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Research Article

Optimal Design of Nonuniform Linear Arrays in Cellular Systems by Out-of-Cell Interference Minimization

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Optimal design of a linear antenna array with nonuniform interelement spacings is investigated for the uplink of a cellular system. The optimization criterion considered is based on the minimization of the average interference power at the output of a conventional beamformer (matched filter) and it is compared to the maximization of the ergodic capacity (throughput). Out-of-cell interference is modelled as spatially correlated Gaussian noise. The more analytically tractable problem of minimizing the interference power is considered first, and a closed-form expression for this criterion is derived as a function of the antenna spacings. This analysis allows to get insight into the structure of the optimal array for different propagation conditions and cellular layouts. The optimal array deployments obtained according to this criterion are then shown, via numerical optimization, to maximize the ergodic capacity for the scenarios considered here. More importantly, it is verified that substantial performance gain with respect to conventionally designed linear antenna arrays (i.e., uniform $\lambda/2$ interelement spacing) can be harnessed by a nonuniform optimized linear array.

1. INTRODUCTION

Antenna arrays have emerged in the last decade as a powerful technology in order to increase the link or system capacity in wireless systems. Basically, the deployment of multiple antennas at either the transmitter or the receiver side of a wireless link allows the exploitation of two contrasting benefits: diversity and beamforming. Diversity relies on fading uncorrelation among different antenna elements and provides a powerful means to combat the impairments caused by channel fluctuations. In [1] it has been shown that a significant increase in system capacity can be achieved by the use of antenna diversity combined with optimum combining schemes. Independence of fading gains associated to the antennas array can be guaranteed if the scattering environment is rich enough and the antenna elements are sufficiently spaced apart (at least 5–10 λ, where λ denotes the carrier wavelength) [2]. On the other hand, when fading is highly correlated, as for sufficiently small antenna spacings, beamforming techniques can be employed in order to mitigate the spatially correlated noise. Interference rejection through beamforming is conventionally performed by designing a uniform linear array with half wavelength interelement spacings, so as to guarantee that the angle of arrivals can be potentially estimated free of aliasing. Moreover, beamforming is effective in propagation environments where there is a strong line-of-sight component and the system performance is interference-limited [3].

In this paper, we consider the optimization of a linear nonuniform antenna array for the uplink of a cellular system. The study of nonuniform linear arrays dates back to the seventies with the work of Saholos [4] on radiation pattern and directivity. In [5] performance of linear and circular arrays with different topologies, number of elements, and propagation models is studied for the uplink of an interference free system so as to optimize the network coverage. The idea of optimizing nonuniform-spaced antenna arrays to enhance the overall throughput of an interference-limited system was firstly proposed in [6]. Therein, for flat fading channels, it is shown that unequally spaced arrays outperform equally spaced array by 1.5–2 dB. Here, different from [6], a more realistic approach that explicitly takes into account the cellular layout (depending on the reuse factor) and the propagation model (that ranges from line-of-sight to richer scattering according to the ring model [7]) is accounted for.

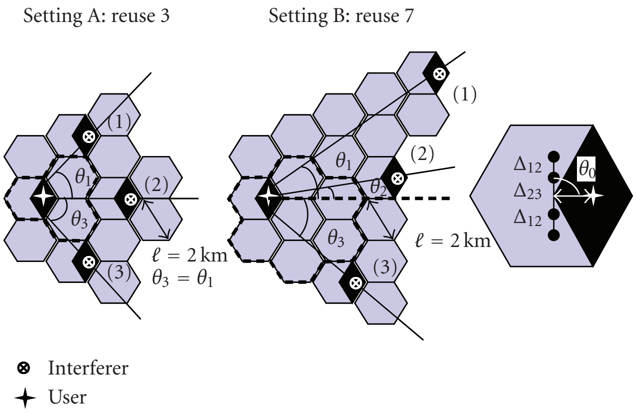


FIGURE 1: Two cellular systems with hexagonal cells and trisectorial antennas at the base stations (reuse factor $F = 3$, setting A, on the right and $F = 7$, setting B, on the left). The array is equipped with $N = 4$ antennas. Shaded sectors denote the allowed areas for user and the three interferers belonging to the first ring of interference (dashed lines identify the cell clusters of frequency reuse).

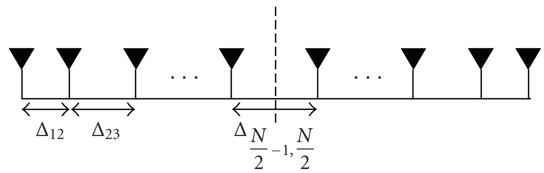


FIGURE 2: Nonuniform symmetric array structure for N even.

For illustration purposes, consider the interference scenarios sketched in Figure 1. Therein, we have two different settings characterized by hexagonal cells and different reuse factors ($F = 3$ for setting A and $F = 7$ for setting B, frequency reuse clusters are denoted by dashed lines). The base station is equipped with a symmetric antenna array[1] containing an even number N of directional antennas ($N = 4$ in the example) to cover an angular sector of 120 deg, other BS antenna array design options are discussed in [8]. Each terminal is provided with one omnidirectional antenna. On the considered radio resource (e.g., time-slot, frequency band, or orthogonal code), it is assumed there is only one active user in the cell, as for TDMA, FDMA, or orthogonal CDMA. The user of interest is located in the respective sector according to the reuse scheme. The contribution of out-of-cell interferers is modelled as spatially correlated Gaussian noise. In Figure 1, the first ring of interference is denoted by shaded cells. The problem we tackle is that of finding the antenna spacings in vector $\boldsymbol{\Delta} = [\Delta_{12}\ \Delta_{23}]^T$ (as shown in the example) so as to optimize given performance metrics, as detailed below.

[1] The symmetric array assumption (as in the array structure of Figure 2) has been made mainly for analytical convenience in order to simplify the optimization problem. However, it is expected that for a scenario with a symmetric layout of interference (such as setting A), the assumption of a symmetric array does not imply any loss of optimality, while, on the other hand, for an asymmetric layout (such as setting B), capacity gains could be in principle obtained by deploying an asymmetric array.

Two criteria are considered, namely, the minimization of the average interference power at the output of a conventional beamformer (matched filter) and the maximization of the ergodic capacity (throughput). Since in many applications the position of users and interferers is not known a priori at the time of the antenna deployment or the in-cell/out-cell terminals are mobile, it is of interest to evaluate the optimal spacings not only for a fixed position of users and interferers but also by averaging the performance metric over the positions of user and interferers within their cells (see Section 2).

