APPROXIMATION COMMON FIXED POINT OF ASYMPTOTICALLY QUASI-NONEXPANSIVE-TYPE MAPPINGS BY THE FINITE STEPS ITERATIVE SEQUENCES

JING QUAN, SHIH-SEN CHANG, AND XIAN JUN LONG

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The purpose of this paper is to study sufficient and necessary conditions for finite-step iterative sequences with mean errors for a finite family of asymptotically quasi-nonexpansive and type mappings in Banach spaces to converge to a common fixed point. The results presented in this paper improve and extend the recent ones announced by Ghost-Debnath, Liu, Xu and Noor, Chang, Shahzad et al., Shahzad and Udomene, Chidume et al., and all the others.

1. Introduction and preliminaries

Throughout this paper, we assume that E is a real Banach space, $F(T)$, $D(T)$, and N denote the set of fixed points of T, the domain of T, and the set of positive integers, respectively.

Definition 1.1. Let $T : D(T) = E \to E$ be a mapping.

(1) T is said to be *quasi-nonexpansive* if $F(T) \neq \emptyset$ and $\|Tx - p\| \leq \|x - p\|$, for all $x \in E$ and $p \in F(T)$.

(2) T is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\}$ of positive real numbers with $k_n \geq 1$ and $\lim_{n\to+\infty} k_n = 1$, such that $\|T^n x - T^n y\| \leq k_n\|x - y\|$, for all $x, y \in E$ and $n \in N$.

(3) T is said to be *asymptotically quasi-nonexpansive* if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\}$ of positive real numbers with $k_n \geq 1$ and $\lim_{n\to+\infty} k_n = 1$ such that $\|T^n x - p\| \leq k_n\|x - p\|$, for all $x \in E$, $p \in F(T)$, and all $n \in N$.

(4) T is said to be *asymptotically nonexpansive* type if

$$\limsup_{n\to\infty}\left\{\sup_{x,y\in E}\left[\left\|T^n x - T^n y\right\|^2 - \|x - y\|^2\right]\right\} \leq 0. \tag{1.1}$$

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(5) T is said to be *asymptotically quasi-nonexpansive* type if

$$\limsup_{n\to\infty}\left\{\sup_{x\in E,\, y\in F(T)}\left[\left\|T^n x - p\right\|^2 - \|x-p\|^2\right]\right\} \le 0. \tag{1.2}$$

From the above definitions, it follows that if $F(T)$ is nonempty, quasi-nonexpensive mappings, asymptotically nonexpensive mappings, asymptotically quasi-nonexpensive mappings, and asymptotically nonexpensive type-mappings are all special cases of asymptotically quasi-nonexpensive-type mappings.

Definition 1.2 (see [2]). Let $T_1, T_2, T_3 : E \to E$ be asymptotically quasi-nonexpansive-type mappings. Let $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ be three given sequences in E and let x_1 be a given point. Let $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$, $\{\eta_n\}$, $\{\xi_n\}$ be sequences in [0,1] satisfying the following conditions:

$$\begin{gathered}\alpha_n + \gamma_n \le 1, \qquad \beta_n + \delta_n \le 1, \qquad \eta_n + \xi_n \le 1, \\ \sum_{n=1}^{\infty} \gamma_n < \infty, \qquad \sum_{n=1}^{\infty} \delta_n < \infty, \qquad \sum_{n=1}^{\infty} \xi_n < \infty.\end{gathered} \tag{1.3}$$

Then the sequence $\{x_n\} \subset E$ defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n-\gamma_n)x_n + \alpha_n T_1^n y_n + \gamma_n u_n, \quad n \ge 1, \\ y_n &= (1-\beta_n-\delta_n)x_n + \beta_n T_2^n z_n + \delta_n v_n, \quad n \ge 1, \\ z_n &= (1-\eta_n-\xi_n)x_n + \eta_n T_3^n x_n + \xi_n w_n, \quad n \ge 1, \end{aligned} \tag{1.4}$$

is called the *three-step iterative sequence with mean errors* of T_1, T_2, T_3.

