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Two novel price-based algorithms for spectrum sharing in cognitive radio networks

Meng-Dung Weng[1], Bih-Hwang Lee[1*] and Jhih-Ming Chen[2]

Abstract

Cognitive radio network is expected to use flexible radio frequency spectrum sharing techniques for achieving more efficient frequency spectrum usage. In this article, we consider the spectrum sharing problem that one primary user (PU) can share its frequency spectrum by renting this spectrum to multiple secondary users (SUs). The pricing scheme is a key issue for spectrum sharing in cognitive radio network. We first propose a nonlinear one-leader–multiple-follower (NLMF) sharing spectrum scheme as a multi-object optimization problem; the prices are offered by PU to SUs at the same time. This problem can be solved using particle swarm optimization (PSO); SUs gradually and iteratively adjust their strategies respectively based on the observations on their opponents' previous strategies until Nash equilibrium is completed. We then present a general nonlinear bilevel one-leader-multiple-follower (NBMF) optimization problem to further consider the revenue of the PU and a new optimal strategic pricing optimization technique which applies bilevel programming and swarm intelligence. A leader-follower game is formulated to obtain the Stackelberg-Nash equilibrium for spectrum sharing that considers not only revenue of a PU but also the SUs utility. We develop a swarm particle algorithm to iteratively solve the problem defined in the NBMF decision model for searching the strategic pricing optimization. The behaviors of two pricing models have been evaluated, and the performance results show that the proposed algorithms perform well to solve the spectrum sharing in a cognitive radio network.

Keywords: Spectrum sharing; Cognitive radio; Nash equilibrium; Bilevel programming; Swarm particle algorithm; Strategic pricing optimization

1. Introduction

According to the regulations of the Federal Communication Commission (FCC) [1], a large portion of the unutilized priced frequency spectrum and the scarcity in spectrum resource should be used by providing tools to utilize spectrum holes [2]. Recently, cognitive radio (CR) provides great flexibility by extending software radio to improve spectrum utilization [2-6], which is now regarded as a hopeful wireless communication system. Primary users (licensed users) are willing to share frequency spectrum with secondary users (unlicensed users) which can adaptively adjust the transmission parameters to satisfy the requirements of quality of service (QoS) according to the environment information and opportunistically access those available frequency bands not occupied by primary users.

By this way, we can use the spectrum resource to enhance the system performance. We consider spectrum sharing as a spectrum trading process; therefore, price not only acts an important role in spectrum trading but also is an effective way to improve system utilization and performance to maximize the primary users' profits. The spectrum trading process indicates the values of both spectrum pricing and purchasing by allowing the spectrum trading between secondary users (SUs) and primary users (PUs). The price paid by SU to PU depends on the satisfaction to use that spectrum, while the price determines the PU's revenue.

In this article, we address the spectrum sharing problem in a cognitive radio environment, which consists of several SUs to compete with each other to demand opportunistic spectrum access to a single PU. We formulate this situation as an oligopoly market, where a few firms (i.e., SUs) compete with each other in terms of amount

* Correspondence: bhlee@mail.ntust.edu.tw
[1]National Taiwan University of Science and Technology, 43, Keelung Rd., Section 4, Taipei 106, Taiwan
Full list of author information is available at the end of the article

of commodity (i.e., the frequency spectrum) supplied to the market (i.e., PU) to gain the greatest profit.

A noncooperative game is used to analyze two situations, one case is that SUs receive the offered price by the PU and maximize their payoffs by the amount of their demand spectrum. Another case is that the main objection is to maximize the profit and revenue for all SUs and the PU, respectively. For these scenarios, we apply a dynamic game in which the selection of strategy by an SU is entirely determined by the pricing information obtained from the PU. Based on this information, each SU gradually adapts the size of its spectrum sharing which is controlled by the price function. To solve the corresponding optimization problem, we search the strategy space using swarm particle to identify the optimum behavior.

Compared with the current research on the spectrum sharing in a cognitive radio environment, the major contributions of this article are as follows. First, the idea is to bring together swarm particle algorithms and Nash strategy, and make the swarm particle algorithm to build the Nash equilibrium (NE). Nash-particle swarm optimization (PSO) is an alternative for multiple-objective optimizations as it is an optimization tool based on noncooperative game theory. Second, by applying bilevel techniques in the cognitive radio markets, we propose the concept and related definitions of nonlinear bilevel one-leader-multiple-follower (NBMF) decision problem and use swarm particle algorithm in designing the spectrum sharing scheme among PU and SUs for CR networks. We also build up a nonlinear NBMF decision model for strategic pricing problems, where the Stackelberg-Nash equilibrium is regarded as the solution.

The rest of this article is organized as follows. Section 2 addresses the related works on dynamic spectrum sharing in cognitive radio networks (CRN). Section 3 proposes the general NBMF decision model and introduces the Stackelberg-Nash equilibrium. We present the system model and describe the spectrum sharing and pricing strategy problem in Section 4. We develop Nash-PSO and NBMF-PSO algorithm as solutions to the NLMF, and the NBMF problems in Section 5. Then, we verify the effectiveness of the proposed algorithm to validate the NLMF and the NBMF decision model using simulation in Section 6, and finally we make conclusions in Section 7.

2. Background and related works

For the development of communication protocols, cognitive radio technology offers tremendous potential to improve the radio spectrum usage by allowing cognitive devices to opportunistically access vast portions of the spectrum. Dynamic spectrum access is a new paradigm, whereby a cognitive radio device opportunistically accesses the unutilized or underutilized spectrum bands. There are many important issues needed to be overcome to achieve the objective of dynamic spectrum access, including spectrum sensing, spectrum allocation, spectrum access/sharing, transmitter-receiver handshaking, and spectrum mobility [7]. In [8], the different spectrum sharing models are categorized as open sharing, hierarchical access, and dynamic exclusive usage models. Some of the spectrum sharing proposals can be identified as being hierarchical access methods, in that there is usually a primary system that owns the spectrum rights and a secondary system that wants to access this spectrum whenever possible. Dynamic spectrum sharing is a challenging in the design of CRNs due to the requirement of peaceful coexistence of both PUs and SUs, as well as the availability of wide range of radio spectrum.

Various techniques were used to model the spectrum sharing problems for CR networks. In [9], the conventional approach based on the centralized control is proposed. However, as the available spectrum holes in the CRN are rapidly changing, it is difficult to use centralized approach in the practical application. Other studies propose distributed approaches [10,11]. The authors of [10] provide better adaptive capacity to cognitive radios, in dynamic communication environments. The efforts of [11] have been absorbed in applying the distributed ARQ in cognitive radio systems to make better the dependability of secondary links. Dynamic spectrum sharing through cognitive radios can significantly enhance the spectrum utilization in a wireless network. In spectrum sharing CR networks, the problem of fair resource allocation among secondary users was investigated in [12]. In [13], the fair, efficient, and power optimized (FEPO) spectrum sharing scheme is proposed to achieve efficient spectrum utilization. In [14], the biologically inspired spectrum sharing algorithm is introduced based on the adaptive task allocation model of an insect colony, which enables each cognitive radio in the same environment to fairly share the licensed or unlicensed spectrum bands.

Game theory [15] has been considered a feasible mathematical tool for solving the resource allocation problems in distributed CRNs. The fundamental concept of game theory is to resolve the competition and cooperation among multiple intelligent rational decision makers. The comparison between cooperative and noncooperative approaches has been presented in [16] through game theoretical analysis. The authors of [17] develop the optimal resource allocation strategies in secondary spectrum access problem using cooperative game theory. This work applies the concept of Nash bargaining solution to guarantee fairness and maximize the utility of the system optimality. In [18], the dynamic spectrum access problem is formulated as a noncooperative game, and the concepts of the correlated equilibrium and regret learning are used to solve the dynamic spectrum access problem. However, the abovementioned works do not consider the pricing issue in spectrum trading.