Even if the ergodic capacity criterion has to be considered to be the most appropriate for array design in interference-limited scenario, the interference power minimization is analytically tractable and highlights the justification for unequal spacings. Therefore, the problem of minimizing the interference power is considered first and a closed-form expression for this criterion is derived as a function of the antenna spacings (Section 4). This analysis allows to get insight into the structure of the optimal array for different propagation conditions and cellular layouts avoiding an extensive numerical maximization of the ergodic capacity. The optimal array deployments obtained according to the two criteria are shown via numerical optimization to coincide for the considered scenarios (Section 5). More importantly, it is verified that substantial performance gain with respect to conventionally designed linear antenna arrays (i.e., uniform $\lambda/2$ interelement spacing) can be harnessed by an optimized array (up to 2.5 bit/s/Hz for the scenarios in Figure 1).

2. PROBLEM FORMULATION

The signal received by the N antenna array at the base station serving the user of interest can be written as

$$\mathbf{y} = \mathbf{h}_0 x_0 + \sum_{i=1}^{M} \mathbf{h}_i x_i + \mathbf{w}, \tag{1}$$

where $\mathbf{h}_0$ is the $N \times 1$ vector describing the channel gains between the user and the N antennas of the base station, x_0 is the signal transmitted by the user, $\mathbf{h}_i$ and x_i are the corresponding quantities referred to the ith interferer ($i = 1, \dots, M$), $\mathbf{w}$ is the additive white Gaussian noise with $E[\mathbf{w}\mathbf{w}^H] = \sigma^2\mathbf{I}$. The channel vectors $\mathbf{h}_0$ and $\{\mathbf{h}_i\}_{i=1}^M$ are uncorrelated among each other and assumed to be zero-mean complex Gaussian (Rayleigh fading) with spatial correlation $\mathbf{R}_0 = E[\mathbf{h}_0\mathbf{h}_0^H]$ and $\{\mathbf{R}_i = E[\mathbf{h}_i\mathbf{h}_i^H]\}_{i=1}^M$, respectively. The correlation matrices are obtained according to a widely employed geometrical model that assumes the scatterers as distributed along a ring around the terminal, see Figure 3. This model was thoroughly studied in [2, 7] and a brief review can be found in Section 3. According to this model, the spatial correlation matrices of the fading channel depend on

(1) the set of $N/2$ antenna spacings (N is even) $\boldsymbol{\Delta} = [\Delta_{12}\ \Delta_{23}\ \cdots\ \Delta_{N/2,N/2+1}]^T$, where Δ_{ij} is the distance between the ith and the jth element of the array (the

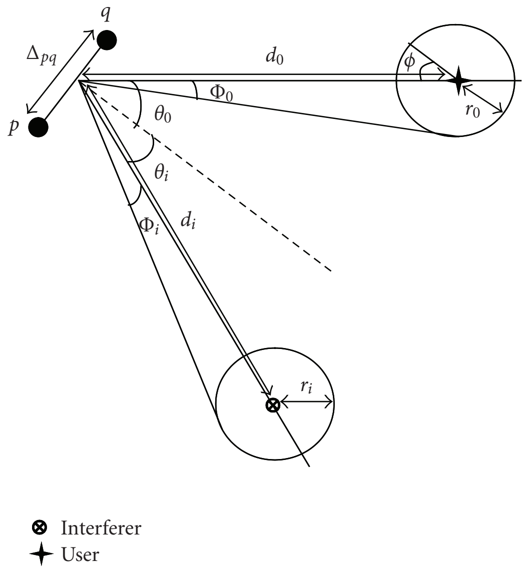


FIGURE 3: Propagation model for user and interferers: the scatterers are distributed on a rings of radii r_i around the terminals.

array is assumed to be symmetric as shown in Figure 2, extension to an odd number of antennas N is straightforward);

(2) the relative positions of user and interferers with respect to the base station of interest (these latter parameters can be conveniently collected into the vector $\boldsymbol{\eta} = [\boldsymbol{\eta}_0^T \ \boldsymbol{\eta}_1^T \ \cdots \ \boldsymbol{\eta}_M^T]^T$, where, as detailed in Figure 3, vector $\boldsymbol{\eta}_0 = [d_0 \ \theta_0]^T$ parametrizes the geometrical location of the in-cell user and vectors $\boldsymbol{\eta}_i = [d_i \ \theta_i]^T$ ($i = 1, \ldots, M$) describe the location of the interferers ($i = 1, \ldots, M$));

(3) the propagation environment is described by the angular spread of the scattered signal received by the base station (ϕ_0 for the user and ϕ_i ($i = 1, \ldots, M$) for the interferers); notice that for ideally $\phi_i \to 0$ all scatterers come from a unique direction so that line-of-sight (LOS) channel can be considered. Shadowing can be possibly modelled as well, see Section 3 for further discussion.

2.1. *Interference power minimization*

From (1), the instantaneous total interference power at the output of a conventional beamforming (matched filter) is [9]

$$\mathcal{P}(\mathbf{h}_0, \boldsymbol{\Delta}, \boldsymbol{\eta}) = \mathbf{h}_0^H \mathbf{Q} \mathbf{h}_0, \tag{2}$$

where

$$\mathbf{Q} = \mathbf{Q}(\boldsymbol{\Delta}, \boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_M) = \sum_{i=1}^{M} \mathbf{R}_i(\boldsymbol{\Delta}, \boldsymbol{\eta}_i) + \sigma^2 \mathbf{I}_N \tag{3}$$

accounts for the spatial correlation matrix of the interferers and for thermal noise with power σ^2. Notice that, for clarity of notation, we explicitly highlighted that the interference correlation matrices depend on the terminals' locations $\boldsymbol{\eta}$ and the antenna spacings $\boldsymbol{\Delta}$ through nonlinear relationships. The first problem we tackle is that of finding the set of optimal spacings $\hat{\boldsymbol{\Delta}}$ that minimizes the average (with respect to fading) interference power, $\mathcal{P}(\boldsymbol{\Delta}, \boldsymbol{\eta}) = E_{\mathbf{h}_0}[\mathcal{P}(\mathbf{h}_0, \boldsymbol{\Delta}, \boldsymbol{\eta})]$, that is,

$$(\text{Problem-1}) : \hat{\boldsymbol{\Delta}} = \arg\min_{\boldsymbol{\Delta}} \mathcal{P}(\boldsymbol{\Delta}, \boldsymbol{\eta}) \tag{4}$$

for a fixed given position $\boldsymbol{\eta}$ of user and interferers. Problem 1 is relevant for fixed system with a known layout at the time of antenna deployment. Moreover, its solution will bring insight into the structure of the optimal array, which can be to some extent generalized to a mobile scenario. In fact, in mobile systems or in case the position of users and interferers is not known a priori at the time of the antenna deployment, it is more meaningful to minimize the average interference power for any arbitrary position of in-cell user ($\boldsymbol{\eta}_0$) and out-of-cells interferers ($\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_M$). Denoting the averaging operation with respect to users and interferers positions by $E_{\boldsymbol{\eta}}[\mathcal{P}(\boldsymbol{\Delta}, \boldsymbol{\eta})]$, the second problem (9) can be can be stated as

$$(\text{Problem-2}) : \hat{\boldsymbol{\Delta}} = \arg\min_{\boldsymbol{\Delta}} E_{\boldsymbol{\eta}}[\mathcal{P}(\boldsymbol{\Delta}, \boldsymbol{\eta})]. \tag{5}$$