Let $T_1, T_2, \dots, T_N : E \to E$ be N asymptotically quasi-nonexpansive-type mappings. Let x_1 be a given point. Then the sequence $\{x_n\}$ defined by

$$\begin{aligned} x_{n+1} &= (1-a_{n1}-b_{n1})x_n + a_{n1} T_1^n y_{n1} + b_{n1} u_{n1}, \\ y_{n1} &= (1-a_{n2}-b_{n2})x_n + a_{n2} T_2^n y_{n2} + b_{n2} u_{n2}, \\ &\vdots \\ y_{nN-2} &= (1-a_{nN-1}-b_{nN-1})x_n + a_{nN-1} T_{N-1}^n y_{nN-1} + b_{nN-1} u_{nN-1}, \\ y_{nN-1} &= (1-a_{nN}-b_{nN})x_n + a_{nN} T_N^n x_n + b_{nN} u_{nN}, \end{aligned} \tag{1.5}$$

is called the *N-step iterative sequence with mean errors* of $T_1, T_2, \dots, T_N$, where $\{u_{n^i}\}_{n=1}^{\infty}$, $i = 1,2,\dots,N$, are N sequences in E, $\{a_{n^i}\}_{n=1}^{\infty}$, $\{b_{n^i}\}_{n=1}^{\infty}$, $i = 1,2,\dots,N$, are N sequences in $[0,1]$ satisfying the following conditions:

$$\begin{gathered} a_{n^i} + b_{n^i} \le 1, \quad n \le 1,\ i = 1,2,\dots,N, \\ \sum_{n=1}^{\infty} b_{n^i} < \infty, \quad i = 1,2,\dots,N. \end{gathered} \tag{1.6}$$

Petryshyn and Williamson [9] proved a sufficient and necessary condition for the Mann iterative sequences to converge to a fixed point for quasi-nonexpansive mappings. Ghosh and Debnath [5] extended the result of [9] and gave a sufficient and necessary condition for the Ishikawa iterative sequence to converge to a fixed point for quasi-nonexpansive mappings. Liu [6–8] extended the above results and proved some sufficient and necessary conditions for the Ishikawa iterative sequence or the Ishikawa iterative sequences with errors for asymptotically quasi-nonexpansive mappings to converge to a fixed point. Chidume et al. [4] obtained a strong convergence theorem to a fixed point of a family of nonself nonexpansive mappings in Banach spaces by an algorithm for nonself-mappings. Shahzad and Udomene [10] established necessary and sufficient conditions for the convergence of the Ishikawa-type iterative sequences involving two asymptotically quasi-nonexpansive mappings to a common fixed point of the mappings defined on a nonempty closed convex subset of a Banach space and a sufficient condition for the convergence of the Ishikawa-type iterative sequences involving two uniformly continuous asymptotically quasi-nonexpansive mappings to a common fixed point of the mappings defined on a nonempty closed convex subset of a uniformly convex Banach space. Alber [1] studied the approximating methods for finding the fixed points of asymptotically nonexpansive mappings.

Recently, Chang et al. [2] complement, improve, and perfect all the above results and obtained some necessary and sufficient conditions for the Ishikawa iterative sequence with mixed errors of asymptotically quasi-nonexpansive-type mappings in Banach spaces to converge to a fixed point in Banach spaces. And also using the N-step iterative sequences (1.5), Chang et al. [3] proved the weak and strong convergence of finite steps iterative sequences with mean errors to a common fixed point for a finite family of asymptotically nonexpansive mappings.

