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Research Article

Optimized Punctured ZCZ Sequence-Pair Set: Design, Analysis, and Application to Radar System

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Based on the zero correlation zone (ZCZ) concept, we present the definitions and properties of a set of new ternary codes, ZCZ sequence-Pair Set (ZCZPS), and propose a method to use the optimized punctured sequence-pair along with Hadamard matrix to construct an optimized punctured ZCZ sequence-pair set (OPZCZPS) which has ideal autocorrelation and cross-correlation properties in the zero correlation zone. Considering the moving target radar system, the correlation properties of the codes will not be severely affected when Doppler shift is not large. We apply the proposed codes as pulse compression codes to radar system and the simulation results show that optimized punctured ZCZ sequence-pairs outperform other conventional pulse compression codes, such as the well-known polyphase code—P4 code.

1. Introduction

Pulse compression is known as a technique to raise the signal to maximum sidelobe (signal-to-sidelobe) ratio to improve the target detection and range resolution abilities of the radar system. This technique allows a radar to simultaneously achieve the energy of a long pulse and the resolution of a short pulse without the high peak power which is required by a high energy short duration pulse [1]. One of the waveform designs suitable for pulse compression is phase-coded waveform design. The phase-coded waveform design is that a long pulse of duration T is divided into N subpulses each of width T_s. Each subpulse has a particular phase, which is selected in accordance with a given code sequence. The pulse compression ratio equals the number of subpulses $N = T/T_s \approx BT$, where the bandwidth is $B \approx 1/T_s$. In general, a phase-coded waveform with longer code word, in other words, higher pulse compression ratio, can have lower sidelobe of autocorrelation, relative to the mainlobe peak, so its main peak can be better distinguished. The relative lower sidelobe of autocorrelation is very important since range sidelobes are so harmful that they can mask main peaks caused by small targets situated near large targets. In addition, the cross-correlation property of the pulse compression codes should be considered in order to reduce the interference among radars when we choose a set of pulse compression codes to work in a Radar Sensor Network (RSN).

Much time and effort was put for designing sequences with impulsive autocorrelation functions (ACFs) and cross-correlation functions (CCFs) for radar target ranging and target detection. On one hand, for aperiodic sequences, it is known that for most binary sequences of length N ($N > 13$) the attainable sidelobe levels are approximately $\sqrt{N}$ [2, 3] and the mutual peak cross-correlations of the same-length sequences are much larger and are usually in the order of $2\sqrt{N}$ to $3\sqrt{N}$. Later, set of binary sequences of length N with autocorrelation sidelobes and cross-correlation peak values of approximately $\sqrt{N}$ are studied in paper [4]. Besides, the small set of Kasami sequences and the Bent sequences could achieve maximum correlation values of approximately $\sqrt{N}$. In addition to binary sequences, polyphase codes, with better Doppler tolerance and lower range sidelobes such as the Frank and P1 codes, the Butler-matrix derived P2 code, the linear-frequency-derived P3 and P4 codes were provided and intensively analyzed in [5–7]. Quadiphase [8] code could also reduce poor fall-off of the radiated spectrum and mismatch loss in the receiver pulse compression filter of biphase codes. Nevertheless, the range sidelobe of the polyphase codes can not be low enough to avoid masking returns from targets. Hence, considerable work has been done to reduce range sidelobes for the radar system. By suffering a small S/N loss, the authors in [9] present several binary pulse compression codes to greatly reduce sidelobes. In the previous paper [10], pulse compression using a digital-analog hybrid technique is studied to achieve very low range sidelobes for potential application to spaceborne rain radar. In the paper [11], time-domain weighting of the transmitted pulse is used and is able to achieve a range sidelobe level of −55 dB or better in flight tests. These sidelobe suppression methods, however, degrade the receiving resolution because of wider mainlobe.

On the other hand, for periodic sequences, the lowest periodic ACF that could be achieved for binary sequences, as in the case of m-sequences [12, 13] or Legendre sequences, is $|R_i(\tau \neq 0) = 1|$. GMW [14] has the same periodic ACF properties, but posses larger linear complexity. Considering the nonbinary case, it is possible to find perfect sequences, such as two valued Golomb sequences, Ipatov ternary sequences, Frank sequences, Chu sequences, and modulatable sequences. However, it should be noted that for both binary and non-binary cases, it is impossible for the sequences to have perfect ACF and CCF simultaneously although ideal CCFs could be achieved alone. One can synthesize a set of non-binary sequences with impulsive ACF and the lower bound of CCF: $R_{ij} = \sqrt{N}$, $\forall \tau, i \neq j$ [15, 16], which is governed by Welch bound and Sidelnikov bound.

So far in the previous work, range sidelobes could hardly reach as low as zero. In addition, it has also been well proven that it is impossible to design a set of codes with ideal impulsive autocorrelation function and ideal zero cross-correlation functions, since the corresponding parameters have to be limited by certain bounds, such as Welch bound [15], Sidelnikov bound [16], Sarwate bound [17], and Levenshtein bound [18]. To overcome these difficulties, the new concepts, generalized orthogonality (GO), also called Zero Correlation Zone (ZCZ) is introduced. Based on ZCZ [19–21] concept, we propose a set of ternary codes, ZCZ sequence-pair set, which can reach zero autocorrelation sidelobe zero mutual cross-correlation peaks during Zero Correlation Zone. We also present and analyze a method to construct such ternary codes and subsequently apply them to a radar detection system. The method is that optimized punctured sequence-pair joins together with Hadamard matrix to construct optimized punctured ZCZ sequence-pairs set. An example is presented, investigated, and studied in the radar targets detection simulation system for the performance evaluation of the proposed ternary codes. Because of the outstanding property performance and well target detection performance in simulation system, the newly proposed codes can be useful candidates for pulse compression application in radar system.

