SM: Strong convergence theorems for strongly continuous semigroups of pseudocontractions. Appl Math Lett. **24**, 1845–1848 (2011). doi:10.1016/j.aml.2011.05.003

doi:10.1186/1687-1812-2012-16
Cite this article as: Quan *et al.*: **Strong and weak convergence of an implicit iterative process for pseudocontractive semigroups in Banach space.** *Fixed Point Theory and Applications* 2012 **2012**:16.

Submit your manuscript to a SpringerOpen journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at ► springeropen.com