The purpose of this paper is to study sufficient and necessary conditions for finite-step iterative sequences with mean errors for a finite family of asymptotically quasi-nonexpansive-type mappings in Banach spaces to converge to a common fixed point. Our result shows that [2, Condidtion (2.1) in Theorem 2.1] can be removed. The results present in this paper improve, extend, and perfect the recent ones announced by Petryshyn and Williamson [9], Ghost and Debnath [5], Liu [6, 7], Xu and Noor [12], Chang [2, 3], Shahzad et al. [4], Shahzad and Udomene [10], Chidume et al. [1], and all the others.

In order to prove our main results, we will need the following lemma.

Lemma 1.3 (see [11]). *Let $\{a_n\},\{b_n\}$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} \le a_n + b_n, \quad n \ge 1. \tag{1.7}$$

If $\sum_{n=1}^{\infty} b_n < \infty$, then $\lim_{n\to\infty} a_n$ exists.

2. Main results

Theorem 2.1. *Let E be a Banach space and $T_i : E \to E$ $(i = 1,2,\dots,N)$ be N asymptotically quasi-nonexpansive-type mappings with a nonempty fixed-point set $F(T) = \bigcap_{i=1}^{N} F(T_i)$, that* *is,*

$$\limsup_{n\to\infty}\left\{\sup_{x\in E,\, p\in F(T)}\left[\left\|T_i^n x - p\right\|^2 - \|x-p\|^2\right]\right\} \le 0, \quad i = 1,2,\dots,N. \tag{2.1}$$

Let $\{u_{n^i}\}$ *be a bounded sequence in E. For any given point* x_1 *in E, generate the sequence* $\{x_n\}$ *defined by (1.5). If* $\sum_{n=1}^{\infty}\alpha_{n^i} < \infty$*, then sequence* $\{x_n\}$ *strongly converges to a common fixed point of* T_i $(i = 1,2,\dots,N)$ *if and only if* $\liminf_{n\to\infty} d(x_n, F(T)) = 0$*, where* $d(y,S)$ *denotes the distance of* y *to set S; that is,* $d(y,S) = \inf_{s\in S}\|y-s\|$.

Proof. (1) For the sake of convenience, we prove the conclusion only for the case of $N = 3$ and then the other cases can be proved by the same way. For the purpose, let $\alpha_n = a_{n^1}$, $\beta_n = a_{n^2}$, $\eta_n = a_{n^3}$, $\gamma_n = b_{n^1}$, $\delta_n = b_{n^2}$, $\xi_n = b_{n^3}$. Then we can consider the sequence $\{x_n\}$ defined by (1.4) and $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ are bounded. For all $p \in F(T)$, let

$$\begin{aligned} M_1 &= \sup\{\|u_n - p\|\} : n \ge 1, \qquad M_2 = \sup\{\|v_n - p\|\} : n \ge 1, \\ M_3 &= \sup\{\|w_n - p\|\} : n \ge 1, \qquad M = \max\{M_i : i = 1,2,3\}. \end{aligned} \tag{2.2}$$

It follows from (2.1) that

$$\begin{aligned} &\limsup_{n\to\infty}\left\{\sup_{x\in E,\, p\in F(T)}\left[\left(\left\|T_i^n x - p\right\| - \|x-p\|\right)\left(\left\|T_i^n x + p\right\| - \|x-p\|\right)\right]\right\} \\ &\quad = \limsup_{n\to\infty}\left\{\sup_{x\in E,\, p\in F(T)}\left[\left\|T_i^n x - p\right\|^2 - \|x-p\|^2\right]\right\} \le 0, \quad i = 1,2,3. \end{aligned} \tag{2.3}$$

Therefore we have

$$\limsup_{n\to\infty}\left\{\sup_{x\in E,\, p\in F(T)}\left[\left\|T_i^n x - p\right\| - \|x-p\|\right]\right\} \le 0, \quad i = 1,2,3. \tag{2.4}$$

This implies that for any given $\epsilon > 0$, there exists a positive integer n_0 such that for $n \ge n_0$, we have

$$\sup_{x\in E,\, p\in F(T)}\left\{\left\|T_i^n x - p\right\| - \|x-p\|\right\} < \epsilon, \quad i = 1,2,3. \tag{2.5}$$