The rest of the paper is organized as follows. Section 2 introduces the definitions and properties of ZCZPS. In Section 3, the optimized punctured ZCZPS is introduced, and a method using optimized punctured sequence-pair and Hadamard matrix to construct such codes is given and proved. In Section 4, the properties and ambiguity function of optimized punctured ZCZPS are simulated and analyzed. The performance of optimized punctured ZCZPS is investigated in radar targets detection system by comparing with P4 code in Section 5. In Section 6, conclusions are drawn on optimized punctured ZCZPS.

2. Definitions and Properties of ZCZ Sequence-Pair Set

Zero Correlation Zone (ZCZ) is a new concept provided by Fan et al. [21, 22] in which the autocorrelation sidelobes and cross-correlation values are zero while the time delay is kept within ZCZ instead of the whole period of time domain. There has been considerable interest in constructing [23–27] new classes of ZCZ sequences in ZCZ and studying their properties [28].

Here, we introduce sequence-pair into the ZCZ concept to construct ZCZ sequence-pair set. We consider ZCZPS $(\mathbf{X}, \mathbf{Y})$, $\mathbf{X}$ is a set of K sequences of length N and $\mathbf{Y}$ is a set of K sequences of the same length N:

$$\begin{aligned} \mathbf{x}^{(p)} &\in \mathbf{X} \quad p = 0, 1, 2, \ldots, K-1, \\ \mathbf{y}^{(q)} &\in \mathbf{Y} \quad q = 0, 1, 2, \ldots, K-1. \end{aligned} \tag{1}$$

The autocorrelation function (ACF) (here we use autocorrelation to stand for the cross-correlation between two different sequences of a sequence-pair to distinguish the cross-correlation between two different sequence-pairs) of sequence-pair $(\mathbf{x}^{(p)}, \mathbf{y}^{(p)})$ is defined by

$$R_{\mathbf{x}^{(p)}\mathbf{y}^{(p)}}(\tau) = \sum_{i=0}^{N-1} x_i^{(p)} y_{(i+m) \bmod N}^{(p)*}, \quad 0 \le m \le N-1. \tag{2}$$

The cross-correlation function of two sequence-pairs $(\mathbf{x}^{(p)}, \mathbf{y}^{(p)})$ and $(\mathbf{x}^{(q)}, \mathbf{y}^{(q)})$, $p \neq q$ is defined by

$$C_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau) = \sum_{i=0}^{N-1} x_i^{(p)} y_{(i+m) \bmod N}^{(q)*}, \quad 0 \le m \le N-1, \tag{3}$$

where $\tau = mT_s$ is the time delay and T_s is the bit duration.

For pulse compression sequences, some properties are of particular concern in the optimization for any design in engineering field. They are the peak sidelobe level, the energy of autocorrelation sidelobes, and the energy of their mutual cross-correlation [4]. Therefore, the peak sidelobe level which represents a source of mutual interference and obscures weaker targets can be presented as $\max_K |R_{x^{(p)}y^{(p)}}(\tau)| = 0$, τ is among the zero correlation zone for ZCZPS. Another optimization criterion for the set of sequence-pairs is the energy of autocorrelation sidelobes joined together with the energy of cross-correlation. By minimizing the energy, it can be distributed evenly, and the peak autocorrelation sidelobe and the cross-correlation level can be minimized as well [4]. Here, the energy of ZCZPS can be employed as

$$E = \sum_{p=0}^{K-1} \sum_{\tau=1}^{Z_0} R^2_{\mathbf{x}^{(p)}\mathbf{y}^{(p)}}(\tau) + \sum_{p=0}^{K-1} \sum_{q=0, q \neq p}^{K-1} \sum_{\tau=0}^{Z_0} C_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau). \tag{4}$$

According to (4), it is obvious to see that the energy can be kept low while minimizing the autocorrelation sidelobes and cross-correlation values of any two sequence-pairs within Zero Correlation Zone.

Hence, the ZCZPS can be constructed by minimizing the autocorrelation sidelobe of a sequence-pair and cross-correlation value of any two sequence-pairs in ZCZPS.

Definition 1. Assume $(\mathbf{X}, \mathbf{Y})$ to be a sequence-pair set of K sequence-pairs and each sequence-pair is of N bit length. If all the sequence-pairs in the set satisfy the following equation:

$$R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau) = \sum_{i=0}^{N-1} x_i^{(p)} y_{(i+m) \bmod (N)}^{(q)*} = \sum_{i=0}^{N-1} y_i^{(p)} x_{(i+m) \bmod (N)}^{(q)*} = \begin{cases} \lambda N, & \text{for } m = 0,\ p = q, \\ 0, & \text{for } m = 0,\ p \neq q, \\ 0, & \text{for } 0 < |m| \leq Z_0, \end{cases} \tag{5}$$

where $p, q = 1, 2, 3, \ldots, K-1$, $i = 0, 1, 2, \ldots, N-1$, $0 < \lambda \leq 1$ and $\tau = mT_s$. Then $(\mathbf{x}^{(p)}, \mathbf{y}^{(p)})$ is called a ZCZ sequence-pair, ZCZP is an abbreviation, and $(\mathbf{X}, \mathbf{Y})$ is called a ZCZ sequence-pair set, ZCZPS(N, K, Z_0) is an abbreviation.