Pricing impacts the incentive of the PUs (or primary service providers) in selling the spectrum and the satisfaction of the SUs in buying the spectrum. A pricing process can maximize the utilities of both PUs and SUs according to the spectrum dynamics. Efficient pricing techniques not only increase the users' performance, but also improve network utilization in consideration of the rapid growth and variety of network demand. Researchers have already proposed and investigated to achieve dynamic spectrum sharing by various pricing and auction mechanisms [19-21]. In [19], the problem of a CDMA operator participating in a dynamic spectrum allocation scheme is addressed in a cooperative framework based on multi-unit Vickrey auction. The operators maximize their revenue based on the users' willingness to pay. The authors of [20] use a power/channel allocation scheme that uses a distributed pricing strategy to improve the network's performance by a noncooperative model. An auction-based spectrum sharing approach is proposed [21], in which many SUs purchase channels from one PU or spectrum broker through an auction process to efficiently share spectrum among the users in interference-limited systems. The abovementioned methods do not include the interaction between PU and SUs.

Cognitive radios have been chosen as a pricing decision platform to realize the cognitive network operators who interact with a group of secondary users [22-29]. In [22], a novel spectrum management policy based on Vickrey auction is proposed to construct the co-win situation that simultaneously satisfies four parties which are involved in CRN operations (i.e., PUs, CR users, operators, and regulators). In [23], a bandwidth auction game is proposed for dynamic spectrum sharing. A game theoretic Cournot model is presented in [24], which considers the problem of spectrum sharing among a PU and multiple SUs. However, it cannot further consider the profit of the primary user. In addition, a Bertrand model is presented in [25], where multiple service providers compete with each other to obtain the Nash equilibrium pricing. In [26], distributed algorithms were presented for three different pricing models (i.e., market equilibrium, competitive, and cooperative) via game theory among the primary users and secondary users. As such, the problem is treated from the point of view of the primary users. The authors of [27] and [28] analyze the PU and SU interactions exclusively without considering the hierarchical structure of the spectrum markets. In [29], the authors study a multiple-level spectrum sharing in cognitive radio among primary, secondary, and tertiary services. Therefore, a spectrum price model plays an important role in the interaction of PUs and SUs.

The bilevel programming problem (BLPP) has a wide variety of applications, and bilevel programming techniques have been applied with remarkable success in different domains such as decentralized resource planning [30], transport system planning [31], civil engineering [32], road network management [33], power market [34], economics, and management [35,36]. The existing methods for solving BP can be categorized as traditional method and heuristic (stochastic) method. The traditional method includes K-best algorithm [37] and branch-and-bound algorithm [38], while the heuristic method includes genetic algorithm-based approach [39], adaptive search method related to the taboo search to solve such problems [40], and global optimization techniques based on convergence analysis [41].

The NLMF model considers the case, where the prices at the same time can be iteratively adapted for the strategies in terms of SUs' requested spectrum size, respectively. The NBMF decision model allows PU to choose its prices for spectrum sharing allocation such that its revenue can be maximized, and each SU competes noncooperatively and independently with each other to maximize its profit, which is determined by the demand spectrum of each SU. The hierarchical relation between them is considered by making the PU and the SUs decide sequentially. To the best of our knowledge, this article is the first to utilize swarm intelligence with game theory to the spectrum sharing scheme among PU and SUs for CR networks.

3. Mathematic descriptions of NBMF problems

3.1 Problem statements

The BLPP is regarded as an uncooperative, two-person game, as introduced by Von Stackelberg [42] in 1952. In the basic model, the decision variables are partitioned among two players who seek to optimize their individual utility functions. The bilevel programming techniques are mainly developed for solving decentralized management problems with decision makers in a hierarchical organization. A decision maker is known as the leader at the upper level, and it is known as the follower at the lower level. Each leader or follower optimizes his objective function with or without considering the objective of the other level, but the decision of each level affects the optimization of the other level.

Usually, in a real-world situation, there is more than one follower in the lower level. This type of the hierarchical structure is called a bilevel multi-follower (BLMF) decision making model. However, the different relationships among these followers might force the leader to use multiple different processes in deriving an optimal solution for leader decision making. Therefore, the leader's decision will be affected not only by the reactions of these followers, but also by the relationships among these followers. In general, there are three kinds of relationships among the followers: cooperative condition, uncooperative condition, and partial cooperative condition [43].

For the cooperative condition, the followers totally share the decision variables in their objectives and constraints. In the uncooperative condition, there is no sharing of decision variables among the followers. In the partial cooperative condition, the followers partially share decision variables in their objectives and/or constraints. There is no exact way to solve the nonlinear bilevel decision problems. For solving the nonlinear bilevel decision problems, heuristic approach may be an alternative in the research community [34,44,45]. In this section, we assume that there are one leader and N followers in a bilevel decision system. Let x and y_i be the decision variables of the leader and the ith follower, for $i = 1, 2, \ldots, N$, respectively. We also assume that the objective functions of the leader and the ith follower are $F(x, y_1,\ldots, y_N)$ and $f_i(x, y_1,\ldots, y_N)$, for $i = 1, 2, \ldots, N$, respectively, while G and g are the vector valued functions of x and the set of $(y_1,\ldots, y_N)$. The sets of X and Y represent the search spaces in the upper and lower bounds on the elements of the vectors x and y_i. An NBMF problem with one leader and N followers is introduced with some related definitions and notations as defined in (1), where

$$x \in X \subset R^n, y_i \in Y_i \subset R^{m_i}, Y = (Y_1, Y_2, \ldots, Y_N)^T, \quad F : X \times Y_1 \times \ldots \times Y_N \rightarrow R^1, f_i : X \times Y_i \rightarrow R^1,$$

and

$$i = 1, 2, \ldots, N, \quad N \geq 2$$

$$\begin{cases} \min\limits_{x \in X} F(x, y_1, \ldots, y_N) \\ \text{subject to } \; G(x, y_1, \ldots, y_N) \leq 0 \\ \min\limits_{y_i \in Y_i} f_i(x, y_1, \ldots, y_N), i = 1, \ldots, N \\ \text{subject to } \; g_i(x, y_1, \ldots, y_N) \leq 0, i = 1, \ldots, N. \end{cases} \tag{1}$$

In order to facilitate further discussion of the properties of the NBMF decision model, the following definitions are introduced [43]:

(a) Denote the constraint region of the NBMF problem by

$$S = \{(x, y_1, \ldots, y_N) \in X \times Y_1 \times \ldots \times Y_N, G(x, y_1, \ldots, y_N) \leq 0, g(x, y_1, \ldots, y_N) \leq 0, i = 1, 2, \ldots, N\}.$$

(b) Projection of S onto the leader's decision space is defined as

$$S(X) = \{x \in X : \exists y_i \in Y_i, G(x, y_1, \ldots, y_N) \leq 0, g(x, y_1, \ldots, y_N) \leq 0, i = 1, 2, \ldots, N\}.$$

(c) Denote the feasible region of the follower's problem for each fixed $x \in S(X)$ by

$$S_i(x) = \{y_i \in Y_i : (x, y_1, \ldots, y_N) \in S, g(x, y_1, \ldots, y_N) \leq 0, i = 1, 2, \ldots, N\}.$$

(d) Let $P_i(x)$ be the follower's rational reaction set for $x \in S(X)$, which is defined as

$$P_i(x) = \left\{ y_i \in Y_i : y_i \in \arg\min \left[f_i\left(x, \hat{y}_i\right) : \hat{y}_i \in S_i(x) \right] \right\}, \quad i = 1, 2, \ldots, N,$$

where

$$\arg\min \left[f_i\left(x, \hat{y}_i\right) : \hat{y}_i \in S_i(x) \right] = \left[y_i \in S_i(x) : f_i(x, y_i) \leq f_i\left(x, \hat{y}_i\right), \hat{y}_i \in S(x) \right], \quad i = 1, 2, \ldots, N.$$

The followers observe the leader's action and simultaneously react by selecting y_i from their feasible set to minimize their objective function.

(e) Denote the inducible region by $IR = \{(x, y_1, \ldots y_N) : (x, y_1, \ldots, y_N) \in S, y_i \in P_i(x), i = 1, \ldots, N\}$.