2.2. *Ergodic capacity maximization*

The instantaneous capacity for the link between the user and the BS reads [2]

$$C(\mathbf{h}_0, \boldsymbol{\Delta}, \boldsymbol{\eta}) = \log_2 (1 + \mathbf{h}_0^H \mathbf{Q}^{-1} \mathbf{h}_0) \ [\text{bit/s/Hz}], \tag{6}$$

and depends on both the antenna spacings $\boldsymbol{\Delta}$ and the terminals' locations $\boldsymbol{\eta}$. For fast-varying fading channels (compared to the length of the coded packet) or for delay-insensitive applications, the performance of the system from an information theoretic standpoint is ruled by the ergodic capacity $\mathcal{C}(\boldsymbol{\Delta}, \boldsymbol{\eta})$. The latter is defined as the ensemble average of the instantaneous capacity over the fading distribution,

$$\mathcal{C}(\boldsymbol{\Delta}, \boldsymbol{\eta}) = E_{\mathbf{h}_0}[C(\mathbf{h}_0, \boldsymbol{\Delta}, \boldsymbol{\eta})]. \tag{7}$$

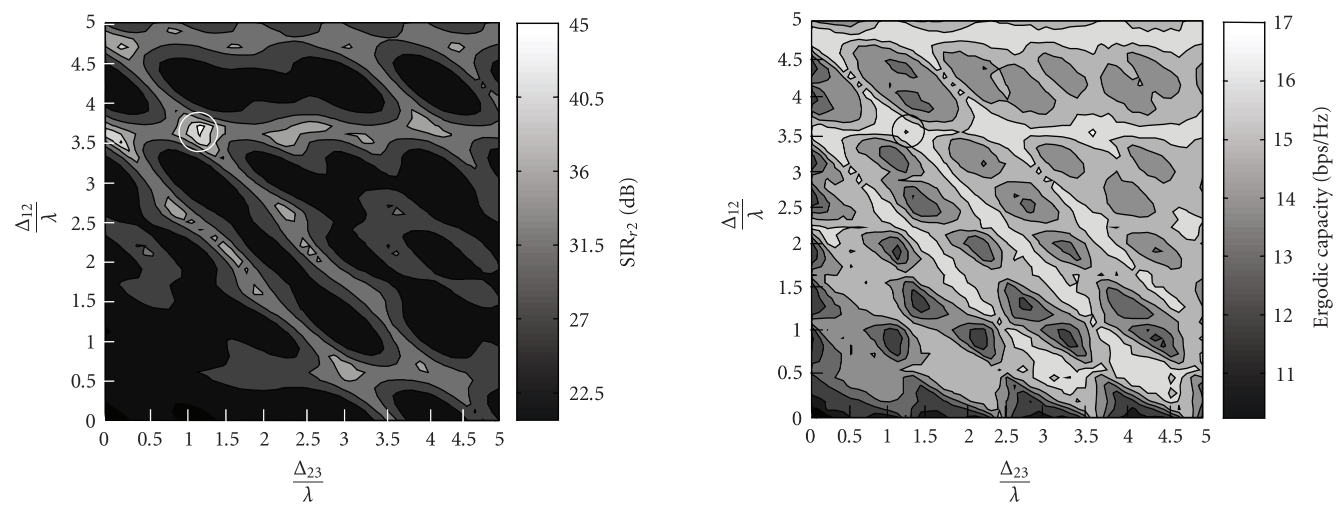
According to the alternative performance criterion herein proposed, the first problem (4) is recasted as

$$(\text{Problem 1}) : \hat{\boldsymbol{\Delta}} = \arg\max_{\boldsymbol{\Delta}} \mathcal{C}(\boldsymbol{\Delta}, \boldsymbol{\eta}), \tag{8}$$

and therefore requires the maximization of the ergodic capacity for a fixed given position $\boldsymbol{\eta}$ of user and interferers. As before, denoting the averaging operation with respect to users and interferers positions by $E_{\boldsymbol{\eta}}[\mathcal{C}(\boldsymbol{\Delta}, \boldsymbol{\eta})]$, the second problem (5) can be modified accordingly:

$$(\text{Problem 2}) : \hat{\boldsymbol{\Delta}} = \arg\max_{\boldsymbol{\Delta}} E_{\boldsymbol{\eta}}[\mathcal{C}(\boldsymbol{\Delta}, \boldsymbol{\eta})]. \tag{9}$$

Different from the interference power minimization approach, in this case, functional dependence of the performance criterion (7) on the antenna spacings $\boldsymbol{\Delta}$ is highly nonlinear (see Section 3 for further details) and complicated by the presence of the inverse matrix $\mathbf{Q}^{-1}$ that relies upon $\boldsymbol{\Delta}$ and $\boldsymbol{\eta}$. This implies both a large-computational complexity for



(a) (b)

Figure 4: Setting-A: rank-2 approximation of the signal-to-interference ratio $\text{SIR}_{r2}(\boldsymbol{\Delta}, \overline{\boldsymbol{\eta}})$ (23) versus Δ_{12}/λ and Δ_{23}/λ (a) compared with ergodic capacity $\mathcal{C}(\boldsymbol{\Delta}, \overline{\boldsymbol{\eta}})$ (b) ($r = 50$ m). Circles denote optimal solutions.

the numerical optimization of (8) and (9), and the impossibility to get analytical insight into the properties of the optimal solution. When the number of antenna array is sufficiently small (as in Section 5), optimization can be reasonably dealt with through an extensive search over the optimization domain and without the aid of any sophisticated numerical algorithm. On the contrary, in case of an array with a larger number of antenna elements, more efficient optimization techniques (e.g., simulated annealing) may be employed to reduce the number of spacings to be explored and thus simplify the optimization process. Below we will prove (by numerical simulations) that the limitations of the above optimization (8)-(9) are mitigated by the criteria (4)-(5) still preserving the final result.

3. SPATIAL CORRELATION MODEL

We consider a propagation scenario where each terminal, be it the user or an interferer, is locally surrounded by a large number of scatterers. The signals radiated by different scatterers add independently at the receiving antennas. The scatterers are distributed on a ring of radius r_0 around the terminal (r_i, $i = 1, \ldots, M$ for the interferers) and the resulting angular spread of the received signal at the base station is denoted by $\phi_0 \simeq r_0/d_0$ (or $\phi_i \simeq r_i/d_i$), as in Figure 3. Because of the finite angular spreads $\{\phi_i\}_{i=0}^{M}$, the propagation model appears to be well suited for outdoor channels.