3. Optimized Punctured ZCZ Sequence-Pair Set

3.1. Definition of Optimized Punctured ZCZ Sequence-Pair Set. Matsufuji and Torii have provided some methods of constructing ZCZ sequences in [29, 30]. In this section, a set of novel ternary codes, namely, the optimized punctured ZCZ sequence-pair set, is constructed by applying the optimized punctured sequence-pair [31] to the Zero Correlation Zone. Here, optimized punctured ZCZPS is a specific kind of ZCZPS.

Definition 2 (see [31]). Sequence $\mathbf{u} = (u_0, u_1, \ldots, u_{N-1})$ is the punctured sequence for $\mathbf{v} = (v_0, v_1, \ldots, v_{N-1})$

$$u_j = \begin{cases} 0, & \text{if } u_j \text{ is punctured}, \\ v_j, & \text{if } u_j \text{ is non-punctured}, \end{cases} \tag{6}$$

where P is the number of punctured bits in sequence $\mathbf{u}$. Suppose $v_j \in (-1, 1)$ and $u_j \in (-1, 0, 1)$, $\mathbf{u}$ is P-punctured binary sequence, $(\mathbf{u}, \mathbf{v})$ is called a punctured binary sequence-pair.

Definition 3 (see [31]). The autocorrelation of punctured sequence-pair $(\mathbf{u}, \mathbf{v})$ is defined as

$$R_{\mathbf{uv}}(\tau) = R_{\mathbf{uv}}(mT_s) = \sum_{i=0}^{N-1} u_i v_{(i+m) \bmod N}, \quad 0 \leq m \leq N-1. \tag{7}$$

If the punctured sequence-pair has the following autocorrelation property:

$$R_{\mathbf{uv}}(mT_s) = \begin{cases} E, & \text{if } m \equiv 0 \bmod N, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

the punctured sequence-pair is called an optimized punctured sequence-pair [31]. Where, $E = \sum_{i=0}^{N-1} u_i v_i = N - P$, is the energy of punctured sequence-pair.

Definition 4. If $(\mathbf{X}, \mathbf{Y})$ in Definition 1 is constructed by optimized punctured sequence-pair and a certain matrix, such as Hadamard matrix or an orthogonal matrix, where

$$x_i^{(p)} \in (-1, 1), \quad i = 0, 1, 2, \ldots, N-1, \qquad y_i^{(q)} \in (-1, 0, 1), \quad i = 0, 1, 2, \ldots, N-1. \tag{9}$$

Then

$$R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau) = \sum_{i=0}^{N-1} x_i^{(p)} y_{(i+m) \bmod N}^{(q)*} = \begin{cases} \lambda N, & \text{for } m = 0,\ p = q, \\ 0, & \text{for } m = 0,\ p \neq q, \\ 0, & \text{for } 0 < |m| \leq Z_0, \end{cases} \tag{10}$$

where $0 < \lambda \leq 1$ and $\tau = mT_s$, then $(\mathbf{X}, \mathbf{Y})$ can be called an optimized punctured ZCZ sequence-pair set. OPZCZPS(N, K, Z_0) is an abbreviation.

3.2. Design of Optimized Punctured ZCZ Sequence-Pair Set. Based on an optimized punctured binary sequence-pair of odd length and a Hadamard matrix, an optimized punctured ZCZPS can be constructed on following steps.

Step 1. Considering an optimized punctured binary sequence-pair $(\mathbf{u}, \mathbf{v})$ of odd length, the length of each sequence is N_1:

$$\mathbf{u} = u_0, u_1, \ldots, u_{N_1-1}, \quad u_i \in (-1, 1), \qquad \mathbf{v} = v_0, v_1, \ldots, v_{N_1-1}, \quad v_i \in (-1, 0, 1), \qquad i = 0, 1, 2, \ldots, N_1 - 1, \quad N_1 \text{ is odd}. \tag{11}$$

Step 2. A Hadamard matrix $\mathbf{B}$ (the Hadamard matrix is made up of a set of Walsh sequences) of order N_2 is used here. N_2, the length of each sequence, is equal to the number of the sequences in the matrix. Here, any Hadamard matrix order is possible and $\mathbf{b}^{(p)}$ is the row vector of the matrix:

$$\mathbf{B} = \left[\mathbf{b}^{(0)}; \mathbf{b}^{(1)}; \ldots; \mathbf{b}^{(N_2-1)}\right], \qquad \mathbf{b}^{(p)} = \left(b_0^{(p)}, b_1^{(p)}, \ldots, b_{N_2-1}^{(p)}\right), \qquad R_{\mathbf{b}^{(p)}\mathbf{b}^{(q)}} = \begin{cases} N_2, & \text{if } p = q, \\ 0, & \text{if } p \neq q. \end{cases} \tag{12}$$

Step 3. Doing bit-multiplication on the optimized punctured binary sequence-pair and each row of the Hadamard matrix $\mathbf{B}$, then sequence-pair set $(\mathbf{X}, \mathbf{Y})$ is obtained,

$$\begin{aligned}
\mathbf{b}^{(p)} &= \left(b_0^{(p)}, b_1^{(p)}, \ldots, b_{N_2-1}^{(p)}\right), \quad p = 0, 1, \ldots, N_2 - 1,\\
x_j^{(p)} &= u_{j \bmod N_1} b_{j \bmod N_2}^{(p)}, \quad 0 \le p \le N_2 - 1,\ 0 \le j \le N - 1,\\
\mathbf{X} &= \left(\mathbf{x}^{(0)}; \mathbf{x}^{(1)}; \ldots; \mathbf{x}^{(N_2-1)}\right),\\
y_j^{(p)} &= v_{j \bmod N_1} b_{j \bmod N_2}^{(p)}, \quad 0 \le p \le N_2 - 1,\ 0 \le j \le N - 1,\\
\mathbf{Y} &= \left(\mathbf{y}^{(0)}; \mathbf{y}^{(1)}; \ldots; \mathbf{y}^{(N_2-1)}\right).
\end{aligned} \tag{13}$$

