Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 90401, 13 pages
doi:10.1155/2007/90401

Research Article

A Simplified Constant Modulus Algorithm for Blind Recovery of MIMO QAM and PSK Signals: A Criterion with Convergence Analysis

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Received 31 October 2006; Revised 18 June 2007; Accepted 3 September 2007

Recommended by Monica Navarro

The problem of blind recovery of QAM and PSK signals for multiple-input multiple-output (MIMO) communication systems is investigated. We propose a simplified version of the well-known constant modulus algorithm (CMA), named simplified CMA (SCMA). The SCMA cost function consists in projection of the MIMO equalizer outputs on one dimension (either real or imaginary part). A study of stationary points of SCMA reveals the absence of any undesirable local stationary points, which ensures a perfect recovery of all signals and a global convergence of the algorithm. Taking advantage of the phase ambiguity in the solution of the new cost function for QAM constellations, we propose a modified cross-correlation term. It is shown that the proposed algorithm presents a lower computational complexity compared to the constant modulus algorithm (CMA) without loss in performances. Some numerical simulations are provided to illustrate the effectiveness of the proposed algorithm.

1. INTRODUCTION

In the last decade, the interest in blind source separation (BSS) techniques has been important. The problem of blind recovery of multiple independent and identically distributed (i.i.d.) signals from their linear mixture in a multiple-input multiple-output (MIMO) system arises in many applications such as spatial division multiple access (SDMA), multiuser communications (such as CDMA for code division multiple access), and more recently Bell Labs layered space-time (BLAST) [1–3]. The aim of blind signals separation is to retrieve source signals without the use of a training sequence, which can be expensive or impossible in some practical situations. Another interesting class of blind methods is blind identification. Unlike blind source separation, the aim of blind identification is to find an estimate of the MIMO channel matrix [4–6]. Once this estimate has been obtained, the source signals can be efficiently recovered using MIMO detection methods, such as maximum likelihood (ML) [7] and BLAST [8] detection methods. The main difference between blind source separation and blind identification is that in the first case, the source signals are recovered directly from the observations, whereas in the second case, a MIMO detection algorithm is needed, which may increase complexity (complexity depends on used methods). Note that, unlike BSS techniques, ML detector is nonlinear but optimum and suffers from high complexity. Sphere decoding [7] allows to reduce considerably ML detector complexity. On the other hand, the performance of MIMO detection methods depends strongly on the quality of the channel estimate which results from blind identification. In this paper, we consider the problem of blind source separation of MIMO instantaneous channel.

In literature, the constant modulus of many communication signals, such as PSK and 4-QAM signals, is a widely used property in blind source separation and blind equalization. The initial idea can be traced back to Sato [9], Godard [10], and Treichler et al. [11, 12]. The algorithms are known as CMAs. The first application after blind equalization was blind beamforming [13, 14] and more recently blind signals separation [15, 16]. In the case of constant modulus signals, CMA has proved reasonable performances and desired convergence requirements. On the other hand, the CMA yields a degraded performance for nonconstant modulus signals such as the quadrature amplitude modulation (QAM) signals, because the CMA projects all signal points onto a single modulus.

In order to improve the performance of the CMA for QAM signals, the so-called modified constant modulus algorithm (MCMA) [17], known as MMA for multimodulus algorithm, has been proposed [18–20]. This algorithm, instead of minimizing the dispersion of the magnitude of the equalizer output, minimizes the dispersion of the real and imaginary parts separately; hence the MMA cost function can be considered as a sum of two one-dimensional cost functions. The MMA provides much more flexibility than the CMA and is better suited to take advantage of the symbol statistics related to certain types of signal constellations, such as nonsquare and very dense constellations [18]. Please notice that both CMA and MMA are two-dimensional (i.e., employ both real and imaginary part of the equalizer outputs). Another class of algorithm has been proposed recently and named constant norm algorithm (CNA), whose CMA represents a particular case [21, 22].

In this paper, we propose a simplified version of the CMA cost function named simplified CMA (SCMA) and based only on one dimension (either real or imaginary part), as opposed to CMA. The major advantage of SCMA is its low complexity compared to that of CMA and MMA. Because, instead of using both real and imaginary parts as in CMA and MMA, only one dimension, the real or imaginary part, is considered in SCMA, which makes it very attractive for practical implementation especially when complexity issue arises such as in user's side. We will demonstrate that only the existing stationary points of the SCMA cost function correspond to a perfect recovery of all source signals except for the phase and permutation indeterminacy. We will show that the phase rotation is not the same for QAM, 4-PSK, and P-PSK ($P \geq 8$). Moreover, in order to reduce the complexity further, we will introduce a modified cross-correlation term by taking advantage of the phase ambiguity of the SCMA cost function for QAM constellations. An adaptive implementation by means of the stochastic gradient algorithm (SGA) will be described. A part of the results presented in this paper (QAM case with its convergence analysis) was previously reported in [23].

The remainder of the paper is organized as follows. In Section 2, the problem formulation and assumptions are introduced. In Section 3, we describe the SCMA criterion. The convergence analysis of the proposed cost function is carried out in Section 4. Section 5 introduces a modified cross-correlation constraint for QAM constellations. In Section 6, we present an adaptive implementation of the algorithm. Finally, Section 7 presents some numerical results.