The rational reaction set $P(x)$ defines the response, while the inducible region IR represents the set, which the leader may optimize his objective. The leader's problem is then to optimize its objective function over the inducible region, the NBMF problem can be written as

$$\min\{F(x, y_1, \ldots, y_N) : (x, y_1, \ldots, y_N) \in IR\}.$$

3.2 Nash equilibrium and Stackelberg-Nash equilibrium

For an optimization problem with N objectives, a Nash strategy consists of N players, each optimizes his own strategy. However, each player has to optimize his strategy, given that all the other strategies are fixed by the rest of the players. When no player can further improve his strategy, it means that the system has reached an equilibrium state called Nash equilibrium (NE). The bilevel programming is a multiple person noncooperative game with leader-follower strategy. In a game with N followers, $y_i \in Y_i$ is a strategy of follower, and $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_N) \in Y_{-i}$ is the set of others' strategy. If $x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_N$ are revealed by the leader and the other followers, then the reaction y_i^* of the ith follower must be the optimal solution of the follower's objection function as shown in (2)

$$\begin{cases} \min\limits_{y_i \in Y_i} f_i(x, y_1, \ldots, y_N), i = 1, \ldots, N \\ \textit{subject to } \; g(x, y_1, \ldots, y_N) \leq 0. \end{cases} \tag{2}$$

For all followers, the profile strategies $\left(y_1^*, \ldots, y_i^*, \ldots, y_N^*\right) \in Y$ with respect to x is the Nash equilibrium of followers as shown in (3), for any $\left(y_1^*, \ldots, y_{i-1}^*, y_i, y_{i+1}^*, \ldots, y_N^*\right) \in Y$ and $i = 1,2,\ldots,N$. There is a unique Nash equilibrium that all the followers might make such equilibrium because no follower can further improve his own objective by altering his strategy unilaterally. The Stackelberg-Nash equilibrium of the bilevel programming is discussed by [44]. Let us define $(x^*, y_1^*, \dots, y_i^*, \dots, y_N^*)$ as the profile strategies with $x^* \in X$, where $(y_1^*, \dots, y_i^*, \dots, y_N^*)$ is a Nash equilibrium of followers with respect to x^*, and the profile strategies $(x^*, y_1^*, \dots, y_i^*, \dots, y_N^*)$ are a Stackelberg-Nash equilibrium to the bilevel programming (1) if and only if (4) is satisfied for any $\hat{x} \in X$ and the Nash equilibrium $(\hat{y}_1, \dots, \hat{y}_i, \dots \hat{y}_N)$ with respect to $\hat{x}$.

$$f_i(x, y_1^*, \dots, y_{i-1}^*, y_i^*, y_{i+1}^*, \dots, y_N^*) \geq f_i(x, y_1^*, \dots, y_{i-1}^*, y_i, y_{i+1}^*, \dots, y_N^*) \quad (3)$$

$$F(x^*, y_1^*, \dots, y_i^*, \dots, y_N^*) \geq F(\hat{x}, \hat{y}_1, \dots, \hat{y}_i, \dots \hat{y}_N). \quad (4)$$

4. System model

We consider the cognitive scenario of dynamic spectrum sharing consisting of only a PU and N SUs as shown in Figure 1. When a frequency spectrum is unoccupied by its corresponding PU, it can sell portions of the available spectrum b_i (e.g., time slots in a time division multiple access (TDMA)-based wireless access system) to the SU_i ($i = 1, 2, \dots, N$) at the offered price p. Both PU and SUs use adaptive modulation for wireless transmission. The spectrum demand of SUs depends on the transmission rate achieved due to the adaptive modulation in the allocated frequency spectrum and the price charged by PU. After obtaining the right to use the spectrum, SU uses an adaptive modulation to improve the performance.

We adopt a widely used transmission model by adaptive modulation, where PU and SUs dynamically adjust their transmission rate based on their corresponding channel qualities. Therefore, the spectral efficiency of the transmission for SU_i, k_i, can be obtained by (5) [46], where γ_i is the received signal-to-noise ratio (SNR) for SU_i; and BER_i^{tar} is the bit error rate (BER) at the target level, which is to guarantee the required quality of transmission.

$$k_i = \log_2\left(1 + \frac{1.5\gamma_i}{\ln\left(0.2/\mathrm{BER}_i^{\mathrm{tar}}\right)}\right). \quad (5)$$

As the owner of spectrum resources, PU has the right to determine the spectrum price. A PU adjusts the price of spectrum as the total requested spectrum changed, so the PU charges all of SUs at the same price. Let $P(\mathbf{b})$ be the price function, which can be obtained as shown in (6) [24], where b_{it} is the size of spectrum demanded by SU_i at time t, and $\mathbf{b} = \{b_i \mid i = 1, \dots, N\}$ is the demand vector. Let us denote l and q as a fixed payment and the elasticity (i.e., slope) of the price function, respectively, while l_t and q_t are the values of l and q at time t, respectively. Therefore, the price function is convex if l_t, q_t, and α are assumed to be positive and greater than one; moreover, the price function is linear if $\alpha = 1$. The item $\left(b_{it} + \sum_{j \neq i} b_{jt}\right)$ is the total sharing spectrum at time t. Consequently, the PU always determines the price function for the shared frequency spectrum in terms of the amount of spectrum demanded by SUs. The demand for the spectrum is larger, and the PU will charge a higher price in cognitive radio environment as $\alpha = 1$. Let us denote w_1 and w_2 to be the worth values of the spectrum for the PU and SU, respectively; then, it is necessary for the condition $w_1 \times \sum_{j \in \mathbf{b}} b_j < P(\mathbf{b}) < w_2 \times \sum_{j \in \mathbf{b}} b_j$ to ensure that the PU is willing to share the spectrum with SUs, while N SUs are willing to buy the spectrum. In the NLMF mode, the price is determined by PU, and the values of w_1 and w_2 are set to 1 and 0, respectively. In the NBMF mode, the PU would never have predominated, and the part of its price will be transferred to the SUs. We get the values of w_1 and w_2 by estimating the values of l_t and q_t. Then, we can refer to this price and use the other spectrum directly by signing the contract without competition. A contract between PU and SUs ensures that PU will not deviate because of selfishness.

$$P(\mathbf{b}) = l_t + q_t \times \left(b_{it} + \sum_{j \neq i} b_{jt}\right)^{\alpha}. \quad (6)$$

SUs define their spectrum demands by maximizing their payoff functions. Each SU's payoff function is defined as the subtraction between the earned revenue and the paid cost for sharing frequency spectrum with the PU. The revenue of SU_i can be obtained by $R_i \times k_i \times b_i$, while the cost of spectrum sharing is $b_i P(\mathbf{b})$, where R_i is the user revenue of achievable per unit transmission

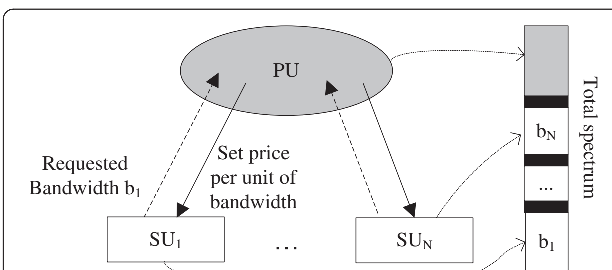

Figure 1 System model of spectrum sharing.

rate. Let us define $U_i(\mathbf{b})$ to be the profit function of each SU_i, which can be obtained by (7) [24].