Here, the optimized punctured binary sequence-pairs are of odd lengths and the lengths of Walsh sequence are 2^n, $n = 1, 2, \ldots$. It is easy to see that $\gcd(N_1, N_2) = 1$, common divisor of N_1 and N_2 is 1, then $N = N_1 * N_2$. The sequence-pair set $(\mathbf{X}, \mathbf{Y})$ is the optimized punctured ZCZPS and $N_1 - 1$ is the Zero Correlation Zone Z_0. The length of each sequence in optimized punctured ZCZPS is $N = N_1 * N_2$ that depends on the product of length of optimized punctured sequence-pair and the length of Walsh sequence in Hadamard matrix. The number of sequence-pairs in optimized punctured ZCZPS rests on the order of the Hadamard matrix. The sequence $\mathbf{x}^{(p)}$ in sequence set $\mathbf{X}$ and the corresponding sequence $\mathbf{y}^{(p)}$ in sequence set $\mathbf{Y}$ construct a sequence-pair $(\mathbf{x}^{(p)}, \mathbf{y}^{(p)})$ that can be used as a pulse compression code.

The correlation property of the sequence-pairs in optimized punctured ZCZPS is

$$\begin{aligned}
R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau) &= R_{\mathbf{uv}}(m \bmod N_1) R_{\mathbf{b}^{(p)}\mathbf{b}^{(q)}}(m \bmod N_2)\\
&= \begin{cases} EN_2, & \text{if } m = 0,\ p = q,\\ 0, & \text{if } 0 < |m| \le N_1 - 1,\ p = q,\\ 0, & \text{if } 0 \le |m| \le N_1 - 1,\ p \ne q, \end{cases}
\end{aligned} \tag{14}$$

where $N_1 - 1$ is the Zero Correlation Zone Z_0 and $\tau = mT_s$.

Proof. (1) When $p = q$,

$$\begin{gathered}
\tau = 0, \quad R_{\mathbf{uv}}(0) = E, \quad R_{\mathbf{b}^{(p)}\mathbf{b}^{(q)}}(0) = N_2,\\
R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(0) = R_{uv}(0) R_{b^{(p)}b^{(q)}}(0) = EN_2,\\
0 < |\tau| \le (N_1 - 1)T_s, \quad R_{\mathbf{uv}}(\tau) = 0,
\end{gathered} \tag{15}$$

$$R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau) = R_{\mathbf{uv}}(m \bmod N_1) R_{\mathbf{b}^{(p)}\mathbf{b}^{(q)}}(m \bmod N_2) = 0.$$

(2) When $p \ne q$,

$$\begin{gathered}
\tau = 0, \quad R_{\mathbf{b}^{(p)}\mathbf{b}^{(q)}}(0) = 0,\\
R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(0) = R_{\mathbf{uv}}(0) R_{b^{(p)}b^{(q)}}(0) = 0,\\
0 < |\tau| \le (N_1 - 1)T_s,\\
R_{\mathbf{uv}}(\tau) = 0,
\end{gathered} \tag{16}$$

$$R_{\mathbf{x}^{(p)}\mathbf{y}^{(q)}}(\tau) = R_{\mathbf{uv}}(m \bmod N_1) R_{b^{(p)}b^{(q)}}(m \bmod N_2) = 0.$$

□

According to Definition 1, the OPZCZPS constructed by the above method is a ZCZPS.

4. Properties of Optimized Punctured ZCZ Sequence-Pair Set

Considering the optimized punctured ZCZPS constructed by the method mentioned in the last section, the autocorrelation and cross-correlation properties can be simulated and analyzed. For example, the optimized punctured ZCZPS $(\mathbf{X}, \mathbf{Y})$ is constructed by 31-length optimized punctured binary sequence-pair $(\mathbf{u}, \mathbf{v})$, $\mathbf{u} = [++++---+-+-+++----+--+--+++-++-]$, $\mathbf{v} = [++++000+0+0+++0000+00+00+++0++0]$ (using "+" and "−" symbols for "1" and "−1") and Hadamard matrix $\mathbf{H}$ of order 4. We follow the three steps presented in Section 3.2 to construct the optimized punctured ZCZPS. The number of sequence-pairs here is 4, and the length of each sequence is $31 * 4 = 124$. The first row of each matrix $\mathbf{X} = [\mathbf{x}^{(1)}; \mathbf{x}^{(2)}; \mathbf{x}^{(3)}; \mathbf{x}^{(4)}]$ and $\mathbf{Y} = [\mathbf{y}^{(1)}; \mathbf{y}^{(2)}; \mathbf{y}^{(3)}; \mathbf{y}^{(4)}]$ constitute a certain optimized punctured ZCZP $(\mathbf{x}^{(1)}, \mathbf{y}^{(1)})$. Similarly, the second row of each matrix $\mathbf{X}$ and $\mathbf{Y}$ constitute another optimized punctured ZCZ sequence-pair $(\mathbf{x}^{(2)}, \mathbf{y}^{(2)})$, and so on:

$$\begin{aligned}
\mathbf{x}^{(1)} = [&++++---+-+-+++----+--+--\\
&+++-++-++++---+-+-+++--\\
&--+--+--+++-++-++++---+\\
&-+-+++----+--+--+++-++-\\
&++++---+-+-+++----+--+-\\
&-+++-++-],\\
\mathbf{y}^{(1)} = [&++++000+0+0+++0000+00+00+++0\\
&++0++++000+0+0+++0000+00+00++\\
&+0++0++++000+0+0+++0000+00+00\\
&+++0++0++++000+0+0+++0000+00\\
&+00+++0++0],\\
\mathbf{x}^{(2)} = [&+-+--+------+--+-+++---\\
&++-+++---+-++-++++++-++-\\
&+---+++--+---+++-+--+--\\
&----+--+-+++---++-+++-\\
&--+-++-++++++-++-+---+\\
&++--+---++],
\end{aligned}$$

$$\mathbf{y}^{(2)} = [+ - + - 000 - 0 - 0 - + - 0000 + 00 - 00 + - + 0$$
$$+ -0 - + - +000 + 0 + 0 + - + 0000 - 00 + 00$$
$$- + - 0 - +0 + - + -000 - 0 - 0 - + - 0000$$
$$+ 00 - 00 + - + 0 + -0 - + - +000 + 0 + 0 + -$$
$$+0000 - 00 + 00 - + - 0 - +0]. \quad (17)$$

Here, optimized punctured ZCZ sequence-pairs $(\mathbf{x}^{(1)}, \mathbf{y}^{(1)})$ and $(\mathbf{x}^{(2)}, \mathbf{y}^{(2)})$ are studied as two examples in the following parts.

4.1. Autocorrelation and Cross-Correlation Properties. The autocorrelation property and cross-correlation property of 124-length sequence-pairs in the optimized punctured ZCZ sequence-pair set $(\mathbf{X}, \mathbf{Y})$ are shown in Figures 1 and 2.

From the Figures 1 and 2, the peak autocorrelation sidelobe of ZCZPS and their cross-correlation value are kept as low as zero while the time delay is kept within $Z_0 = N_1 - 1 = 30$ (Zero Correlation Zone). And it is always true that the cross-correlation values of optimized punctured ZCZPS and the autocorrelation sidelobe could be kept as low as zero during ZCZ.

We still have to confess that the energy loss of the proposed codes is no less than 1.7 db due to reference mismatch. However, the perfect periodic ACF and CCF achieved simultaneously during the ZCZ zone and the codes' structure could make up for it. It is known that a suitable criterion for evaluating code of length N is the ratio of the peak signal mainlobe divided by the peak signal sidelobe (PSR) of their autocorrelation function, which can be bounded by [32]

$$[\text{PSR}]_{\text{dB}} \leq 20 \log_2 N = [\text{PSR}_{\max}]_{\text{dB}}. \quad (18)$$

The only aperiodic uniform phase codes that can reach the $\text{PSR}_{\max}$ are the Barker codes whose length is equal or less than 13. Considering the periodic sequences, the m-sequences or Legendre sequences could achieve the lowest periodic ACF of $|R_i(\tau \neq 0) = 1|$. For non-binary sequences, it is possible to find perfect sequences of ideal ACF. Golomb codes are a kind of two valued (biphase) perfect codes which obtain zero periodic ACF but result in large mismatch power loss. The Ipatov code shows a way of designing code pairs with perfect periodic autocorrelation (the cross-correlation of the code pair) and minimal mismatch loss. In addition, zero periodic autocorrelation function for all nonzero shifts could be obtained by polyphase codes, such as Frank and Zadoff codes. However, for both binary and non-binary periodic sequences, it is not possible for the sequences to have perfect ACF and CCF simultaneously although ideal CCFs could be achieved alone. Comparing with the above codes, the proposed ternary codes could obtain perfect periodic ACF during the ZCZ and the reference sequence is made of $(-1, 0, 1)$ which is much less complicated than other perfect ternary codes such as Ipatvo code. The reference code for Ipatov code is of a three-element alphabet which might not always be integer.

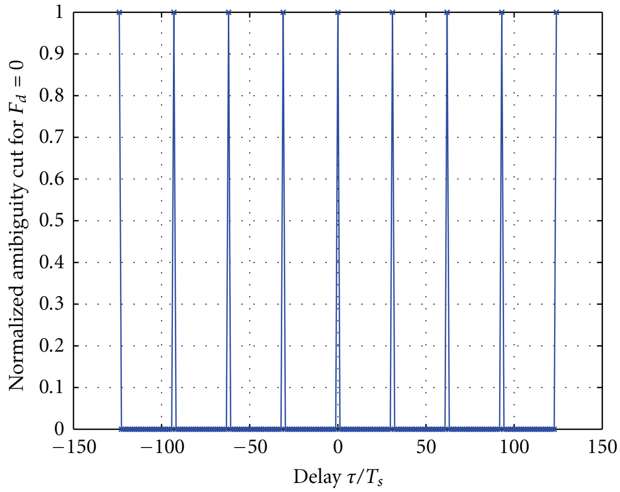


Figure 1: Periodic autocorrelation property of optimized punctured ZCZPS.