2. PROBLEM FORMULATION

We consider a linear data model which takes the following form:

$$\mathbf{y}(n) = \mathbf{H}\mathbf{a}(n) + \mathbf{b}(n), \tag{1}$$

where $\mathbf{a}(n) = [a_1(n),\ldots,a_M(n)]^T$ is the $(M \times 1)$ vector of the source signals, $\mathbf{H}$ is the $(N \times M)$ MIMO linear memoryless channel, $\mathbf{y}(n) = [y_1(n),\ldots,y_N(n)]^T$ is the $(N\times 1)$ vector of the received signals, and $\mathbf{b}(n) = [b_1(n),\ldots,b_N(n)]^T$ is the $(N\times 1)$ noise vector. M and N represent the number of transmit and receive antennas, respectively.

In the case of the MIMO frequency selective channel (convolutive model), the system can be reduced to the model in (1) tanks to the linear prediction method presented in [5]. Afterwards, blind source separation methods can be applied. The following assumptions are considered:

(1) $\mathbf{H}$ has full column rank M,
(2) the noise is additive white Gaussian independent from the source signals,
(3) the source signals are independent and identically distributed (i.i.d), mutually independent $E[\mathbf{a}\mathbf{a}^H] = \sigma_a^2\mathbf{I}_M$, and drawn from QAM or PSK constellations.

Please notice that these assumptions are not very restrictive and satisfied in BLAST scheme whose corresponding model is given in (1). Moreover, throughout this paper by QAM constellation we mean only square QAM constellation. In order to recover the source signals, the received signal $\mathrm{y}(n)$ is processed by an $(N \times M)$ receiver matrix $\mathbf{W} = [\mathbf{w}_1,\ldots,\mathbf{w}_M]$. Then, the receiver output can be written as

$$\begin{aligned}\mathbf{z}(n) &= \mathbf{W}^T\mathbf{y}(n) = \mathbf{W}^T\mathbf{H}\mathbf{a}(n) + \mathbf{W}^T\mathbf{b}(n)\\ &= \mathbf{G}^T\mathbf{a}(n) + \widetilde{\mathbf{b}}(n),\end{aligned} \tag{2}$$

where $\mathbf{z}(n) = [z_1(n),\ldots,z_M(n)]^T$ is the $(M \times 1)$ vector of the receiver output, $\mathbf{G} = [\mathbf{g}_1,\ldots,\mathbf{g}_M] = \mathbf{H}^T\mathbf{W}$ is the $(M \times M)$ global system matrix, and $\widetilde{\mathbf{b}}(n)$ is the filtered noise at the receiver output.

The purpose of blind source separation is to find the matrix $\mathbf{W}$ such that $\mathbf{z}(n) = \hat{\mathbf{a}}(n)$ is an estimate of the source signals.

Please note that in blind signals separation, the best that can be done is to determine $\mathbf{W}$ up to a permutation and scalar multiple [3]. In other words, $\mathbf{W}$ is said to be a separation matrix if and only if

$$\mathbf{G}^T = \mathbf{W}^T\mathbf{H} = \mathbf{P}\mathbf{\Lambda}, \tag{3}$$

where $\mathbf{P}$ is a permutation matrix and $\mathbf{\Lambda}$ a nonsingular diagonal matrix.

Throughout this paper, we use small and capital boldface letters to denote vectors and matrices, respectively. The symbols $(\cdot)^*$ and $(\cdot)^T$ denote the complex conjugate and transpose, respectively, $(\cdot)^H$ is the Hermitian transpose, and $\mathbf{I}_p$ is the $(p \times p)$ identity matrix.

3. THE PROPOSED CRITERION

Unlike the CMA algorithm [10], whose aim consists in constraining the modulus of the equalizer outputs to be on a circle (projection onto a circle), we suggest to project the equalizer outputs onto one dimension (either real or imaginary part). To do so, we suggest to penalize the deviation of the square of the real (imaginary) part of the equalizer outputs from a constant.