In [7], the spatial correlation matrix of the resulting Rayleigh distributed fading process at the base station is computed by assuming a parametric distribution of the scatterers along the ring, namely, the von Mises distribution (variable $0 \leq \vartheta < 2\pi$ runs over the ring, see Figure 3):

$$f(\vartheta) = \frac{1}{2\pi I_0(\kappa)} \exp\left[\kappa \cos(\vartheta)\right]. \tag{10}$$

By varying parameter κ, the distribution of the scatterers ranges from uniform ($f(\vartheta) = 1/(2\pi)$ for $\kappa = 0$) to a Dirac delta around the main direction of the cluster $\vartheta = 0$ (for $\kappa \to \infty$). Therefore, by appropriately adjusting parameter κ and the angular spreads for each user and interferers ϕ_i, a propagation environment with a strong line-of-sight component ($\phi_i \simeq 0$ and/or $\kappa \to \infty$) or richer scattering (larger ϕ_i with κ small enough) can be modelled. The (normalized) spatial correlation matrix has the general expression for both user and interferers (for the (p, q)th element with $p, q = 1, \ldots, N$ and $i = 0, 1, \ldots, M$) [7]:

$$\left[\overline{\mathbf{R}}_i(\theta_i, \Delta)\right]_{pq} = \exp\left[j\frac{2\pi}{\lambda}\Delta_{pq}\sin(\theta_i)\right] \cdot \frac{I_0\left(\sqrt{\kappa^2 - \left((2\pi/\lambda)\Delta_{pq}\phi_i\cos(\theta_i)\right)^2}\right)}{I_0(\kappa)}. \tag{11}$$

It is worth mentioning that spatial channel models based on different geometries such as elliptical or disk models [10, 11] may be considered as well by appropriately modifying the spatial correlation (11). Effects of mutual coupling (not addressed in this paper) between the array elements may be included in our framework too, see [12, 13].

From (11), the spatial correlation matrices $\mathbf{R}_i$ of the user and interferers can be written as

$$\mathbf{R}_i(\boldsymbol{\eta}_i, \boldsymbol{\Delta}) = \rho_i \overline{\mathbf{R}}_i(\theta_i, \boldsymbol{\Delta}) \quad \text{with } \rho_i = \frac{K}{d_i^{\alpha}}, \tag{12}$$

where K is an appropriate constant that accounts for receiving and transmitting antenna gain and the carrier frequency, and α is the path loss exponent. The contribution of shadowing in (12) will be considered in Section 5.3 as part of an additional log-normal random scaling term.

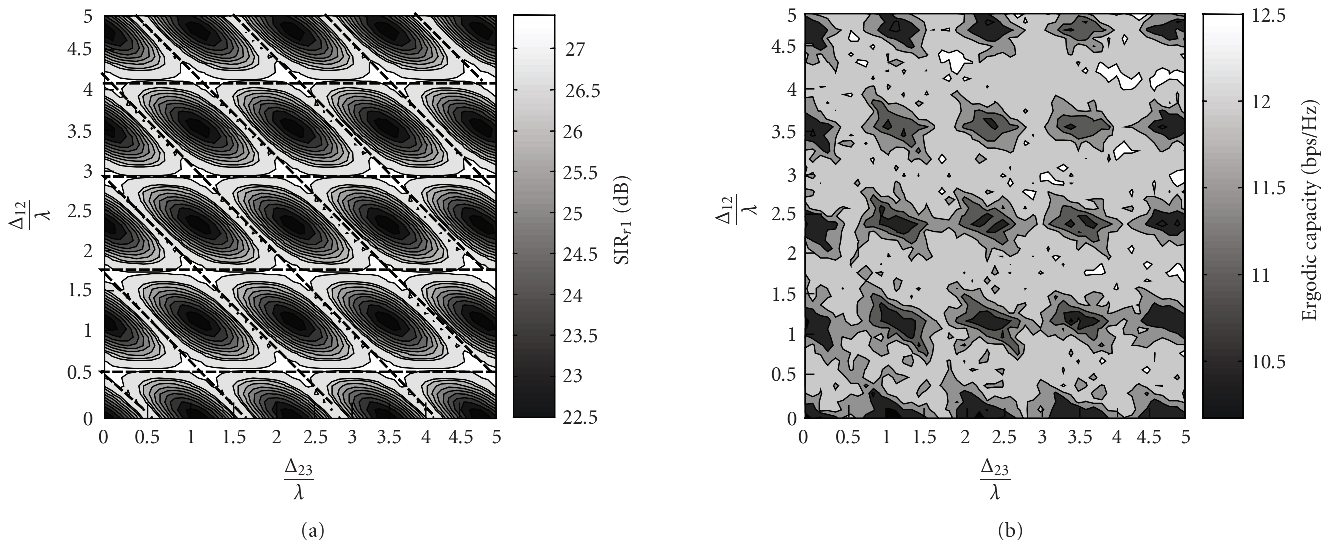


FIGURE 5: Setting-A: rank-1 approximation (a) of the signal-to-interference ratio, $\text{SIR}_{r1}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$, versus Δ_{12}/λ and Δ_{23}/λ. Dashed lines denote the optimality conditions (24) obtained by the rank-1 approximation. As a reference, ergodic capacity is shown (b), for an angular spread approaching zero.

4. REDUCED-RANK APPROXIMATION FOR THE INTERFERENCE POWER

According to a reduced-rank approximation for the spatial correlation matrices of user and interferers $\mathbf{R}_i$ for $i = 0, 1, \ldots, M$, in this section, we derive an analytical closed form expression for the interference power (2) to ease the optimization of the antenna spacings $\mathbf{\Delta}$. In Section 4.1, we consider the case where the angular spread for users and interferers ϕ_i is small so that a rank-1 approximation of the spatial correlation matrices can be used. This first case describes line-of-sight channels. Generalization to channel with richer scattering is given in Section 4.2.

4.1. Rank-1 approximation (line-of-sight channels)

If the angular spread is small for both user and interferers[2] (i.e., $\phi_i \ll 1$ for $i = 0, 1, \ldots, M$), the associated spatial correlation matrices $\{\mathbf{R}_i\}_{i=0}^{M}$, can be conveniently approximated by enforcing a rank-1 constraint. For $\phi_i \ll 1$, the following simplification holds in (11): $I_0(\sqrt{\kappa^2 - ((2\pi/\lambda)\Delta_{pq}\phi_i\cos(\theta_i))^2})/I_0(\kappa) \simeq 1$. Therefore, the spatial correlation matrices (12) can be approximated as (we drop the functional dependency for simplicity of notation)

$$\mathbf{R}_i \simeq \rho \cdot \mathbf{v}_i\mathbf{v}_i^H, \tag{13}$$

where

$$\mathbf{v}_i(\mathbf{\Delta}, \mathbf{J}) = \Big[1 \ \exp\big[-j\omega(\theta_i)\Delta_{12}\big] \ \cdots \ \exp\big[-j\omega(\theta_i)\Delta_{1N}\big]\Big]^T \tag{14}$$

and $\omega_i(\theta) = 2\pi/\lambda\sin(\theta_i)$.