Since $\{x_n\},\{y_n\},\{z_n\} \subset E$, we have

$$\left\|T_1^n y_n - p\right\| - \left\|y_n - p\right\| < \epsilon, \quad \forall p \in F(T),\ \forall n \ge n_0, \tag{2.6}$$

$$\left\|T_2^n z_n - p\right\| - \left\|z_n - p\right\| < \epsilon, \quad \forall p \in F(T),\ \forall n \ge n_0, \tag{2.7}$$

$$\left\|T_3^n x_n - p\right\| - \left\|x_n - p\right\| < \epsilon, \quad \forall p \in F(T),\ \forall n \ge n_0. \tag{2.8}$$

Thus for any $p \in F(T)$, using (1.4) and (2.6), we have

$$\begin{aligned} ||x_{n+1}-p|| &= ||(1-\alpha_n-\gamma_n)(x_n-p)+\alpha_n(T_1^n y_n-p)+\gamma_n(u_n-p)|| \\ &\leq (1-\alpha_n-\lambda_n)||x_n-p||+\alpha_n(||T_1^n y_n-p||-||y_n-p||) \\ &\quad +\alpha_n||y_n-p||+\gamma_n||u_n-p|| \\ &\leq (1-\alpha_n-\lambda_n)||x_n-p||+\alpha_n\epsilon+\alpha_n||y_n-p||+\gamma_n M. \end{aligned} \tag{2.9}$$

Consider the third term in the right-hand side of (2.9), using (1.4) and (2.7), we have that

$$\begin{aligned} ||y_n-p|| &= ||(1-\beta_n-\delta_n)(x_n-p)+\beta_n(T_2^n z_n-p)+\delta_n(v_n-p)|| \\ &\leq (1-\beta_n-\delta_n)||x_n-p||+\beta_n(||T_2^n z_n-p||-||z_n-p||) \\ &\quad +\beta_n||z_n-p||+\delta_n||v_n-p|| \\ &\leq (1-\beta_n-\delta_n)||x_n-p||+\beta_n\epsilon+\beta_n||z_n-p||+\delta_n M. \end{aligned} \tag{2.10}$$

Consider the third term in the right-hand side of (2.10), using (1.4) and (2.8), we have that

$$\begin{aligned} ||z_n-p|| &= ||(1-\eta_n-\xi_n)(x_n-p)+\eta_n(T_3^n x_n-p)+\xi_n(w_n-p)|| \\ &\leq (1-\eta_n-\xi_n)||x_n-p||+\eta_n(||T_3^n x_n-p||-||x_n-p||) \\ &\quad +\eta_n||x_n-p||+\xi_n||w_n-p|| \\ &\leq (1-\xi_n)||x_n-p||+\eta_n\epsilon+\xi_n M. \end{aligned} \tag{2.11}$$

Substituting (2.11) into (2.10) and simplifying, we have

$$||y_n-p|| \leq (1-\beta_n\xi_n-\delta_n)||x_n-p||+\beta_n\epsilon(1+\eta_n)+\beta_n\xi_n M+\delta_n M. \tag{2.12}$$

Substituting (2.12) into (2.9) and simplifying, we have

$$\begin{aligned} ||x_{n+1}-p|| &\leq (1-\gamma_n-\alpha_n\beta_n\xi_n-\alpha_n\delta_n)||x_n-p||+\alpha_n\epsilon+\alpha_n\beta_n\epsilon(1+\eta_n) \\ &\quad +\alpha_n\delta_n M+\alpha_n\beta_n\xi_n M+\gamma_n M \\ &\leq ||x_n-p||+\alpha_n(1+\beta_n+\beta_n\eta_n)\epsilon+(\gamma_n+\delta_n+\xi_n)M \\ &\leq ||x_n-p||+3\alpha_n\epsilon+(\gamma_n+\delta_n+\xi_n)M. \end{aligned} \tag{2.13}$$