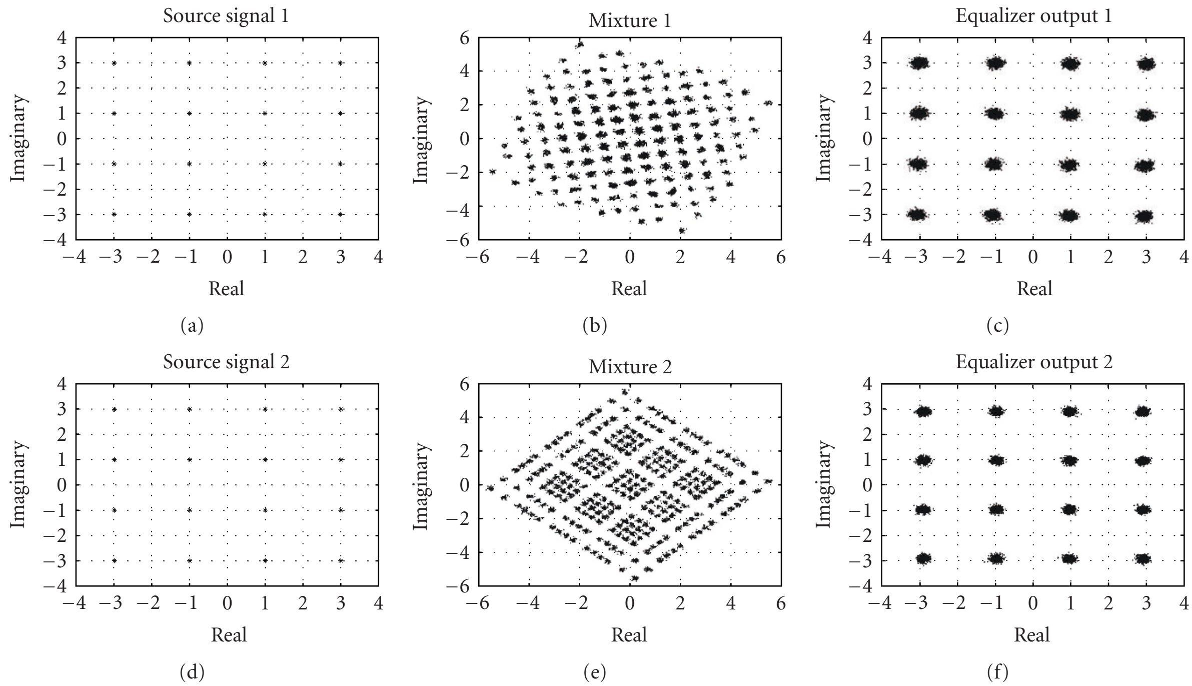

Figure 1: 16-QAM constellation. Left column: the constellations of the transmitted signals, middle column: the constellations of the received signals (mixtures), right column: the constellations of the recovered signals.

For the ℓth equalizer, we suggest to optimize the following criterion:

$$\min_{\mathbf{w}_\ell} \mathcal{J}(\mathbf{w}_\ell) = E\Big[\big(z_{R,\ell}(n)^2 - R\big)^2\Big], \quad \ell = 1,\dots,M, \tag{4}$$

where $z_{R,\ell}(n)$ denotes the real part of the ℓth equalizer output $z_\ell(n) = \mathbf{w}_\ell^T \mathbf{y}(n)$ and R is the dispersion constant fixed by assuming a perfect equalization with respect to the zero forcing (ZF) solution, and is defined as

$$R = \frac{E\big[a_R(n)^4\big]}{E\big[a_R(n)^2\big]}, \tag{5}$$

where $a_R(n)$ denotes the real part of the source signal $a(n)$.

The term on the right side of the equality (4) prevents the deviation of the square of the real part of the equalizer outputs from a constant. The minimization of (4) allows the recovery of only one signal at each equalizer output (see proof in Section 4). But the algorithm minimization (4) does not ensure the recovery of all source signals because it may converge in order to recover the same source signal at many outputs. In order to avoid this problem, we suggest to use a cross-correlation term due to its computational simplicity. Then (4) becomes

$$\min_{\mathbf{w}_\ell} \mathcal{J}(\mathbf{w}_\ell) = E\Big[\big(z_{R,\ell}(n)^2 - R\big)^2\Big] + \alpha \sum_{i=1}^{\ell-1} \big| r_{\ell i}(n) \big|^2, \quad \ell = 1,\dots,M, \tag{6}$$

where $\alpha \in \mathbb{R}^+$ is the mixing parameter and $r_{\ell i}(n) = E[z_\ell(n) z_i^*(n)]$ is the cross-correlation between the ℓth and the ith equalizer outputs and prevents the extraction of the same signal at many outputs. Then the first term in (6) ensures the recovery of only one signal at each equalizer output and the cross-correlation term ensures that each equalizer output is different from the other ones; this results in the recovery of all source signals (see Section 4). In the following sections, we name (6) the cross-correlation simplified CMA (CC-SCMA) criterion. In (6) we could also use the imaginary part thanks to the symmetry of the QAM and PSK constellations. Since the analysis is the same for the imaginary part, throughout this paper, we only consider the real part.

4. CONVERGENCE ANALYSIS

Theorem 1. *Let M be i.i.d. and mutually independent signals $a_i(n), i = 1,\dots,M$, which share the same statistical properties, are drawn from QAM or PSK constellations and are transmitted via an $(M \times N)$ MIMO linear memoryless channel and without the presence of noise. Provided that the weighting factor α is chosen to satisfy $\alpha \geq 2E[a_R^4]/\sigma_a^4 d^2$ (where $d = 1$ for QAM and P-PSK ($P \geq 8$) constellations and $d = \sqrt{2}$ for 4-PSK constellation), the algorithm in (6) will converge to a setting that corresponds, in the absence of any noise, to a perfect recovery of all transmitted signals, and the only stable minima are the Dirac-type vector taking the following form: $\mathbf{g}_\ell = [0,\dots,0, d_\ell e^{j\phi_\ell}, 0,\dots,0]^T$, where $\mathbf{g}_\ell$ is the ℓth column vector of $\mathbf{G}$, d_ℓ is the amplitude, and ϕ_ℓ is the phase rotation of the nonzero element. The pair (d_ℓ, ϕ_ℓ) is given by $\{1, \text{modulo}\,(\pi/2)\}$, $\{\sqrt{2}, \text{modulo}\,[(2k+1)\pi/4]\}$, and $\{1, \text{arbitrary in}\,[0, 2\pi]\}$ for QAM, 4-PSK, and P-PSK ($P \geq 8$) constellations, respectively.*