From (13), the channel vectors for user and interferers can be written as $\mathbf{h}_i = \gamma\sqrt{\rho_i}\mathbf{v}_i$, where $\gamma \sim \mathcal{CN}(0,1)$. Therefore, within the rank-1 approximation, the interference power reads (the additive noise contribution $\sigma^2\mathbf{I}_N$ has been dropped since it is immaterial for the optimization problem)

$$\mathcal{P}_1(\mathbf{\Delta}, \boldsymbol{\eta}) = \mathbf{v}_0^H\left(\sum_{i=1}^{M}\rho_i\mathbf{v}_i\mathbf{v}_i^H\right)\mathbf{v}_0, \tag{15}$$

therefore, optimal spacings with respect to Problem 1 (4) can be written as

$$\hat{\mathbf{\Delta}} = \arg\min_{\mathbf{\Delta}} \mathcal{P}_1(\mathbf{\Delta}, \boldsymbol{\eta}), \tag{16}$$

where the subscript is a reminder of the rank-1 approximation. The advantage of the rank-1 performance criterion (15) is that it allows to derive an explicit expression as a function of the parameters of interest. In particular, after tedious but straightforward algebra, we get

$$\mathcal{P}_1(\mathbf{\Delta}, \boldsymbol{\eta}) = \sum_{i=1}^{M}\left[\rho_i N + \rho_i\sum_{j=1}^{L}4S(l_j, \theta_0, \theta_i) + \rho_i\sum_{k=1}^{C}2S(c_k, \theta_0, \theta_i)\right], \tag{17}$$

[2] Rank-1 approximation for the out-of-cell interferers is quite accurate when considering large reuse factors as the angular spread experienced by the array is reduced by the increased distance of the out-of-cell interferers.

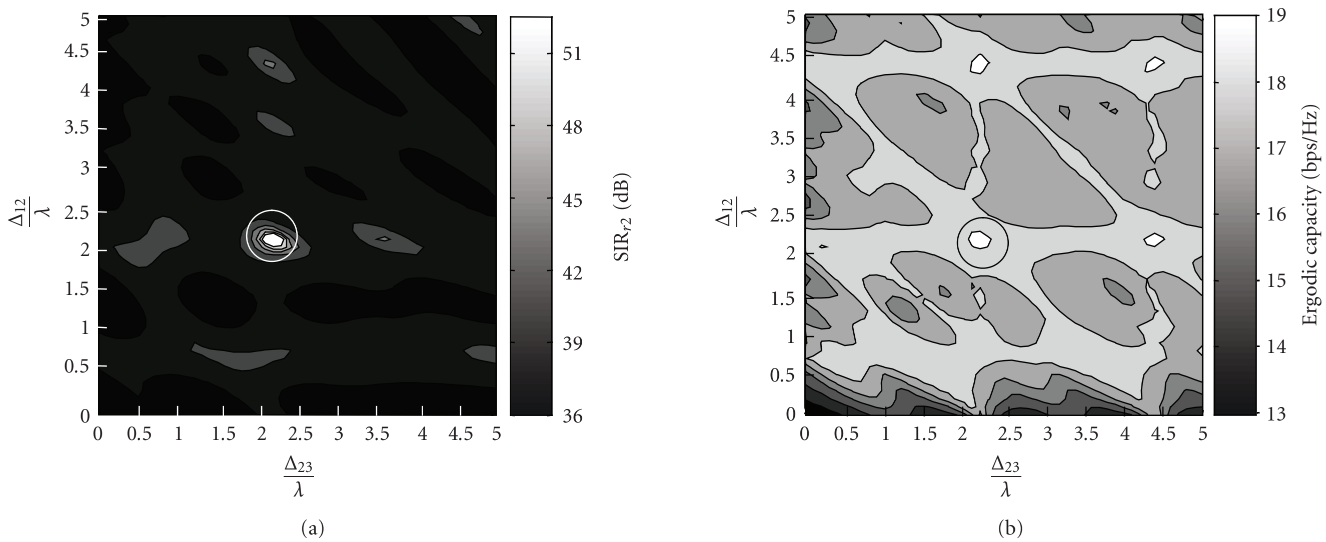


Figure 6: Setting B: rank-2 approximation of the signal-to-interference ratio, $\mathrm{SIR}_{r2}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$, (23) versus Δ_{12}/λ and Δ_{23}/λ (a) compared with ergodic capacity $\mathcal{C}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$ (b) ($r = 50$ m). Circles denote optimal solutions.

where $S(x, \theta_n, \theta_m) = \cos[2\pi x(\sin(\theta_n) - \sin(\theta_m))]$, $L = \left(\binom{N}{2} - N/2\right)/2$ is the number of "lateral spacings" $l_i = \Delta_{i,j}$ for $i = 1, \ldots, N/2 - 1$, $j = i+1, \ldots, N-i$, $C = N/2$ is the number of "central spacings" $c_i = \Delta_{i,N-i}$ for $i = 1, \ldots, N/2$.

As a remark, notice that if there exists a set of antenna spacing $\widetilde{\mathbf{\Delta}}$ such that the user vector $\mathbf{v}_0$ is orthogonal to the M interference vectors $\{\mathbf{v}_i\}_{i=1}^{M}$, then this nulls the interference power, $\mathcal{P}_1(\mathbf{\Delta}, \boldsymbol{\eta}) = 0$, and thus implies that $\widetilde{\mathbf{\Delta}}$ is a solution to (16) (and therefore to (4)).