To determine the optimal shared spectrum sizes for the SUs, their profit functions should be maximized with respect to the shared spectrum sizes b_i for $i = 1, \ldots, N$. To obtain Nash equilibrium, we have to mathematically solve the marginal profit of SU_i. That is, we can derive the profit function with respect to the shared spectrum size and set it to zero as shown in (8). Each SU then solves its own equation to find the amount of its spectrum demand:

$$U_i(\mathbf{b}) = R_i k_i b_i - b_i P(\mathbf{b}), \qquad i = 1, 2, \ldots, N \tag{7}$$

$$\frac{\partial U_i(\mathbf{b})}{\partial b_i} = R_i k_i - l - q\left(\sum_{b_j \in \mathbf{b}} b_j\right)^{\alpha} - q b_i \alpha \left(\sum_{b_j \in \mathbf{b}} b_j\right)^{\alpha-1} = 0 \qquad i = 1, 2, \ldots, N. \tag{8}$$

The Nash equilibrium is considered to be the solution of the game to ensure all SUs satisfied, where the Nash equilibrium is obtained using the best response (BR) function that is the best strategy of one player, given the others' strategies. Let us denote $\mathbf{b}_{-i}$ to be the set of strategies adopted by all SUs except SU_i, where $\mathbf{b}_{-i} = \{\mathbf{b}_j \mid j = 1, 2, \ldots, N \text{ for } j \neq i\}$ and $\mathbf{b} = \mathbf{b}_{-i} \cup \{\mathbf{b}_i\}$. BR_i is denoted as the BR function of SU_i, given the size of the spectrum sharing by other SU's $\mathbf{b}_j$, which is defined as (9). The set $\mathbf{b}^* = \{b_1^*, \ldots, b_N^*\}$ denotes the Nash equilibrium of any game if and only if (10) is satisfied, where $\mathbf{b}_{-i}^*$ denotes the set of BR_j for all SUs except SU_i:

$$\mathrm{BR}_i(\mathbf{b}_{-i}) = \arg\max_{b_i} U_i(\mathbf{b}_{-i} \cup \{b_i\}) \tag{9}$$

$$b_i^* = \mathrm{BR}_i\left(\mathbf{b}_{-i}^*\right) \qquad \forall i. \tag{10}$$

A static game model is presented for the ideal case, where all SUs can entirely acquire the strategies and profit of the other SUs. Some numerical methods are required to solve Equation (8) to obtain Nash equilibrium. We formulate the following optimization problem with the objective, $\min \sum_{i=1}^{N} |b_i - \mathrm{BR}_i(\mathbf{b}_{-i})|$. The algorithm reaches the Nash equilibrium when the minimum value of the objective function is zero. Subsequently, to relax the supposition, a dynamic game model is presented for which the information of the other SUs is unknown to a specific SU. In this scenario, we regard nonlinear one-leader-multiple-follower (NLMF) model as a nonlinear multiple-objective optimization problem, so it can be modeled as a dynamic game by Nash-PSO algorithms to adjust the requested spectrum size. The objective of the NLMF model is to maximize the profit function of all respective SUs, which is defined as (11). When the channel quality is getting better, the transmission rate can be higher; accordingly, the demand and the offered price for channels will increase. The value of $b_{t\max}$ cannot make the supply short of demand, while the value of $b_{t\min}$ is zero as the channel quality is very bad, which implies that the SU has no demand for the channel.

$$\begin{cases} \max\limits_{b_{ti}} f_i = f_i(l_t, q_t, b_{t1}, \ldots b_{tN}) = \sum\limits_{t=1}^{T} (R_i k_{ti} b_{ti} - b_{ti} P(\mathbf{b})), \quad i = 1, 2, \ldots, N \\ 0 < \sum\limits_{i=1}^{N} b_{ti} < B^{\mathrm{tot}}, \quad b_{t\min} < b_{ti} < b_{t\max}. \end{cases} \tag{11}$$

A PU sells its spectrum resource at a fixed price per unit time; the revenue is the sold spectrum multiplied by the price. The PU sells a fraction of the spectrum to SUs according to the demand, where the revenue of the PU can be obtained by (12). In the game model, SUs purchase a fraction of the spectrum by the given price, and the PU decides the price to maximize its revenue; hence, it can be modeled to maximize its revenue as shown in (13). Under different channel qualities, SUs have a variety of demands. The parameters l_t and q_t are introduced to measure the negative impact from SUs to PU and find a feasible pricing region to guarantee the primary service and satisfactory for SUs. If l_t and q_t are given among a predefined range, a feasible pricing region can be found to guarantee that it may not produce the negative values from the payoffs of the SUs, respectively:

$$F = P(\mathbf{b}) \times \sum_{i=1}^{N} b_i \tag{12}$$

$$\begin{cases} \max\limits_{x_t, y_t} F = F(l_t, q_t, b_{t1}, \ldots b_{tN}) = \sum\limits_{t=1}^{T} \left(P(\mathbf{b}) \times \sum\limits_{i=1}^{N} b_{ti} \right), \\ l_{t\min} \leq l_t \leq l_{t\max}, \qquad q_{t\min} \leq q_t \leq q_{t\max}. \end{cases} \tag{13}$$

Because spectrum sharing and pricing strategy involve two hierarchical optimizations, there exists a game relationship among PU and SUs to compete selfishly in a noncooperative Nash game to maximize their individual utilities. A pricing mechanism is to guide every player toward rational behaviors, and any one's decision may affect the others', which is a typical bilevel decision problem with one PU and multiple SUs. By combining the NLMF model defined in (11) with the strategic pricing model defined in (13), we propose an NBMF decision model for a competitive strategic spectrum allocation in a pricing-based cognitive radio market as shown in (14):

$$\begin{cases} \max\limits_{x_t, y_t} F = F(l_t, q_t, b_{t1}, \ldots b_{tN}) = \sum\limits_{t=1}^{T} \left(P(\mathbf{b}) \times \sum\limits_{i=1}^{n} b_{ti} \right), \\ l_{t\min} \le l_t \le l_{t\max}, \qquad q_{t\min} \le q_t \le q_{t\max} \\ \max\limits_{b_{ti}} f_i = f_i(l_t, q_t, b_{t1}, \ldots b_{tN}) = \sum\limits_{t=1}^{T} (R_i k_{ti} b_{ti} - b_{ti} P(\mathbf{b})), \quad i = 1, 2, \ldots, N \\ 0 < \sum\limits_{i=1}^{N} b_{ti} < B^{tot}, \qquad b_{t\min} < b_{ti} < b_{t\max}. \end{cases} \tag{14}$$

For the spectrum trading, we consider two different pricing models, NLMF and NBMF, which are to hierarchically describe the strategic pricing problems in competitive cognitive radio markets. The NLMF model first considers the case that PU offers his price to SUs simultaneously. We use the Nash-PSO algorithm to build Nash equilibrium for multiple-objective optimization. In the NBMF model, SUs can observe the pricing strategy of PU and adapt their strategies accordingly from a bilevel angle. Therefore, we use an NBMF-PSO algorithm to estimate and adapt the strategies of SUs to achieve the best response and the strategy of PU to obtain the optimal solution. The Nash-PSO and NBMF-PSO algorithms will be discussed in the next section.

5. NASH-PSO and NBMF-PSO algorithms

Kennedy and Eberhart first introduce the PSO in 1995 as a new heuristic method [47]. PSO works by flying a population of cooperating potential solutions called particles through a problem's solution space, accelerating particles toward better solutions [48,49]. PSO has a good convergent performance and has been applied in many optimization problems including neural network training, multi-object optimization, and some project applications. In PSO, each particle is treated as a point in an n-dimensional space, where its mth particle can be represented by a vector $X_m = (x_{m1}, x_{m2}, \ldots, x_{mn})$, and it is treated as a potential solution that explores the search space by rate of position change called velocity, denoted by $V_m = (v_{m1}, v_{m2}, \ldots, v_{mn})$. Let pb_m be represented as the personal best position of any particle, i.e., $\text{pb}_m = (\text{pb}_{m1}, \text{pb}_{m2}, \ldots, \text{pb}_{mn})$, which is in accord with the position in search space where particle had the lowest (or highest) value as determined by the fitness function. In addition, gb is defined to be the global best position of particle in swarm, which yields the best position of all particles in its neighborhood. The particle update method lies in accelerating each particle towards the optimum value based on its present velocity, its previous experience, and the experience of its neighbors within a reasonable time limit. Let us denote M and w to be the size of the swarm and the inertia weight used to balance the global and local search abilities, respectively. The position and velocity of each particle can be updated according to (15) and (16) [41], where m ($m = 1, 2, \ldots, M$) and d ($d = 1, 2, \ldots, n$) are for the mth particle and the d-dimensional vector, respectively, while c_1 and c_2 are the positive constants; r_1 and r_2 are two uniformly distributed random numbers in the range [0,1]; and k denotes the iteration number. For simplicity and immunity to the global optimum, the PSO algorithm is employed in this article to develop a Nash-PSO algorithm and an NBMF-PSO algorithm to reach an optimal solution for the NLMF and NBMF strategic pricing problem in an oligopoly market:

$$\begin{aligned} v_{md}(k+1) = {} & w v_{md}(k) + c_1 r_1(k) \\ & \times (\text{pb}_{md}(k) - x_{md}(k)) + c_2 r_2(k) \\ & \times (\text{gb}_{md}(k) - x_{md}(k)), \end{aligned} \tag{15}$$

$$x_{md}(k+1) = x_{md}(k) + v_{md}(k+1). \tag{16}$$

5.1 Nash-PSO algorithm

Nash presents a multiple-objective optimization problem which originated from the game theory and economics in 1950 [50,51]. SUs define their spectrum demands from the single PU based on the NLMF model. It uses Nash-PSO algorithm to find the Nash equilibrium and maximizes its payoffs in a distributed fashion. In a practical cognitive radio environment, each SU has the knowledge of its payoffs and costs, but it does not know about the strategies and profits of the other SUs. The obtained profit of each SU is calculated based on the opponent's previous strategies about the optimal strategies which observe the pricing information from the PU; hence, we have to achieve the Nash equilibrium for each SU based on the interaction with the PU only. In this case, each SU can communicate with the PU to obtain the discriminated price function for different strategies. It is supposed if all SUs are intelligent, then they can apply the proposed approach to be aware of their opponent's payoff function and try to maximize their revenue by acknowledging the opponent's strategies. In the Nash-PSO algorithm, we first fix the X variables for leader (i.e., PU) and initiate a swarm to produce the followers' (i.e., SUs') decision variable (Y-particles), each of which has a velocity. Both their numbers are randomly distributed among a pre-defined range. The proposed Nash-PSO algorithm is an iterative algorithm which is to search the Nash equilibrium from the SUs by solving the NLMF model (11) as summarized in Algorithm 1. The global Nash equilibrium of the problem can be obtained if the iterations converge to a single point, because none of the players can gain more profit just by changing his strategic variable.

Algorithm 1: Nash-PSO algorithm

Step1: Randomly generate the position and velocity for *m*th particle among a pre-defined range, $m = 1,\ldots, M$.

Step2: $i = 1$

Step3: For each SU_i, $i = 1,\ldots, N$.

Step4: Take the strategies of all players from the previous iteration.

Step5: Run the PSO algorithm to update every SU's optimal strategy by fixing opponents' strategies at their previous iteration values and acquire the optimal response respectively.

Step5.1: Initiate the SU_i's (i.e., follower's) loop counter $g_f = 0$.

Step5.2: Compute the objection function f_i in (11) of each particle, and select the previously visited best positions pb value for all *m*-particles and the best one gb value among *m*-particles.

Step5.3: Use (15) and (16) to calculate the new velocity vector and update the position for m-particles.

Step5.4: $g_f \leftarrow g_f + 1$

Step5.5: If it finds SU_i's optimal strategy in response to the other SUs' optimal strategy, then go to Step 6. Otherwise, we go to Step5.2.

Step6: If it already finds all SU_i's optimal strategy in response to the other SUs' optimal strategy, then go to Step7. Otherwise, $i \leftarrow i + 1$, then we go to Step3.

Step7: If no SU would change its optimal strategy, then we obtain the Nash equilibrium solution. Otherwise, we go to Step2.

5.2 NBMF-PSO algorithm

By this PSO strategy, the framework of the NBMF-PSO algorithm is shown as in Figure 2. The important notations used in this article are summarized in Table 1. In a real-world NBMF problem, SUs have their individual variables, objectives, and constraints. However, a decision from any particular SU will inevitably be made by guessing the other SUs' strategies. In this case, the lower-level optimization problem is a kind of game problem, and the whole problem becomes an NBMF game problem.

In this NBMF-PSO algorithm, we first initiate a swarm to produce the decision variable (*X*-particles) for PU and generate a population (*Y*-particles) for the followers, while the corresponding velocities are random numbers distributed among a pre-defined range. We then bring the *X*-particles to the lower-level problem of the NBMF model and use the Nash-PSO algorithm to generate the Nash equilibrium point from the followers by solving (14). After obtaining the best responses of the *Y*-particles from the followers, the leader's objective values for each decision variable of the *X*-particles can be calculated. To utilize the PSO strategy again, we obtain the leader's optimal strategy and find the solution changes for several consecutive generations which are smaller than a predefined value; hence, the Stackelberg-Nash equilibrium for the whole NBMF problem can be obtained. The detailed NBMF-PSO algorithm consists of two parts, Algorithm 2 and Algorithm 3, which generate the best response from SUs and the optimal strategies for PU, respectively, as specified as follows:

Algorithm 2: Generate the responses from all followers

Step1: Input the values of x_m from the leader.

Step2: For each x_m, randomly generate the position y_{mpi} and the corresponding velocities v_{mpi} for pth particle among a pre-defined range, $p = 1,\ldots, M_f$.

Step3: $i = 1$.

Step4: For each follower i, $i = 1,\ldots,N$.

Step5: Take the strategies of all followers from the previous iteration.

Step6: Run the Nash-PSO algorithm to update every follower's optimal strategy by fixing opponents' strategies at their previous iteration values and acquire the optimal response respectively.

Step6.1: Initiate the follower's loop counter $g_f = 0$.

Step6.2: Compute the followers' objection function f_i in (14) of each particle and select the previously visited best positions pb_{mpi} value for all p-particles and the best one y_i^* value among p-particles.

Step6.3: Calculate the new velocity vector and update the position.

$$v_{mpi}(k+1) = wv_{mpi}(k) + c_1 r_1(k)(\mathrm{pb}_{mpi}(k) - y_{mpi}(k)) + c_2 r_2(k)(y_i^*(k) - y_{mpi}(k)),$$
$$y_{mpi}(k+1) = y_{mpi}(k) + v_{mpi}(k+1)$$

Step6.4: $g_f \leftarrow g_f + 1$

Step6.5: If it find follower i's optimal strategy in response to the other followers' optimal strategy, then go to Step7. Otherwise, we go to Step6.3.

Step7: If it already finds all follower i's optimal strategy in response to the other followers' optimal strategy, then go to Step8. Otherwise, $i \leftarrow i + 1$, then we go to Step4.

Step8: If no SU would change its optimal strategy, then output y_i^* as the response from the ith follower. Otherwise, we go to Step3.

Algorithm 3: Generate optimal strategies for a leader

Step1: Randomly generate the position x_m and the corresponding velocities v_m for mth particle among a pre-defined range, $m = 1,\ldots, M_l$.

Step2: Initiate the leader's loop counter $g_l = 0$

Step3: For mth particle, $m = 1,\ldots,M_l$, calculate the optimal response from ith follower by Algorithm 2, $i = 1,\ldots,N$

Step4: Compute the leader's objection function F in (14) of each particle, and select the previously visited best position pb_m value for all m-particles and the best one x^* value among m-particles.

Step5: Calculate the new velocity vector and update the position

$$v_m(k+1) = wv_m(k) + c_1 r_1(k)(\mathrm{pb}_m(k) - x_m(k)) + c_2 r_2(k)(x^* - x_m(k))$$

$$x_m(k+1) = x_m(k) + v_m(k+1)$$

Step6: $g_l \leftarrow g_l + 1$.

Step7: If $g_l \geq \max\_g_l$ or the solution changes for several consecutive generations are small enough, then we use stretching technology obtain the global solution for the current leader's solution. Otherwise, we go to Step3.