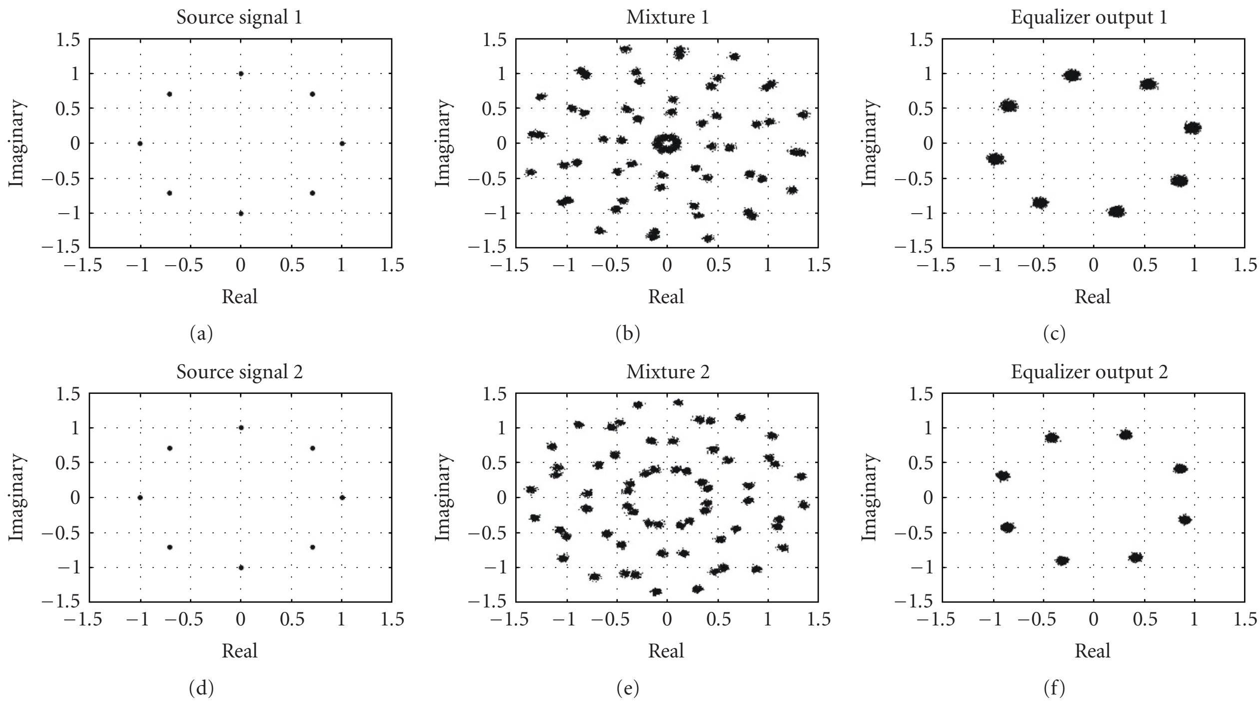

Figure 2: 8-PSK constellation. Left column: the constellations of the transmitted signals, middle column: the constellations of the received signals (mixtures), right column: the constellations of the recovered signals.

Proof. For simplicity, the analysis is restricted to noise-free case, that is,

$$\mathbf{z}(n) = \mathbf{G}^T\mathbf{a}(n). \tag{7}$$

Note that due to the assumed full column rank of $\mathbf{H}$, all results in the $\mathbf{G}$ domain will translate to the $\mathbf{W}$ domain as well. For convenience, the stationary points study will be carried out in the $\mathbf{G}$ domain [24].

Considering (7), the cross-correlation term in (6) can be simplified as

$$\begin{aligned} E\left[z_\ell(n)z_i^*(n)\right] &= E\left[\mathbf{w}_\ell^T\mathbf{H}\mathbf{a}(n)\mathbf{w}_i^H\mathbf{H}^*\mathbf{a}(n)^*\right] \\ &= \mathbf{g}_\ell^T E\left[\mathbf{a}(n)\mathbf{a}(n)^H\right]\mathbf{g}_i^* = \sigma_a^2\mathbf{g}_\ell^T\mathbf{g}_i^* = \sigma_a^2\mathbf{g}_i^H\mathbf{g}_\ell, \end{aligned} \tag{8}$$

where we use the fact that $\mathbf{w}_i^T\mathbf{H} = \mathbf{g}_i^T$.

Using (8) in (6), we get

$$\begin{aligned} \mathcal{J}(\mathbf{g}_\ell) &= E\left[\left(z_{R,\ell}(n)^2 - R\right)^2\right] \\ &\quad + \alpha\sigma_a^4\sum_{i=1}^{\ell-1}\left|\mathbf{g}_i^H\mathbf{g}_\ell\right|^2, \quad \ell = 1,\dots,M. \end{aligned} \tag{9}$$

From (9), we first notice that the adaptation of each $\mathbf{g}_\ell$ depends only on $\mathbf{g}_1,\dots,\mathbf{g}_{\ell-1}$. Then, we can begin by the first output, because $\mathbf{g}_1$ is optimized independently from all the other vectors $\mathbf{g}_2,\dots,\mathbf{g}_M$. Hence, for the first equalizer, $\mathbf{g}_1$, we have

$$\min_{\mathbf{g}_1}\mathcal{J}(\mathbf{g}_1) = E\left[\left(z_{R,1}(n)^2 - R\right)^2\right]. \tag{10}$$

By developing (10), we get (for notation convenience, in the following, we will omit the time index n)

$$\mathcal{J}(\mathbf{g}_1) = E\left[z_{R,1}^4\right] - 2\mathrm{RE}\left[z_{R,1}^2\right] + R^2. \tag{11}$$

Because the development is not the same for QAM, 4-PSK, and P-PSK ($P \geq 8$) constellations, we will enumerate the proof of each case separately. □

4.1. QAM case

After a straightforward development of the terms in (11) with respect to statistical properties of QAM signals (see Appendix A), (11) can be written as

$$\begin{aligned} \mathcal{J}(\mathbf{g}_1) &= E\left[a_R^4\right]\left(\sum_{k=1}^{M}\left|g_{k1}\right|^2 - 1\right)^2 \\ &\quad + \beta\left[\left(\sum_{k=1}^{M}\left|g_{k1}\right|^2\right)^2 - \sum_{k=1}^{M}\left|g_{k1}\right|^4\right] \\ &\quad + 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2 - E\left[a_R^4\right] + R^2, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \mathbf{g}_1 &= \left[g_{11},\dots,g_{M1}\right]^T, \\ g_{k1} &= g_{R,k1} + jg_{I,k1}, \\ \beta &= 3E^2\left[a_R^2\right] - E\left[a_R^4\right] = -\kappa_{a_R} > 0, \end{aligned} \tag{13}$$

and $\kappa_{a_R} = E[a_R^4] - 3E^2[a_R^2]$ represents the kurtosis of the real parts of the symbols. It is always negative in the case of PSK and QAM signals.