4.2. Rank-a ($a > 1$) approximation

In a richer scattering environment, the conditions on the angular spread $\phi_i \ll 1$ that justify the use of rank-1 approximation can not be considered to hold. Therefore, a rank-a approximation with $a > 1$ should be employed (in general) for the spatial correlation matrix of both user and interferers:

$$\mathbf{R}_i \simeq \sum_{k=1}^{a} \rho_i^{(k)} \cdot \mathbf{v}_i^{(k)} \mathbf{v}_i^{(k)H} \tag{18}$$

for $i = 0, 1, \ldots, M$. The set of vectors $\{\mathbf{v}_i^{(k)}\}_{k=1}^{a}$ in (18) is required to be linearly independent. In this paper, we limit the analysis to the case $a = 2$, which will be shown in Section 5 to account for a wide range of practical environments. The expression of vectors $\mathbf{v}_i^{(k)}$ from (11) with respect to the antenna spacings is not trivial as for the rank-1 case. However, in analogy with (14), we could set

$$\mathbf{v}_i^{(k)} = \left[1 \;\; \exp\left[-j\omega_i^{(k)} \;\; \Delta_{12}\right] \;\; \cdots \;\; \exp\left[-j\omega_i^{(k)} \;\; \Delta_{1N}\right]\right]^T, \tag{19}$$

where the wavenumbers $\boldsymbol{\omega}_i = [\omega_i^{(1)}, \omega_i^{(2)}]$ for user and interferers have to be determined according to different criteria. In order to be consistent with the rank-1 case considered in the previous section, here we minimize the Frobenius norm of approximation error matrix $\|\mathbf{R}_i - \sum_{k=1}^{a} \rho_i^{(k)} \cdot \mathbf{v}_i^{(k)} \mathbf{v}_i^{(k)H}\|^2$ with respect to $\boldsymbol{\omega} = [\omega_i^{(1)}, \omega_i^{(2)}]$ vector and $\boldsymbol{\rho} = [\rho_i^{(1)}, \rho_i^{(2)}]$ vectors. For instance, for a uniform distribution of the scatterers along the ring (i.e., $\kappa = 0$), it can be easily proved that the optimal rank-2 approximation (for $i = 0, \ldots, M$) results in

$$\omega_i^{(1)} = \omega_i(\theta_i) + \varphi_i, \qquad \omega_i^{(2)} = \omega_i(\theta_i) - \varphi_i, \tag{20}$$

where $\varphi_i = 2\pi/\lambda \cdot \phi_i \cos(\theta_i)$ and $\rho_i^{(1)} = \rho_i^{(2)} = \rho_i/2$.

As for the rank-1 case in (17), after some algebraic manipulations, the performance criterion $\mathcal{P}_2(\mathbf{\Delta}, \boldsymbol{\eta}) = E_{\mathbf{h}_0}[\mathbf{h}_0^H \mathbf{Q} \mathbf{h}_0]$ admits an explicit expression in terms of the parameters of interest:

$$\begin{aligned} \mathcal{P}_2(\mathbf{\Delta}, \boldsymbol{\eta}) = \sum_{i=1}^{M} \Bigg[& \rho_i N + \rho_i \sum_{j=1}^{L} 4S(l_j, \theta_0, \theta_i) \cdot T_i(l_j) \\ & + \rho_i \sum_{k=1}^{C} 2S(c_k, \theta_0, \theta_i) T_i(c_k) \Bigg], \end{aligned} \tag{21}$$

where $T_i(x) = \cos(\varphi_0 x)\cos(\varphi_i x)$; notice that in practical environments, the angular spread for the in-cell user, φ_0, is larger than the out-of-cell interferers angular spreads, $\varphi_1, \ldots, \varphi_M$ (see Section 5). Therefore, the optimization problem (4) can be stated as

$$\widehat{\mathbf{\Delta}} = \arg\min_{\mathbf{\Delta}} \mathcal{P}_2(\mathbf{\Delta}, \boldsymbol{\eta}). \tag{22}$$

5. NUMERICAL RESULTS

In this section, numerical results related to the layouts in Figure 1 ($N = 4$, $M = 3$, $F = 3$ for setting A and $F = 7$ for setting B with a cell diameter $\ell = 2$ km) are presented. Both the interference power minimization problems (4), (5)

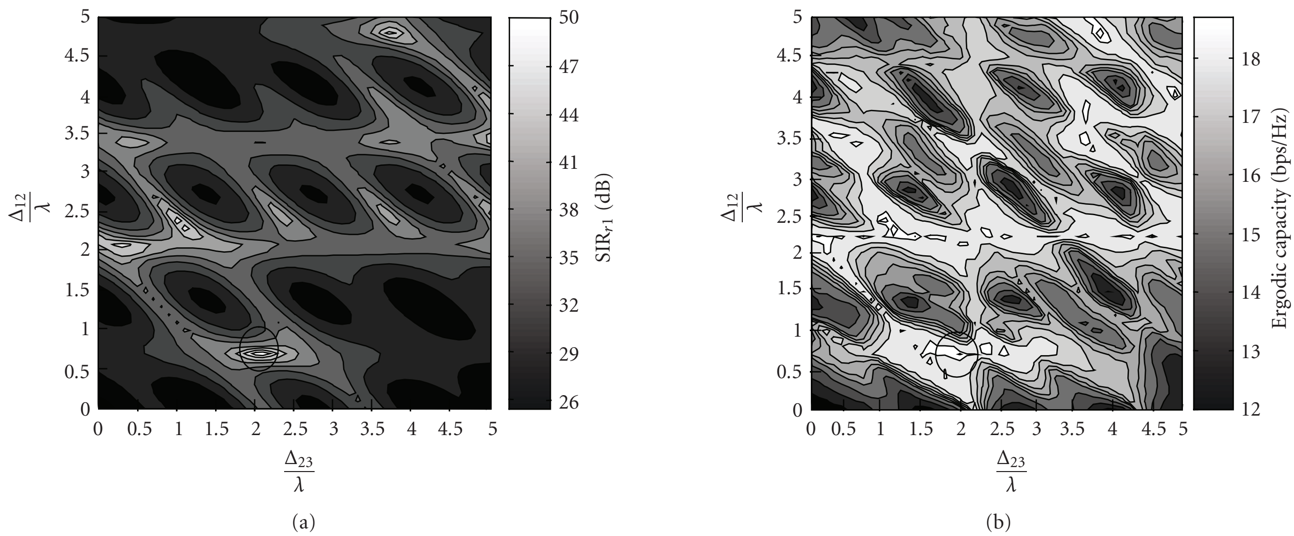


FIGURE 7: Setting-B: rank-1 approximation (a) of the signal-to-interference ratio $\text{SIR}_{r1}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$ versus Δ_{12}/λ and Δ_{23}/λ. Circular marker denotes the optimal solution (24) obtained by the rank-1 approximation. As a reference, ergodic capacity is shown (b), for an angular spread approaching zero.

and the ergodic capacity optimization problems (8), (9) for Problems 1 and 2, respectively, are considered and compared for various propagation environments. For Problem 1, user and interferers are located at the center of their respective allowed sectors ($\overline{\boldsymbol{\eta}}$, as in Figure 1), instead, for Problem 2 average system performances are computed over the allowed positions (herein uniformly distributed) of users and interferers.

Exploiting the rank-a-based approximation (rank-1 and rank-2 approximations in (17) and (21), resp.), the interference power (for fixed user and interferers position $\overline{\boldsymbol{\eta}}$ as for Problem 1, or averaged over terminal positions as for Problem 2) is minimized with respect to the array spacings and the resulting optimal solutions are compared to those obtained through maximization of ergodic capacity. Herein, we show that the proposed approach based on interference power minimization is reliable in evaluating the optimal spacings that also maximize the ergodic capacity of the system. Since the number of antenna array is limited to $N = 4$, ergodic capacity optimization can be carried out through an extensive search over the optimization domain.