Let $A_n = 3\alpha_n\epsilon+(\gamma_n+\delta_n+\xi_n)M$. Then $A_n \geq 0$. It follows from (1.3) and $\sum_{n=1}^{\infty}\alpha_{n^i} < \infty$ that $\sum_{n=1}^{\infty} A_n < \infty$. Then by (2.13), we have

$$||x_{n+1}-p|| \leq ||x_n-p||+A_n. \tag{2.14}$$

It follows from (2.14) and $\sum_{n=1}^{\infty} A_n < \infty$ that

$$d(x_{n+1},F(T)) \leq d(x_n,F(T))+A_n. \tag{2.15}$$

By Lemma 1.3, we know that $\lim_{n\to\infty} d(x_n,F(T))$ exists. Because $\liminf_{n\to\infty} d(x_n,F(T)) = 0$, then we have

$$\lim_{n\to\infty} d(x_n,F(T)) = 0. \tag{2.16}$$

Next we prove that $\{x_n\}$ is a Cauchy sequence in E.

It follows from (2.14) that for any $m \geq 1$, for all $n \geq n_0$, for all $p \in F(T)$,

$$\begin{aligned} ||x_{n+m} - p|| &\leq ||x_{n+m-1} - p|| + A_{n+m-1} \\ &\leq ||x_{n+m-2} - p|| + (A_{n+m-1} + A_{n+m-2}) \\ &\leq \cdots \leq ||x_n - p|| + \sum_{k=n}^{n+m-1} A_k. \end{aligned} \tag{2.17}$$

So by (2.17), we have

$$||x_{n+m} - x_n|| \leq ||x_{n+m} - p|| + ||x_n - p|| \leq 2||x_n - p|| + \sum_{k=n}^{\infty} A_k. \tag{2.18}$$

By the arbitrariness of $p \in F(T)$ and (2.18), we know that

$$||x_{n+m} - x_n|| \leq 2d(x_n,F(T)) + \sum_{k=n}^{\infty} A_k, \quad \forall n \geq n_0. \tag{2.19}$$

For any given $\bar{\epsilon} > 0$, there exists a positive integer $n_1 \geq n_0$ such that for any $n \geq n_1$, $d(x_n,F(T)) < \bar{\epsilon}/4$ and $\sum_{k=n}^{\infty} A_k < \bar{\epsilon}/2$. Thus when $n \geq n_1$, $\|x_{n+m} - x_n\| < \bar{\epsilon}$. So we have that

$$\lim_{n\to\infty} ||x_{n+m} - x_n|| = 0. \tag{2.20}$$

This implies that $\{x_n\}$ is a Cauchy sequence in E. Since E is complete, there exists a $p^* \in E$ such that $x_n \to p^*$ as $n \to \infty$.

Now we have to prove that p^* is a common fixed point of T_i, $i = 1,2,\ldots,N$, that is, $p^* \in F(T)$.

By contradiction, we assume that p^* is not in $F(T)$. Since $F(T)$ is closed in Banach spaces, $d(p^*,F(T)) > 0$. So for all $p \in F(T)$, we have

$$||p^* - p|| \leq ||p^* - x_n|| + ||x_n - p||. \tag{2.21}$$

By the arbitrary of $p \in F(T)$, we know that

$$d(p^*, F(T)) \leq ||p^* - x_n|| + d(x_n,F(T)). \tag{2.22}$$

By (2.16), above inequality and $x_n \to p^*$ as $n \to \infty$, we have

$$d(p^*, F(T)) = 0, \tag{2.23}$$

which contracts $d(p^*, F(T)) > 0$. This completes the proof of Theorem 2.1. □

Corollary 2.2. *Suppose the conditions in Theorem 2.1 are satisfied. Then the N-step iterative sequence $\{x_n\}$ generated by (1.5) converges to a common fixed point $p \in E$ if and only if there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ which converges to p.*