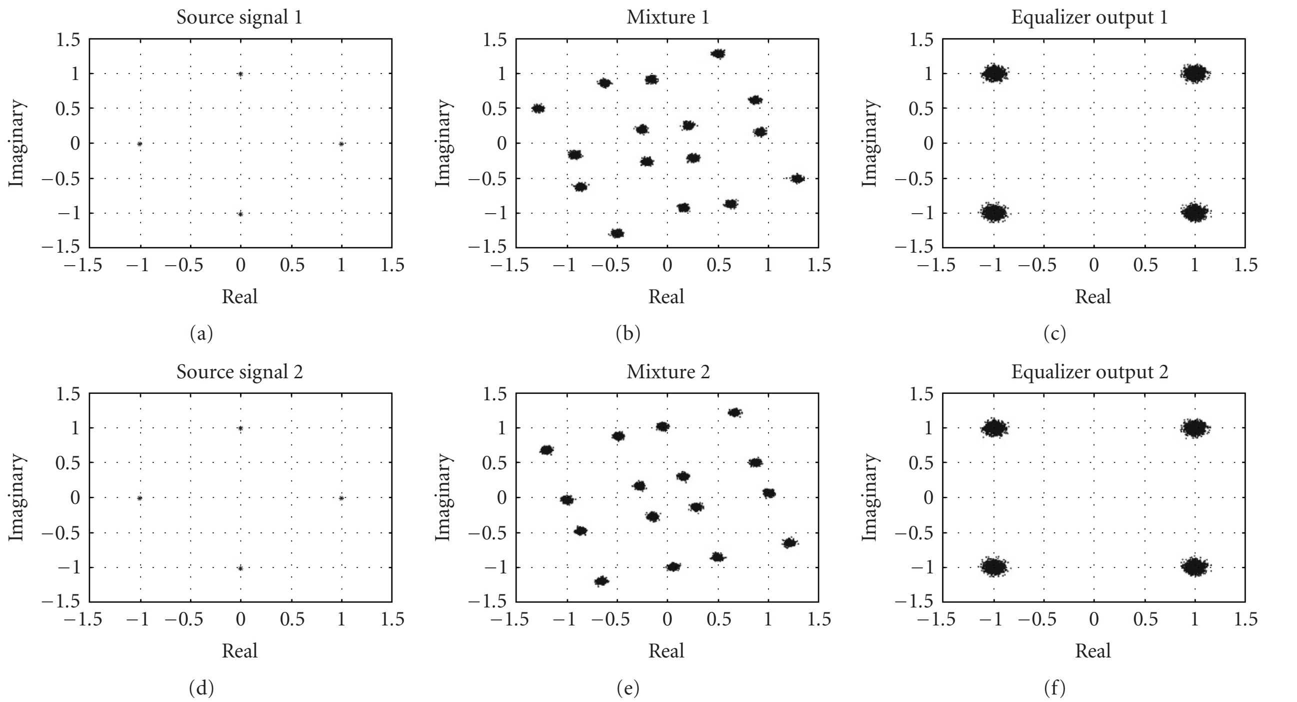

Figure 3: 4-PSK constellation. Left column: the constellations of the transmitted signals, middle column: the constellations of the received signals (mixtures), right column: the constellations of the recovered signals.

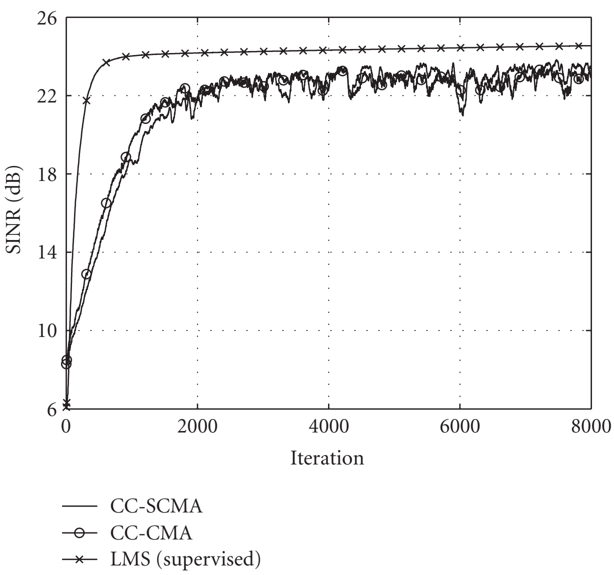

Figure 4: Performance comparison in term of SINR of the proposed algorithm (CC-SCMA) with CC-CMA and supervised LMS.