6. Performance evaluation

In this section, we present two different spectrum sharing models. We employ a strategic pricing problem in a CR market to test the NLMF model with Nash-PSO algorithm and the NBMF decision model with NBMF-PSO algorithm developed in this article. We consider the cognitive radio environment with one PU and two SUs sharing a frequency spectrum of 15 MHz. We use the same parameters and the same method as [24] compared with the Nash-PSO algorithm. We obtain the same result with [24] dynamic game.

6.1 The Nash-PSO algorithm

We consider a cognitive radio environment with one PU and three SUs sharing a frequency spectrum $B^{\mathrm{tot}} = 25$ MHz.

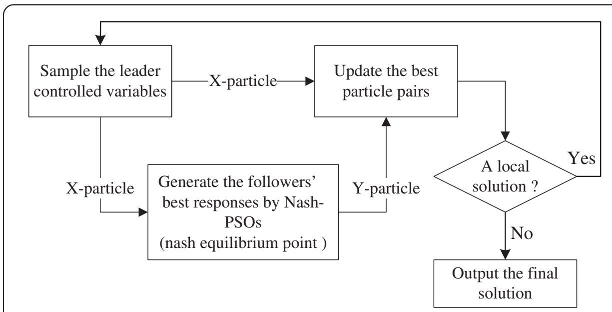

Figure 2 The framework of NBMF-PSO algorithm.

The target BER ($\mathrm{BER}^{\mathrm{tar}}$) is equal to 10^{-4} for three SUs. The revenue of an SU per unit transmission rate (R_i) is 12 for each user. For the price function of PU, we use $l = 0$, $q = 1$, $w_1 = 1$, $w_2 = 0$, $b_{t\min} = 0$, and $b_{t\max} = 10$.

Figures 3 and 4 show the best response of three SUs in the NLMF model obtained by (11). Figure 3 uses the parameters $\mathrm{SU}_1(\gamma_1 = 7$ dB), $\mathrm{SU}_2(\gamma_2 = 8$ dB), and $\mathrm{SU}_3(\gamma_3 = 9$ dB), while Figure 4 uses the parameters $\mathrm{SU}_1(\gamma_1 = 8$ dB), SU_2 ($\gamma_2 = 9$ dB), and $\mathrm{SU}_3(\gamma_3 = 10$ dB). The best response of each SU is a linear function of the other user's strategy. For the three user scenario, the best response function for each player is a plane, and the Nash equilibrium is located at the intersection point of three planes. For different channel quality, the Nash equilibrium will locate at the different places. SU can obtain higher transmission rate from the same spectrum size by adaptive modulation; hence, an SU with better spectral efficiency prefers to have a larger spectrum size to gain higher profit. In addition, the trajectory of spectrum sharing is shown for Nash-PSO best strategies in all iterations converging to the Nash equilibrium, which is considered to be the solution of the spectrum sharing scheme.

The adaptation of NE under different channel qualities is presented in Figure 5, while the SUs' revenues with respect to channel quality are shown in Figure 6. Again, by improving channel quality, SUs increase demands to earn

Table 1 Important notations used in this article

Notation	Meaning
M_l	The number of particles for the leader
M_f	The number of particles for followers
x_m	The m_{th} particles for the leader, $x_m = (x_{m1}, x_{m2}, \ldots, x_{mn})^T$, $m = 1, \ldots, M_l$
v_m	The velocity of x_m, $v_m = (v_{m1}, v_{m2}, \ldots, v_{mn})^T$, $m = 1, \ldots, M_l$
y_m	The follower's choice for each x_m from the leader, $y_m = (y_{m1}, y_{m2}, \ldots, y_{mw})^T$
y_{mp}	The p_{th} particles by the follower for the choice x_m from the leader, $y_{mp} = (y_{mp1}, v_{mp2}, \ldots, v_{mpw})^T$, $p = 1, \ldots, M_f$
v_{mp}	The velocity of y_{mp}, $v_{mp} = (v_{mp1}, v_{mp2}, \ldots, v_{mpw})^T$, $p = 1, \ldots, M_f$
pb_m	The best previously visited solution of x_m, $\mathrm{pb}_m = (\mathrm{pb}_{m1}, \mathrm{pb}_{m2}, \ldots, \mathrm{pb}_{mn})^T$
pb_{mp}	The best previously visited solution of y_{mp}, $\mathrm{pb}_{mp} = (\mathrm{pb}_{mp1}, \mathrm{pb}_{mp2}, \ldots, \mathrm{pb}_{mpw})^T$
x^*	Current best solution for particle x_m from the leader
y^*	Current best solution for particle y_{mp} from the follower
y_{mpi}	The y_{mp} value of the ith follower, $i = 1, \ldots, N$
v_{mpi}	The v_{mp} value of the ith follower, $i = 1, \ldots, N$
y_i^*	The y^*value of the ith follower, $i = 1, \ldots, N$
g_l	Current iteration number for the upper-level problem
g_f	Current iteration number for the lower-level problem
max_g_l	The predefined max iteration number for g_l
max_g_f	The predefined max iteration number for g_f

more payoffs. As expected, SU_1 shares a larger spectrum size with the PU and achieves a higher revenue when its channel quality becomes better. In these figures, we considered a fixed channel quality for SU_2 ($\gamma_2 = 7$ dB) and SU_3 ($\gamma_3 = 8$ dB), while the quality of SU_1 changes from 5 to 11 dB. The size of shared spectrum and revenue proposed by SU_2 are higher than those proposed by SU_1, as the channel quality of SU_1 is less than 7 dB. Similarly, the size of the shared spectrum and revenue proposed by SU_3 are higher than those proposed by SU_1, as the channel quality of SU_1 is less than 8 dB. By improving the channel quality of SU_1, the shared spectrum sizes and revenue offered by the SU_1 will be higher, as the channel quality of SU_1 is greater than 8 dB. The channel quality of an SU will impact the allocated spectrum size and revenue for the other SUs, which replies the impact of competitive strength among the SUs' strategies.

Figure 7 shows the evolution of the best response for three SUs converging to the Nash equilibrium point by the Nash-PSO algorithm. We considered a fixed channel quality for SU_1 ($\gamma_1 = 8$ dB), SU_2 ($\gamma_2 = 9$ dB), and SU_3 ($\gamma_3 = 10$ dB). The spectrum sharing gradually converges to the Nash equilibrium, where the Nash equilibrium point is at (1.7038, 4.0268, 6.5675).

6.2 The NBMF-PSO algorithm

We consider a cognitive radio environment with one PU and three SUs sharing a frequency spectrum $B^{\mathrm{tot}} = 25$ MHz. The $\mathrm{BER}^{\mathrm{tar}}$ is equal to 10^{-4} for three SUs. The revenue of an SU per R_i is 12 for each user. As an example, we assume that PU is fixed at (0, 0), and SU_1, SU_2, and SU_3 are movable and begin at (0,102), (104, 0), and (0, −106), respectively. At $t = 0$, SU_1, SU_2, and SU_3 move along a straight line at the same velocity of 0.085 m/s starting from (0, 102), (104, 0), and (0, −106) to (91.8, 102), (104, −91.8), and (−91.8, −106), respectively, by applying the NBMF-PSO algorithm as shown in Figure 8.

Generally, path loss is proportional to the reciprocal of the fourth power of the distance between the transmitter and receiver if the time-varying fading is not considered. The transmit power of an SU is equal to 0.01 W, and the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2 = 10^{-11}$ W is considered at the input of the receiver. For SU_i, its SNR can be calculated

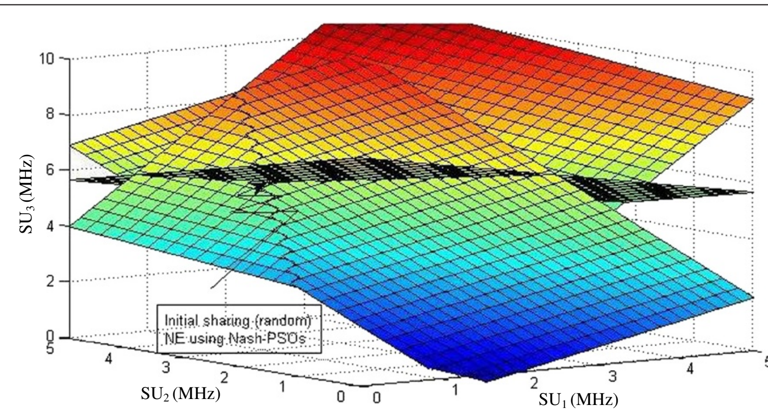

Figure 3 Best response and trajectories to NE when $\gamma_1 = 7$ dB, $\gamma_2 = 8$ dB, $\gamma_3 = 9$ dB.