The channels of user and interferers are assumed to be characterized by the same scatterer radius $r_i = r$ (for the rank-2 case) and $r \to 0$ (for the rank-1 case) with $\kappa = 0$. Furthermore, the path loss exponent is $\alpha = 3.5$. The signal-to-background noise ratio (for the ergodic capacity simulations) is set to $N\rho_0/\sigma^2 = 20\,\text{dB}$. For the sake of visualization, the rank-a approximation of the interference power is visualized (in dB scale) as the signal-to-interference ratio (SIR):

$$\text{SIR}_{\text{ra}}(\mathbf{\Delta}, \boldsymbol{\eta}) = \left[\frac{\rho_0}{\mathcal{P}_a(\mathbf{\Delta}, \boldsymbol{\eta})}\right]_{\text{dB}}. \tag{23}$$

5.1. Setting A ($F = 3$)

Assuming at first fixed position $\overline{\boldsymbol{\eta}}$ for user and interferers (Problem 1), Figure 4(b) shows the exact ergodic capacity $\mathcal{C}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$ for $r = 50\,\text{m}$ (and thus the angular spread is $\phi_0 = 5.75\,\text{deg}$, $\phi_1 = \phi_3 = 0.87\,\text{deg}$, $\phi_2 = 0.82\,\text{deg}$) and Figure 4(a) shows the rank-2 SIR approximation $\text{SIR}_{r2}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$ (23) versus Δ_{12} and Δ_{23} for setting A. According to both optimization criteria, the optimal array has external spacing $\widehat{\Delta}_{12} \simeq 1.26\,\lambda$ and internal spacing $\widehat{\Delta}_{23} \simeq 3.6\,\lambda$. It is interesting to compare this result with the case of a line-of-sight channel that is shown in Figure 5. In this latter scenario, the optimal spacings are easily found by solving the rank-1 approximate problem (16) as ($k = 0, 1, \ldots$)

$$\widehat{\Delta}_{12} = (2k+1)\Psi(\theta_1) \quad \text{with any } \widehat{\Delta}_{23} \geq 0 \tag{24a}$$

$$\text{or } \widehat{\Delta}_{12} + \widehat{\Delta}_{23} = (2k+1)\Psi(\theta_1), \tag{24b}$$

where $\Psi(\theta_1) = \lambda/(2\sin(\theta_1)) \simeq 0.6\,\lambda$ as $\theta_1 = \theta_2 = 52\,\text{deg}$. Conditions (24) guarantee that the channel vector of the user is orthogonal to the channel vectors of the first and third interferers (the second is aligned so that mitigation of its interference is not feasible). Moreover, the optimal spacings for the line-of-sight scenario (24) form a grid (see Figure 5(a)) that contains the optimal spacings for the previous case in Figure 4 with larger angular spread. Notice that, for every practical purpose, the solutions to the ergodic capacity maximization (Figure 5(b)) are well approximated by $\text{SIR}_{r1}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$ maximization in (23). As a remark, we might observe that with line-of-sight channels, there is no advantage of deploying more than two antennas ($\widehat{\Delta}_{12} = 0$ or $\widehat{\Delta}_{23} = 0$ satisfy the optimality conditions (24)) to exploit the interference reduction capability of the array. Instead, for larger angular spread than the line-of-sight case, we can conclude that

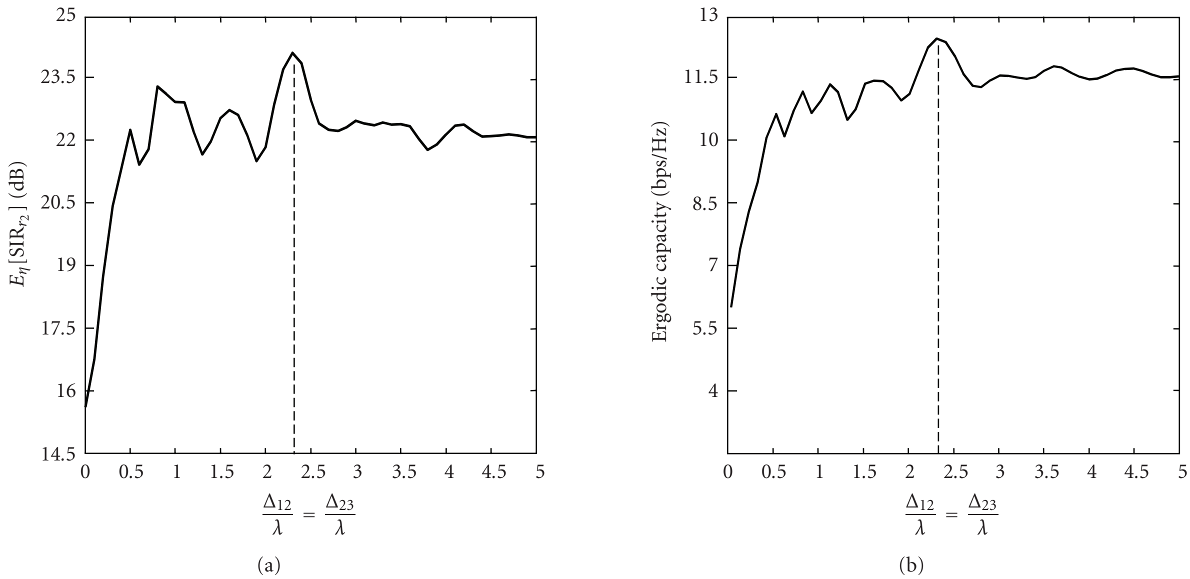


Figure 8: Setting B: rank 2 approximation of the signal-to-interference ratio $E_{\boldsymbol{\eta}}[\mathrm{SIR}_{r2}(\boldsymbol{\Delta}, \boldsymbol{\eta})]$ (a) and ergodic capacity $E_{\eta}[\mathcal{C}(\boldsymbol{\Delta}, \eta)]$ (b) averaged with respect to the position of user and interferers within the corresponding sectors for $\Delta_{12} = \Delta_{23}$.

(i) large enough spacings have to be preferred to accommodate diversity; (ii) contrary to the line-of-sight case, there is great advantage of deploying more than two antennas (approximately 5-6 bit/s/Hz) whereas the benefits of deploying more than three antennas are not as relevant (0.6 bit/s/Hz for an optimally designed three-element array with uniform spacing 3.6 λ); (iii) compared to the $\lambda/2$-uniformly spaced array, optimizing the antenna spacings leads to a performance gain of approximately 2.5 bit/s/Hz.