The minimum of (11) can be found easily by replacing the equalizer output in (12) by any of the transmitted signals, it is given by

$$J_{\min} = E\left[\left(a_R^2 - R\right)^2\right] = E\left[a_R^4\right] - 2\mathrm{R}E\left[a_R^2\right] + R^2. \quad (14)$$

From (5), we have

$$\mathrm{R}E\left[a_R^2\right] = E\left[a_R^4\right]. \quad (15)$$

Then

$$J_{\min} = -E\left[a_R^4\right] + R^2. \quad (16)$$

Comparing (12) and (16), we can write

$$\begin{aligned}\mathcal{J}(\mathbf{g}_1) = J_{\min} &+ \beta\left[\left(\sum_{k=1}^{M}\left|g_{k1}\right|^2\right)^2 - \sum_{k=1}^{M}\left|g_{k1}\right|^4\right] \\ &+ E\left[a_R^4\right]\left(\sum_{k=1}^{M}\left|g_{k1}\right|^2 - 1\right)^2 + 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2.\end{aligned} \quad (17)$$

Since $\beta > 0$ and that

$$\left(\sum_{k=1}^{M}\left|g_{k1}\right|^2\right)^2 \geq \sum_{k=1}^{M}\left|g_{k1}\right|^4, \quad (18)$$

we have

$$\beta\left[\left(\sum_{k=1}^{M}\left|g_{k1}\right|^2\right)^2 - \sum_{k=1}^{M}\left|g_{k1}\right|^4\right] \geq 0. \quad (19)$$

According to (19), $\mathcal{J}(\mathbf{g}_1)$ is composed only of positive terms. Then, minimizing $\mathcal{J}(\mathbf{g}_1)$ is equivalent to finding $\mathbf{g}_1$, which minimizes all terms simultaneously. One way to find the minimum of (17) is to look for a solution that cancels the gradients of each term separately. From (16), we know that

$J_{\min}$ is a constant ($\partial J_{\min}/\partial g_{\ell 1}^* = 0$), hence we only deal with the reminder terms. For that purpose, let us have

$$\mathcal{J}(\mathbf{g}_1) = J_{\min} + J_1(\mathbf{g}_1) + J_2(\mathbf{g}_1) + J_3(\mathbf{g}_1), \tag{20}$$

where

$$\begin{aligned} J_1(\mathbf{g}_1) &= \beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right],\\ J_2(\mathbf{g}_1) &= E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2 - 1\right)^2,\\ J_3(\mathbf{g}_1) &= 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2. \end{aligned} \tag{21}$$

When we compute the derivatives of $J_1(\mathbf{g}_1)$, $J_2(\mathbf{g}_1)$, and $J_3(\mathbf{g}_1)$ with respect to $g_{\ell 1}^*$, we find

$$\begin{aligned} \frac{\partial J_1(\mathbf{g}_1)}{\partial g_{\ell 1}^*} &= 2\beta g_{\ell 1}\left(\sum_{k=1}^{M}|g_{k1}|^2 - |g_{\ell 1}|^2\right) = 0\\ &\Longrightarrow \sum_{k=1,k\neq\ell}^{M}|g_{k1}|^2 = 0, \end{aligned} \tag{22}$$

$$\begin{aligned} \frac{\partial J_2(\mathbf{g}_1)}{\partial g_{\ell 1}^*} &= 2E[a_R^4]g_{\ell 1}\left(\sum_{k=1}^{M}|g_{k1}|^2 - 1\right) = 0\\ &\Longrightarrow \sum_{k=1}^{M}|g_{k1}|^2 = 1, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{\partial J_3(\mathbf{g}_1)}{\partial g_{\ell 1}^*} &= 2\beta\left(g_{R,\ell 1}g_{I,\ell 1}^2 + jg_{R,\ell 1}^2 g_{I,\ell 1}\right) = 0\\ &\Longrightarrow g_{R,\ell 1} = 0 \quad\text{or}\quad g_{I,\ell 1} = 0 \quad\text{or}\quad g_{R,\ell 1} = g_{I,\ell 1} = 0. \end{aligned} \tag{24}$$

Equation (22) implies that only one entry, $g_{\ell 1}$, of $\mathbf{g}_1$ is nonzero and the others are zeros. Equation (23) indicates that the modulus of this entry must be equal to one ($|g_{\ell 1}|^2 = 1$). Finally, from (24) either the real part or the imaginary part must be equal to zero. As a result of (23), the squared modulus of the nonzero part is equal to one, that is, either $g_{R,\ell 1}^2 = 1$ and $g_{I,\ell 1}^2 = 0$ or $g_{R,\ell 1}^2 = 0$ and $g_{I,\ell 1}^2 = 1$. Therefore, the solution $g_{\ell 1}$ is either a pure real or a pure imaginary with modulus equal to one, which corresponds to

$$g_{\ell 1} = e^{jm_1(\pi/2)}, \tag{25}$$

where m_1 is an arbitrary integer.

This solution shows that the minimization of $\mathcal{J}(\mathbf{g}_1)$ forces the equalizer output to form a constellation that corresponds to the source constellation with a modulo $\pi/2$ phase rotation.

From (22), (23), (24), and (25), we can conclude that the only stable minima for $\mathbf{g}_1$ take the following form: $\mathbf{g}_1 = [0,\ldots,0,e^{jm_1(\pi/2)},0,\ldots,0]^T$, that is, only one entry is nonzero, pure-real, or pure-imaginary with modulus equal to one, which can be at any of the M positions and all the other ones are zeros. This solution corresponds to the recovery of only one source signal and cancels the others. For the second equalizer $\mathbf{g}_2$, from (9), we have

$$\mathcal{J}(\mathbf{g}_2) = E\left[\left(z_{R,2}(n)^2 - R\right)^2\right] + \alpha\sigma_a^4\left|\mathbf{g}_1^H\mathbf{g}_2\right|^2, \tag{26}$$

this means that the adaptation of $\mathbf{g}_2$ depends on $\mathbf{g}_1$.