Nevertheless, considering multi targets in the system, multiple peaks of the autocorrelation function of the proposed codes might affect on the range resolution. The range resolution could be limited as $T_s < \tau < N_1 T_s$ or $\tau > NT_s$. Here, T_s is one bit duration, N_1 is the length of an optimized punctured sequence-pair and N is the length of an optimized punctured ZCZ sequence-pair. In the Figure 1, $N_1 = 31$. Otherwise, some digital signal processing methods could also be introduced to distinguish the peaks. On the other hand, there may also be the concern that multiple peaks of single transmitting signal reflected from one target may affect determining the main peak of ACF. As a matter of fact, the matched filter here could shift at the period of ZCZ length to track each peak instead of shifting bit by bit after the first peak is acquired. Hence, in this way could it be working more efficiently. Alike the tracking technology in synchronization of CDMA system, checking several peaks instead of only one peak guarantee the precision of P_D and avoidance of P_{FA}. In addition, those obtained peaks could be averaged before the detection in order to reduce the effect of random noise in the channel so that the detection performance could be improved.

To sum up, the new code could achieve perfect ACF and CCF in the ZCZ simultaneously according to Figures 1 and 2, and its PSR can be as large as infinite.

4.2. Ambiguity Function. When the transmitted impulse is reflected by a moving target, the reflected echo signal includes a linear phase shift which corresponds to a Doppler shift F_d [32]. As a result of the Doppler shift F_d, the main peak of the autocorrelation function is reduced. The SNR is degraded and the sidelobe structure is also changed because of the Doppler shift.

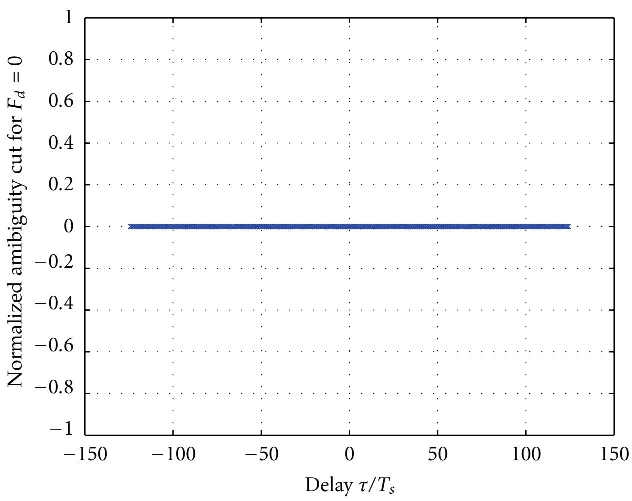


Figure 2: Periodic cross-correlation property of optimized punctured ZCZPS.

The ambiguity function which is usually used to analyze the radar performance within Doppler shift and time delay is defined in [32]:

$$A(\tau, F_D) \equiv \left| \int_{-\infty}^{\infty} x(s) e^{j2\pi F_D s} x^*(s-\tau) ds \right| \equiv \left| \hat{A}(\tau, F_D) \right|, \quad (19)$$

where τ is the time delay between transmitting signal and matched filter, and F_D is the Doppler shift.

In [33], Periodic Ambiguity Function (PAF) is introduced by Levanon as an extension of the periodic autocorrelation for Doppler shift. And the single-periodic complex envelope is [34]

$$\begin{aligned} A_{\text{periodic}}(\tau, F_D) &\equiv \left| \frac{1}{T} \int_0^T x\left(s + \frac{\tau}{2}\right) e^{j2\pi F_D s} x^*\left(s - \frac{\tau}{2}\right) ds \right| \\ &\equiv \left| \hat{A}_{\text{periodic}}(\tau, F_D) \right|, \end{aligned} \quad (20)$$

where T is one period of the signal.

We are studying sequence-pairs in this research, so we use different codes for transmitting part and receiving part. The single-period ambiguity function for ZCZPS can be rewritten as

$$\begin{aligned} A_{\text{pair}}(\tau, F_D) &\equiv \left| \hat{A}_{\text{pair}}(\tau, F_D) \right| \\ &= \left| \frac{1}{T} \int_0^T x^{(p)}\left(s + \frac{\tau}{2}\right) e^{j2\pi F_D s} y^{(q)*}\left(s - \frac{\tau}{2}\right) ds \right|, \end{aligned} \quad (21)$$

where $p, q = 0, 1, 2, \ldots, K-1$, $T = NT_s$ is one period of the signal and T_s is one bit duration. At the same time, when $p = q$, (21) can be used to analyze the autocorrelation property within Doppler shift, and when $q \neq p$, (21) can be used to analyze the cross-correlation performance within Doppler shift. Equation (21) is plotted in Figure 3 in a three-dimensional surface plot to analyze the radar performance of optimized punctured ZCZPS within Doppler shift. Here, maximal time delay is 1 unit (normalized to length of the code, in units of NT_s) and maximal Doppler shift is 5 units for cross-correlation and 3 units for autocorrelation (normalized to the inverse of the length of the code, in units of $1/NT_s$).

In Figure 3(a), there is relative uniform plateau suggesting low and uniform sidelobes. This low and uniform sidelobes minimize target masking effect in Zero Correlation Zone of time domain, where $Z_0 = 30$, $-30\tau_c \leq \tau \leq 30\tau_c$. From Figure 3(b), considering cross-correlation property between any two optimized punctured ZCZ sequence-pairs of the ZCZPS, we can see that the optimized punctured ZCZPS is tolerant of Doppler shift when Doppler shift is not large. When the Doppler shift is zero, or the target is not moving, cross-correlation of our proposed code is zero during ZCZ.