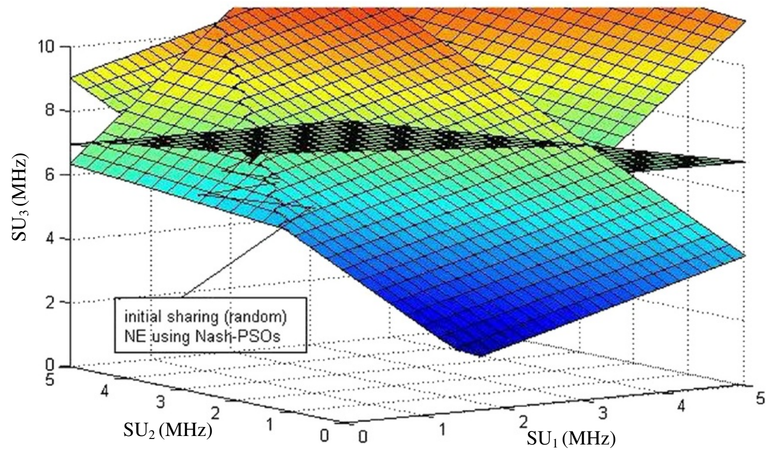

Figure 4 Best response and trajectories to NE when $\gamma_1 = 8$ dB, $\gamma_2 = 9$ dB, $\gamma_3 = 10$ dB.

by (17), where P_i is the transmit power of SU$_i$, and d_i is the distance between SU$_i$ and PU:

$$\gamma_i = P_i d_i^{-4} / \sigma^2. \tag{17}$$

In the simulation, we consider an 18-interval dynamic game for simulations. To simplify the computation, the limit of the coefficients does not vary by different time slots. To design the values of $l_{t\min}$, $l_{t\max}$, $q_{t\min}$, and $q_{t\max}$, it is necessary to ensure that the PU is willing to share spectrum with the SUs, where we set $w_1 = 0.62$, $w_2 = 0.72$, $l_{t\min} = 0.154$, $l_{t\max} = 0.679$, $q_{t\min} = 0.523$, $q_{t\max} = 0.627$, $b_{t\min} = 0$, and $b_{t\max} = 10$ to satisfy this condition. However, this article does not discuss about the lendable time of the spectrum price, but it discusses the time variety of different channel qualities between the different prices of the NLMF and NBMF models. In the NBMF model, the price is $13.30 per unit spectrum as the channel quality is the best, while the price is $4.63 per unit spectrum as the channel quality is the worst.

When the cost of the spectrum offered by the PU is not higher than the revenue gained from the allocated spectrum, an SU is willing to stay the shared spectrum with the competition. Since the location of PU is fixed, the channel quality varies with time as SU$_1$, SU$_2$, and SU$_3$ move, and its SNR and spectral efficiency vary as well. This variation in spectral efficiency affects the amount of spectrum demanded in each interval by SU$_1$, SU$_2$, and SU$_3$.

We use Nash-PSO and NBMF-PSO to compare the NLMF model (11) with the NBMF model (14), respectively, at the same condition. According to (6), the price for the unit of the shared frequency spectrum is an ascendant function of the sharing spectrum size. In the NLMF model, the price function has the fixed $l = 0$ and $q = 1$ values, only to acquire the optimum utility function (11) of SUs. In the NBMF model, we further consider the profit of PU. By adapting the values of the coefficients l and q of the price function, the PU's slope of the price function can be changed, which helps PU achieve more demands from SUs in each interval of the dynamic spectrum

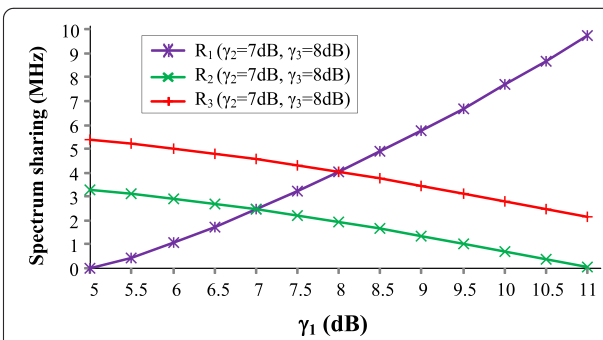

Figure 5 NE of spectrum sharing under different channel qualities.

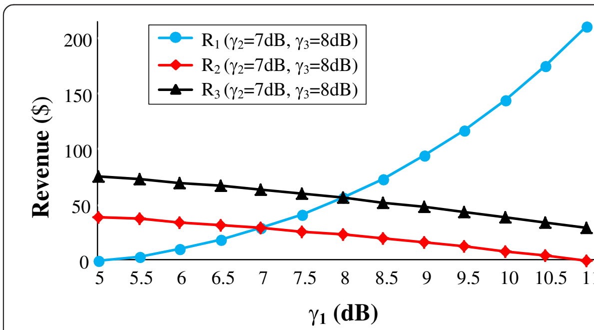

Figure 6 NE of revenue under different channel qualities.

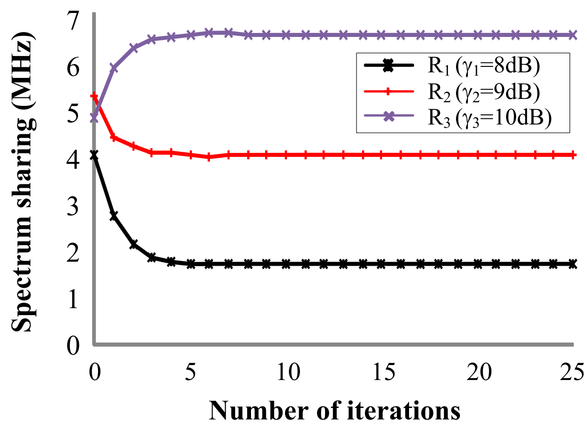

Figure 7 Convergence process of three SUs by applying the Nash-PSO algorithm.

sharing game. With the aim of maximizing both PU and SUs' profits, PU decides how many spectrums are to be lent based on the price each SU pays for the spectrum. The NBMF model obtains more size of sharing frequency than the NLMF model, and enhances the utility of the resource. Figure 9 shows the running results for l_t and q_t in the NBMF model.

Figures 10 and 11 show the shared spectrum sizes and price, respectively, for unit of the shared frequency spectrum under different pricing models. In Figure 10, an SU shares a smaller spectrum size with PU, because

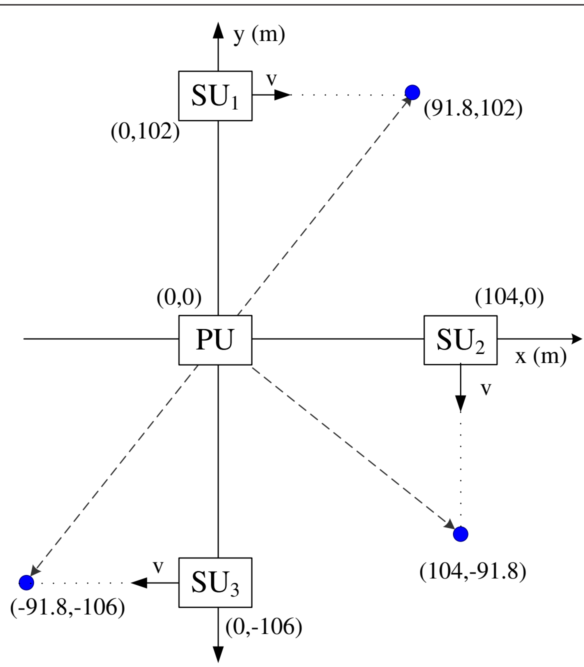

Figure 8 As an example, one PU and three SUs move by applying the NBMF-PSO algorithm.