We examine the convergence of $\mathbf{g}_2$ once $\mathbf{g}_1$ has converged to one signal, because the adaptation of $\mathbf{g}_1$ is realized independently from the other $\mathbf{g}_i$. For the sake of simplicity, and without loss of generality, we consider that $\mathbf{g}_1$ has converged to the first signal, that is,

$$\mathbf{g}_1 = \left[de^{j\varphi} \mid 0,\ldots,0\right]^T, \quad d = 1,\ \varphi = m_1\frac{\pi}{2}, \tag{27}$$

then

$$\left|\mathbf{g}_1^H\mathbf{g}_2\right|^2 = d^2\left|g_{12}\right|^2. \tag{28}$$

Using this and the result in Appendix A, $\mathcal{J}(\mathbf{g}_2)$ in (26) can be expressed as follows:

$$\begin{aligned} \mathcal{J}(\mathbf{g}_2) = {}& E[a_R^4]\left(\sum_{k=1}^{M}|g_{k2}|^2 - 1\right)^2\\ &+ \beta\left[\left(\sum_{k=1}^{M}|g_{k2}|^2\right)^2 - \sum_{k=1}^{M}|g_{k2}|^4\right] + 2\beta\sum_{k=1}^{M}g_{R,k2}^2 g_{I,k2}^2\\ &- E[a_R^4] + R^2 + \alpha\sigma_a^4 d^2\left|g_{12}\right|^2. \end{aligned} \tag{29}$$

If we differentiate (29) directly, with respect to g_{12}^*, and then cancel the operation result, we get

$$\begin{aligned} \frac{\partial\mathcal{J}(\mathbf{g}_2)}{\partial g_{12}^*} = {}& 2E[a_R^4]g_{12}\left(\sum_{k=1}^{M}|g_{k2}|^2 - 1\right) + 2\beta g_{12}\sum_{k=2}^{M}|g_{k2}|^2\\ &+ 2\beta\left(g_{R,12}g_{I,12}^2 + jg_{R,12}^2 g_{I,12}\right) + \alpha\sigma_a^4 d^2 g_{12} = 0. \end{aligned} \tag{30}$$

By canceling both real and imaginary parts of (30), we have

$$\begin{aligned} g_{R,12} = 0 \quad&\text{or}\quad \chi + 2\beta g_{I,12}^2 = 2E[a_R^4] - \alpha d^2\sigma_a^4,\\ g_{I,12} = 0 \quad&\text{or}\quad \chi + 2\beta g_{R,12}^2 = 2E[a_R^4] - \alpha d^2\sigma_a^4, \end{aligned} \tag{31}$$

where $\chi = 2E[a_R^4]\sum_{k=1}^{M}|g_{k2}|^2 + 2\beta\sum_{k=2}^{M}|g_{k2}|^2 \geq 0$.

However, since $\chi + 2\beta g_{I,12}^2 \geq 0$ and $\chi + 2\beta g_{I,12}^2 \geq 0$, the theorem's condition $2E[a_R^4] - \alpha\sigma_a^4 d^2 \leq 0$ requires that $g_{R,12} = 0$ and $g_{I,12} = 0$, that is, $g_{12} = 0$.

Hence, $\mathbf{g}_2$ will take the form

$$\mathbf{g}_2 = \left[0 \mid \overline{\mathbf{g}}_2^T\right]^T, \tag{32}$$

which results in

$$\mathbf{g}_1^H\mathbf{g}_2 = 0. \tag{33}$$

Therefore, (26) is reduced to

$$\min_{\overline{\mathbf{g}}_2}\ \mathcal{J}(\overline{\mathbf{g}}_2) = E\left[\left(z_{R,2}^2(n) - R\right)^2\right], \tag{34}$$

where the second equalizer output $z_2 = \mathbf{g}_2^T\mathbf{a} = \overline{\mathbf{g}}_2^T\overline{\mathbf{a}}$, with $\overline{\mathbf{a}} = [a_2, \dots, a_M]^T$.

Equation (34) has the same form as (10). Hence the analysis is exactly the same as described previously. Consequently, the stationary points of (34) will take the form $\overline{\mathbf{g}}_2 = [0, \dots, 0, e^{jm_2(\pi/2)}, 0, \dots, 0]^T$, which corresponds to $\mathbf{g}_2 = [0 \mid 0, \dots, 0, e^{jm_2(\pi/2)}, 0, \dots, 0]^T$. Hence $\mathbf{g}_2$ will recover perfectly a different signal than the one already recovered by $\mathbf{g}_1$. Without loss of generality, again, we assume that the single nonzero element of $\mathbf{g}_2$ is in its second position, that is, $\mathbf{g}_2 = [0, e^{jm_2(\pi/2)}, 0, \dots, 0]^T$.

If we continue in the same manner for each $\mathbf{g}_i$, we can see that each $\mathbf{g}_i$ converges to a setting, in which zeros have the positions of the already recovered signals and its remaining entries contain only one nonzero element; this corresponds to the recovery of a different signal, and this process continues until all signals have been recovered.

On the basis of this analysis, we can conclude that the minimization of the suggested cost function in the case of QAM signals ensures a perfect recovery of all source signals and that the recovered signals correspond to the source signals with a possible permutation and a modulo $\pi/2$ phase rotation.