Since synchronizing techniques develop exponentially in the industrial world, time delay between transmitting signal and matched filter can, to some extent, be precisely estimated. Therefore, it is necessary to investigate the property of our proposed code when we have the output of the matched filter at the expected time $\tau = 0$. When $\tau = 0$, the ambiguity function can be expressed as

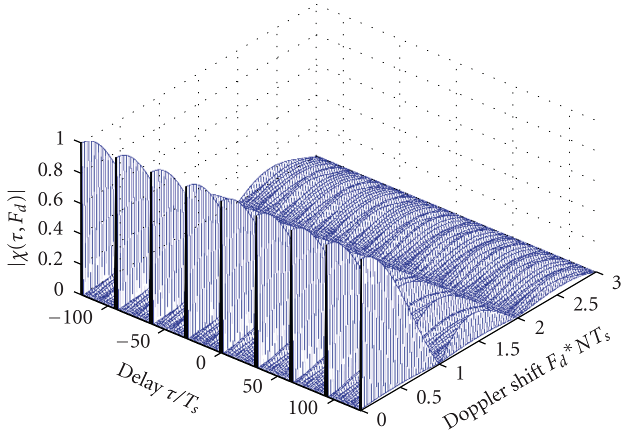
$$\left| \hat{A}_{\text{pair}}(0, F_D) \right| = \left| \frac{1}{T} \int_0^T x^{(p)}(s) y^{(q)*}(s) e^{(j2\pi F_D s)} ds \right|. \quad (22)$$

And the Doppler shift performance without time delay is presented in the Figure 4.

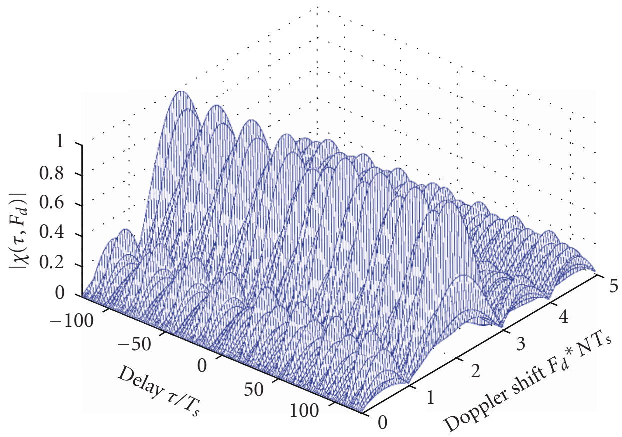
Figure 4(a) illustrates that without time delay of matched filter but having the Doppler shift less than 1 unit, the autocorrelation value of optimized punctured ZCZPS falls sharply during one unit, and the trend of the amplitude over the whole frequency domain decreases as well. Figure 4(b) shows that there are some convex surfaces in the cross-correlation performance. From Figures 4(a) and 4(b), when Doppler frequencies equal to multiples of the pulse repetition frequency (PRF = 1/PRI = $1/Ts$), all the ambiguity values turn to zero except when Doppler frequency is equal to 2 PRF for cross-correlation. That is the same as many widely used pulse compression binary code such as the Barker code. Overall, the ambiguity function performances of optimized punctured ZCZP can be as efficient as conventional pulse compression binary code.

5. Application to Radar System

According to [32], Probability of Detection (P_D), Probability of False Alarm (P_{FA}) and Probability of Miss (P_M) are three probabilities of most interest in the radar system. Note that $P_M = 1 - P_D$. Therefore, we simulated the above three probabilities of using 124-length optimized punctured ZCZ sequence-pair in radar system in this section. The performance of radar system using 124-length P4 code is also studied in order to compare with the performance of optimized punctured ZCZ sequence-pairs of corresponding

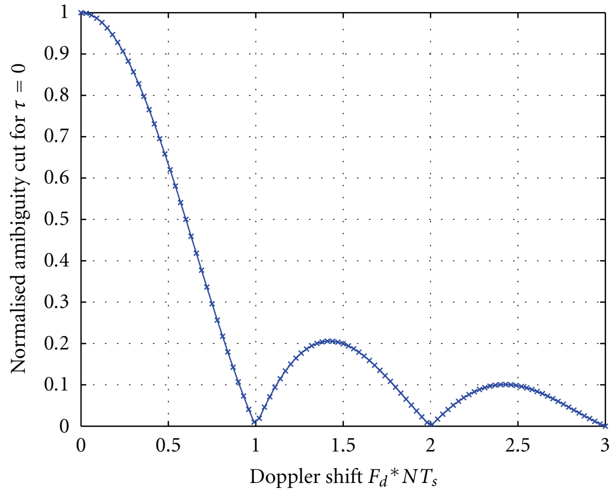


(a)

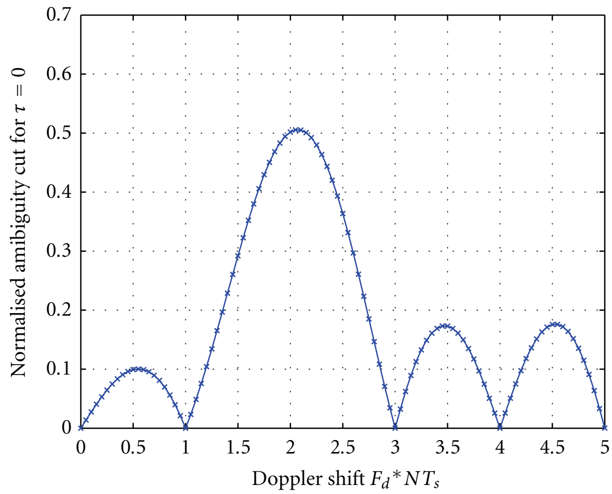


(b)

Figure 3: Ambiguity function of 124-length ZCZPS: (a) autocorrelation, (b) cross-correlation.