Theorem 2.3. *Let E be a Banach space and let $T_i : E \to E$ $(i = 1,2,\dots,N)$ be N asymptotically quasi-nonexpansive mappings with a nonempty fixed-point set $F(T) = \bigcap_{i=1}^N F(T_i)$. Let $\{u_{n^i}\}$ be a bounded sequence in E. For any given point x_1 in E, generate the sequence $\{x_n\}$ by (1.5). If $\sum_{n=1}^{\infty} \alpha_{n^i} < \infty$, then sequence $\{x_n\}$ strongly converges to a common fixed point of T_i $(i = 1,2,\dots,N)$ if and only if $\liminf_{n\to\infty} d(x_n, F(T)) = 0$, where $d(y,S)$ denotes the distance of y to set S.*

Proof. Since T_i are asymptotically quasi-nonexpansive mappings with a nonempty fixed-point set $F(T) = \bigcap_{i=1}^N F(T_i)$, by [3, Proposition 1] or [13], we know that there must exist a sequence $\{k_n\} \subset [1,\infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\left\| T_i^n x - p \right\| \le k_n \|x - p\|, \quad \forall p \in F(T), \forall x \in E, n \ge 1. \tag{2.24}$$

This implies that

$$\left\| T_i^n x - p \right\|^2 - (k_n)^2 \|x - p\|^2 \le 0, \quad \forall p \in F(T), \forall x \in E, n \ge 1. \tag{2.25}$$

Therefore we have

$$\limsup_{n\to\infty} \left\{ \sup_{x \in D, p \in F(T)} \left[\left\| T_i^n x - p \right\|^2 - \|x - p\|^2 \right] \right\} \le 0, \quad i = 1,2,\dots,N. \tag{2.26}$$

This implies that T_i, $i = 1,2,\dots,N$, are N asymptotically quasi-nonexpansive-type mappings with a nonempty fixed-point set $F(T) = \bigcap_{i=1}^N F(T_i)$. Theorem 2.3 can be proved by Theorem 2.1 immediately. □

Theorem 2.4. *Let E be a Banach space and let $T_i : E \to E$ $(i = 1,2,\dots,N)$ be N asymptotically nonexpansive mappings with a nonempty fixed-point set $F(T) = \bigcap_{i=1}^N F(T_i)$. Let $\{u_{n^i}\}$ be a bounded sequence in E. For any given point x_1 in E, generate the sequence $\{x_n\}$ by (1.5). If $\sum_{n=1}^{\infty} \alpha_{n^i} < \infty$, then sequence $\{x_n\}$ strongly converges to a common fixed point of T_i $(i = 1,2,\dots,N)$ if and only if $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.*

Remarks 2.5. We would like to point out that Theorems 2.1, 2.3, and 2.4 generalize and improve the corresponding results of Petryshyn and Williamson [9], Ghost and Debnath [5], Liu [6, 7], and Xu and Noor [12]. These theorems especially improve Chang's results [2] in the following aspects.

(1) We removed the condition (2.1) "*there exists constant $L > 0$ and $\alpha > 0$ such that* $\|Tx - p\| \le L\|x - p\|^{\alpha}$, $\forall x \in E$, $\forall p \in F(T)$" in [2].

(2) "*The Ishikawa iterative sequence with mixed errors*" is extended to *N-step iterative sequence with mean errors*, and so we obtain the common fixed point of N asymptotically nonexpansive-type mappings.

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Jing Quan: Department of Mathematics, Chongqing Normal University, Chongqing 400047, China
E-mail address: quanjingcq@163.com

Shih-Sen Chang: Department of Mathematics, Yibin University, Yibin, Sichuan 644007, China
E-mail address: sszhang_1@yahoo.com.cn

Xian Jun Long: Department of Mathematics, Chongqing Normal University,
Chongqing 400047, China
E-mail addresses: longxj12345@163.com; xianjunlong@hotmail.com