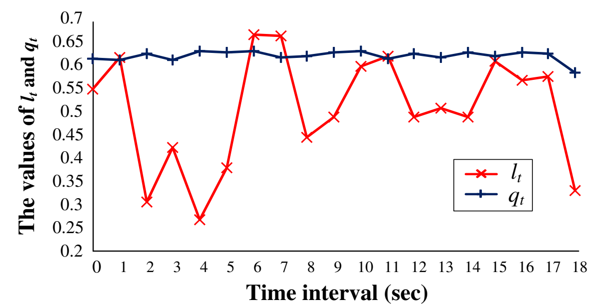

Figure 9 The running results of l_t and q_t for the NBMF model.

its channel quality becomes worse as the time and the distance become longer. In Figure 11, the PU's offered price for unit of the shared spectrum decreases, because the shared spectrum sizes decrease as the channel quality becomes worse. In the NBMF model, it is apparent that the sharing spectrum size increases fast at good channel quality. Because q is small in the NBMF model, the demand of the sharing spectrum size becomes larger by each SU, and PU can gain higher price for the shared spectrum under more competition. The NBMF model performs better than NLMF model in the same situation by having larger size of sharing spectrum for each SU and higher unit price of the sharing spectrum for PU.

Figures 12 and 13 show the total shared spectrum sizes and the revenue of the unit of frequency spectrum, respectively, for the different pricing models. In Figure 12, SUs can share more spectral from PU, and consequently, the total size of the spectrum offered by PU increases in the NBMF model. Similarly, in Figure 13, the revenue is relatively high in the NBMF model. To increase PU's revenue, PU is more willing to lend spectrum to SUs at a higher price. As a result, SUs have to compete with each other only through increasing b_i for the spectrum demand. Once the spectrum demand is improved, the frequency allocation is enhanced for SUs, and the price may become higher for PU.

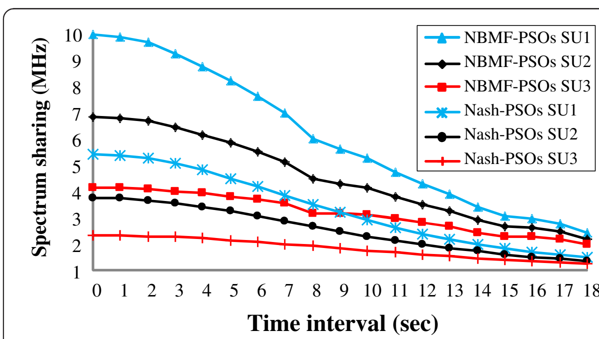

Figure 10 Three SUs compete spectrum sharing for the NLMF and NBMF models.

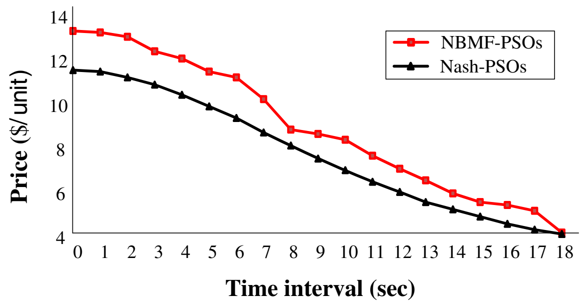

Figure 11 Price per unit spectrum for the NLMF and NBMF models.

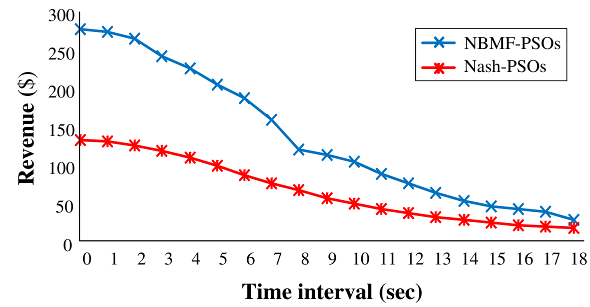

Figure 13 PU's revenue for the NLMF and NBMF models.

Therefore, the NBMF model has larger revenue than the NLMF model.

Simulation results show that our proposed NLMF model utilizing swarm particle algorithm converges fast to the Nash equilibrium, and the NBMF model satisfies both groups of PU and SUs to enlarge the PU's revenue and acquire larger shared spectrum sizes for SUs. The NBMF model pays a lower price than the NLMF model at the same shared spectrum sizes. The shared spectrum sizes may be increased by improving the channel quality, and PU can offer larger spectrum sizes with higher price. PU provides reasonable price to SUs when the number of SUs increases.

7. Conclusions

Pricing is an important issue not only to maximize the revenue of PU, but also to allocate the radio spectrum sharing with SUs efficiently. In this article, we discussed the challenges in designing resource allocation and pricing in cognitive radio network. We have proposed a competitive spectrum sharing and pricing scheme based on noncooperative game for a cognitive radio network consisting of a PU and N SUs. We presented a dynamic game in which an SU adapts its spectrum sharing strategy by observing only the strategy which is a function of spectrum price offered by the PU. By analyzing the strategic pricing behavior of PU, we created an NLMF model and a specific NBMF decision model for oligopoly market in cognitive radio network. The NBMF model has applied NBMF-PSO algorithms to iteratively obtain the solution of this game for searching for the optimal solution of bilevel programming models.

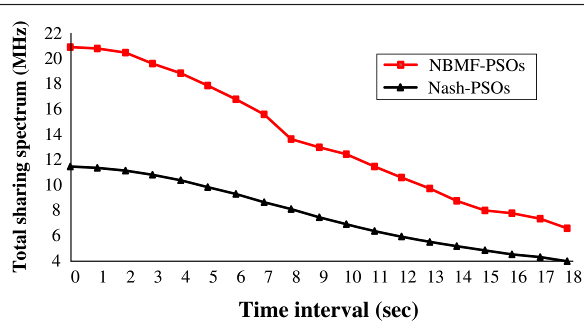

Figure 12 Total size of sharing frequency spectrum for the NLMF and NBMF models.

Numerical studies were carried out to evaluate the performances of the two different pricing models. In the proposed NBMF model, PU can share more spectrum sizes with higher price for unit of shared frequency spectrum to SUs, while PU achieves higher revenue and SU acquires more size of the sharing spectrum by the same price. The NBMF model is more satisfactory for both groups of PU and SUs than NLMF, and enlarges the PU's revenue and provides reasonable price to SUs.

This article applies a strategic pricing problem in a cognitive radio market to help for both PU and SUs to make the strategic decisions by the Nash-PSO and NBMF-PSO algorithms. We combine swarm particle algorithms with Nash strategy based on noncooperative game theory to obtain the Nash equilibrium in multiple-objective optimization problems. By thinking of the gaming and bilevel relationship between PU and SUs, the NBMF decision model can better reflect the features of the real-world strategic pricing problems in the cognitive radio markets and format these problems more practically. The proposed NBMF-PSO algorithm is quite effective for solving the strategic pricing problems defined by the NBMF decision model. In the literature, no other algorithm exists hierarchically for the strategic pricing problems when both the gaming and bilevel relationships are considered between PU and SUs. The NBMF-PSO algorithm makes SUs using Nash-PSO algorithm to gain their rational reactions and reach the Nash equilibrium, and PU obtains the optimal pricing and acquires the highest profit. Further research work will focus on building the optimal strategic pricing models for the multiple PUs and multiple SUs in cognitive radio network.

Competing interests

The authors declare that they have no competing interests.

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Author details

[1]National Taiwan University of Science and Technology, 43, Keelung Rd., Section 4, Taipei 106, Taiwan. [2]Ling Tung University, Taichung 408, Taiwan.

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