Let us now turn to the solution of Problem 2 (9). In this case, the optimal set of spacings $\widehat{\boldsymbol{\Delta}}$ should guarantee the best performance on average with respect to the positions of user and interferers within the corresponding sectors. It turns out that the optimal spacings are $\widehat{\Delta}_{12} = \widehat{\Delta}_{23} \simeq 1.9\,\lambda$ for both optimization criteria (not shown here), and the (average) performance gain with respect to the conventional adaptive arrays with $\widehat{\Delta}_{12} = \widehat{\Delta}_{23} = \lambda/2$ has decreased to approximately 0.5 bit/s/Hz. This conclusion is substantially different for scenario B as discussed below.

5.2. Setting B ($F = 7$)

For Problem 1, the exact ergodic capacity $\mathcal{C}(\boldsymbol{\Delta}, \overline{\boldsymbol{\eta}})$ for $r = 50$ m (and angular spread $\phi_0 = 5.75$ deg, $\phi_1 = 0.34$ deg, $\phi_2 = 0.56$ deg, and $\phi_3 = 0.58$ deg) and the rank-2 approximation $\mathrm{SIR}_{r2}(\boldsymbol{\Delta}, \overline{\boldsymbol{\eta}})$ (23) are shown versus Δ_{12} and Δ_{23}, in Figure 6, for setting B. In this case, the optimal linear minimum length array consists, as obtained by both optimization criteria, by uniform 2.2 λ spaced antennas. Optimal design of linear minimal length array leads to a 2.5 bit/s/Hz capacity gain with respect to the capacity achieved through an array provided with four uniformly $\lambda/2$ spaced antennas. Similarly as before, we compare this result with the case of a line-of-sight channel (Figure 7(a)), where the optimal spacings, solution to the rank-1 approximate problem (16), are $\widehat{\Delta}_{12} = \Psi(\theta_3) \simeq 0.7\,\lambda$ (external spacing) and $\widehat{\Delta}_{23} = 3\Psi(\theta_3) \simeq 2.2\,\lambda$ (internal spacing) $\theta_3 = 43.5$ deg. In this case, the solutions (confirmed by the ergodic capacity maximization, see Figure 7(b)) guarantee that the channel vector of the user is orthogonal to the channel vector of the third (predominant) interferer (the second is almost aligned so that mitigation of its interference is not feasible, the first one has a minor impact on the overall performances). As pointed out before, a larger angular spread than the line-of-sight case requirelarger spacings to exploit diversity.

As for Problem 2 (9), in Figure 8, we compare the analytical rank-2 approximation $E_{\boldsymbol{\eta}}[\mathrm{SIR}_{r2}(\boldsymbol{\Delta}, \boldsymbol{\eta})]$ averaged over the position of users and interferers with the exact averaged ergodic capacity for a uniform-spaced antenna array. The minimal length optimal solutions turn out again to be $\widehat{\Delta}_{12} = \widehat{\Delta}_{23} \simeq 2.2\,\lambda$. Moreover, we can conclude that in this interference layout the capacity gain with respect to the capacity achieved through an array provided with four uniformly $\lambda/2$ spaced antennas is 2.5 bit/s/Hz.

5.3. Impact of nonequal power interfering due to shadowing effects

In this section, we investigate the impact of nonequal interfering powers caused by shadowing on the optimal antenna spacings. This amounts to include in the spatial correlation model (12) a log-normal variable for both user and interferers as $\rho_i = (K/d_i^{\alpha}) \cdot 10^{G_i/10}$ and $G_i \sim \mathcal{N}(0, \sigma_{G_i}^2)$ for $i = 0, 1, \ldots, M$. All shadowing variables $\{G_i\}_{i=0}^{M}$ affect receiving power levels and are assumed to be independent. Figure 9 shows the ergodic capacity averaged over the shadowing processes for setting B and $r = 50$ m (as in Figure 6), when the standard deviation of the fading processes are $\sigma_{G_0} = 3$ dB for the user (e.g., as for imperfect power control) and $\sigma_{G_i} = 8$ dB ($i = 1, \ldots, M$) for the interferers. By comparing Figure 9 with

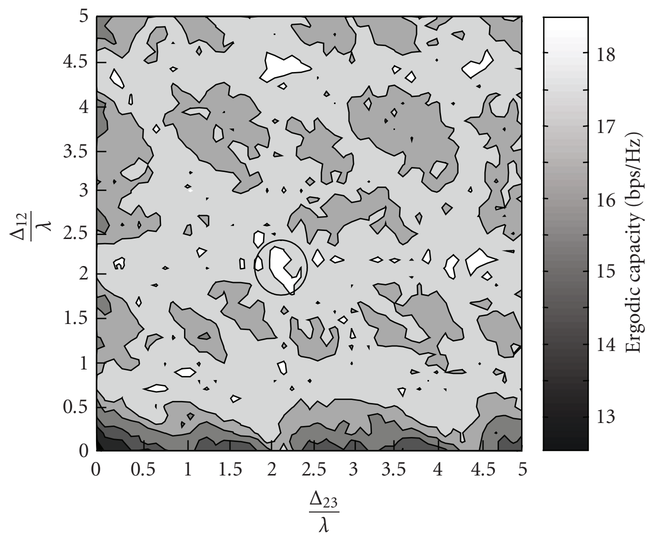


FIGURE 9: Setting B: ergodic capacity $\mathcal{C}(\mathbf{\Delta}, \overline{\boldsymbol{\eta}})$ averaged with respect to the distribution of shadowing ($r = 50$ m). Circle denotes the optimal solution.

Figure 6, we see that the overall effect of shadowing is that of reducing the ergodic capacity but not to modify the optimal antenna spacings; similar results can be attained by analyzing the interference power (not shown here).

6. CONCLUSION

In this paper, we tackled the problem of optimal design of linear arrays in a cellular systems under the assumption of Gaussian interference. Two design problems are considered: maximization of the ergodic capacity (through numerical simulations) and minimization of the interference power at the output of the matched filter (by developing a closed form approximation of the performance criterion), for fixed and variable positions of user and interferers. The optimal array deployments obtained according to the two criteria are shown via numerical optimization to coincide for the considered scenarios. The analysis has been validated by studying two scenarios modelling cellular systems with different reuse factors. It is concluded that the advantages of an optimized antenna array as compared to a standard design depend on both the interference layout (i.e., reuse factor) and the propagation environment. For instance, for an hexagonal cellular system with reuse factor 7, the gain can be on average as high as 2.5 bit/s/Hz. As a final remark, it should be highlighted that optimizing the antenna array spacings in such a way to improve the quality of communication (by minimizing the interference power) may render the antenna array unsuitable for other applications where some features of the propagation are of interest, such as localization of transmitters based on the estimation of direction of arrivals.

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