(a)



(b)

Figure 4: Doppler shift of 124-length ZCZPS ($\tau = 0$): (a) autocorrelation (b) cross-correlation.

length. In the simulation model, 10^5 times of Monte-Carlo simulation has been run for each SNR value. The Doppler shift frequency is a random variable that is kept less than 1 unit (normalized to the inverse of the length of the code, in units of $1/NT_s$), and the expected peak time of the output of the matched filter is at $\tau = 0$.

From Figure 5, the probabilities of miss target detection P_M of the system using 124-length optimized punctured ZCZP are lower than 124-length P4 code especially when the SNR is not high. When SNR is higher than 18 dB, both probabilities of miss targets of the system approach zero. However, the probabilities of miss targets of P4 code fall more quickly than optimized punctured ZCZP.

We plotted the detection probability P_D versus false alarm probability P_{FA} of the coherent receiver. We have simulated the performance at different SNR values. Because of the limited space, we only chose SNR at 12 db and 14 dB. Figure 6 shows performance of 124-length optimized punctured ZCZP and performance of the same length P4 code when the SNR is 12 dB and 14 dB. Within the same SNR value either 12 dB or 14 dB, the detection probabilities of optimized punctured ZCZ sequence-pair are much larger than detection probabilities of P4 code, and meanwhile P_{FA} of the first code are also smaller than P_{FA} of the latter code. Stating differently, optimized punctured ZCZ sequence-pair has higher target detection probability while keeping a lower false alarm probability. Furthermore, observing Figure 6, 124-length optimized punctured ZCZ sequence-pair even has much better performance at 12 dB SNR than P4 code of corresponding length at 14 dB SNR.

6. Conclusions

The definition and properties of a set of newly provided ternary codes-ZCZ sequence-pair set were discussed in this paper. Based on optimized punctured sequence-pair and Hadamard matrix, we have investigated a constructing method for a specific ZCZPS-optimized punctured ZCZPS made up of a set of optimized punctured ZCZPs along with studying its properties. The significant advantage of the

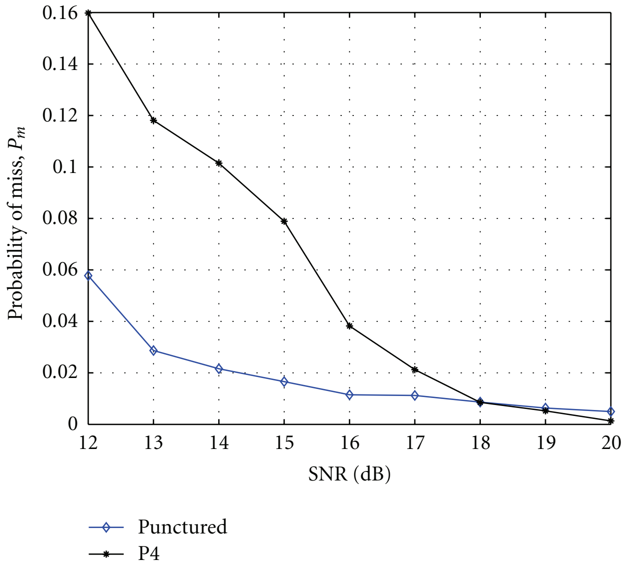


Figure 5: Probability of miss targets detection: 124-length optimized punctured ZCZ sequence-pair versus 124-length P4 code.

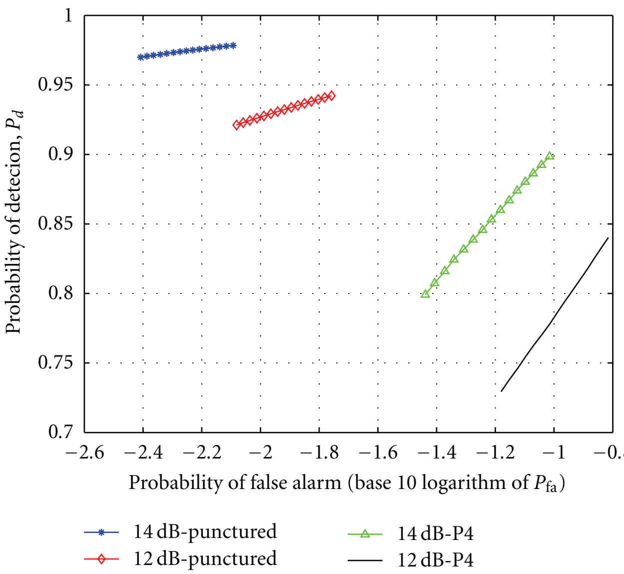


Figure 6: Probability of detection versus probability of false alarm of the coherent receiver: 124-length optimized punctured ZCZ sequence-pair versus 124-length P4 code.

optimized punctured ZCZPS is the considerably reducedn autocorrelation sidelobe and zero mutual cross-correlation value during ZCZ. According to the radar system simulation results shown in Figures 5 and 6, it is easy to observe that 124-length optimized punctured ZCZPS has better performance than P4 code of the same length when the target is not moving very fast in the system. A general conclusion can be drawn that the optimized punctured ZCZPS consisting of optimized punctured ZCZ sequence-pairs can effectively increase the variety of candidates for pulse compression codes. Because of the ideal cross-correlation properties of optimized punctured ZCZPS, our future work would focus on the application of the optimized punctured ZCZPS in multiple radar systems.

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