4.2. *P*-PSK case ($P \geq 8$)

On the basis of the results in Appendix B, we have

$$\mathcal{J}(\mathbf{g}_1) = E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2 - 1\right)^2 + \beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] - E[a_R^4] + R^2. \tag{35}$$

And from (16), $J_{\min} = -E[a_R^4] + R^2$ is a constant. If we cancel the derivatives of the first and second terms on the right side of (35), we obtain

$$\sum_{k=1}^{M}|g_{k1}|^2 = 1, \tag{36}$$

$$\sum_{k=1,k\neq\ell}^{M}|g_{k1}|^2 = 0. \tag{37}$$

Therefore, (36) and (37) dictate that the solution must take the form

$$\mathbf{g}_1 = [0, \dots, 0, e^{j\phi_\ell}, 0, \dots, 0]^T, \tag{38}$$

where $\phi_\ell \in [0, 2\pi]$ is an arbitrary phase in the ℓth position of $\mathbf{g}_1$ which can be at any of the M possible positions.

The solution $\mathbf{g}_1$ has only one nonzero entry with a modulus equal to one, and all the other ones are zeros. This solution corresponds to the recovery of only one source signal and cancels the other ones. With regard to the other vectors, the analysis is exactly the same as the one in the case of QAM signals.

Then, we can say that the minimization of the SCMA criterion, in the case of P-PSK ($P \geq 8$) signals, ensures the recovery of all signals except for an arbitrary phase rotation for each recovered signal.

4.3. 4-PSK case

On the basis of the results found in Appendix C, $J(\mathbf{g}_1)$ can be written as

$$\mathcal{J}(\mathbf{g}_1) = E^2[a_R^2]\left[3\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4 - 4\sum_{k=1}^{M}|g_{k1}|^2 - 4\sum_{k=1}^{M}g_{R,k1}^2 g_{I,k1}^2\right] + R^2. \tag{39}$$

In order to find the stationary points of (39), we cancel its derivative

$$\frac{\partial\mathcal{J}(\mathbf{g}_1)}{\partial g_{\ell 1}^*} = E^2[a_R^2]\left[6g_{\ell 1}\sum_{k=1}^{M}|g_{k1}|^2 - 2g_{\ell 1}|g_{\ell 1}|^2 - 4g_{\ell 1} - 4\left(g_{R,\ell 1}g_{I,\ell 1}^2 + jg_{R,\ell 1}^2 g_{I,\ell 1}\right)\right] = 0. \tag{40}$$

By canceling both real and imaginary parts of (40), we have

$$\begin{aligned} &3g_{R,\ell 1}\sum_{k=1}^{M}|g_{k1}|^2 - g_{R,\ell 1}|g_{\ell 1}|^2 - 2g_{R,\ell 1} - 2g_{R,\ell 1}g_{I,\ell 1}^2 = 0, \\ &3g_{I,\ell 1}\sum_{k=1}^{M}|g_{k1}|^2 - g_{I,\ell 1}|g_{\ell 1}|^2 - 2g_{I,\ell 1} - 2g_{I,\ell 1}g_{R,\ell 1}^2 = 0. \end{aligned} \tag{41}$$

According to (41),

$$g_{R,\ell 1} = g_{I,\ell 1}. \tag{42}$$

Then (41) can be reduced to

$$6\sum_{k=1,k\neq\ell}^{M}g_{R,k1}^2 + 2g_{R,\ell 1}^2 - 2 = 0. \tag{43}$$

Thus

$$g_{R,\ell 1}^2 = -3\sum_{k=1,k\neq\ell}^{M}g_{R,k1}^2 + 1. \tag{44}$$

Finally, we find

$$g_{R,\ell 1}^2 = -3(p-1)g_{R,\ell 1}^2 + 1, \tag{45}$$

where $1 \leq p \leq M$ is the number of nonzero elements in $\mathbf{g}_1$, which gives

$$g_{R,\ell 1}^2 = g_{I,\ell 1}^2 = \begin{cases} \dfrac{1}{(3p-2)}, & \text{if } \ell \in F_p, \\ 0, & \text{otherwise,} \end{cases} \quad \forall p = 1, \dots, M, \tag{46}$$

where F_p is any p-element subset of $\{1, \dots, M\}$.

Now, we study separately the stationary points for each value of p.

(i) $p = 1$: in this case, $\mathbf{g}_1$ has only one non zero entry, with

$$g_{R,\ell 1} = \pm 1, \qquad g_{I,\ell 1} = \pm 1, \tag{47}$$

that is,

$$g_{\ell 1} = c_\ell e^{j\phi_\ell}, \tag{48}$$

where $c_\ell = \sqrt{2}$ and $\phi_\ell = (2q+1)(\pi/4)$, with q is an arbitrary integer. Therefore, $\mathbf{g}_1 = [0, \dots, 0, c_\ell e^{j\phi_\ell}, 0, \dots, 0]^T$ is the global minimum.

(ii) $p \geq 2$: in this case, the solutions have at least two nonzero elements in some positions of $\mathbf{g}_1$. All nonzero elements have the same squared amplitude of $2/(3p-2)$.

Let us consider the following perturbation:

$$\mathbf{g}_1' = \mathbf{g}_1 + \mathbf{e}, \tag{49}$$

where $\mathbf{e} = [e_1, \dots, e_M]^T$ is an $(M \times 1)$ vector whose norm $\|\mathbf{e}\|^2 = \mathbf{e}^H\mathbf{e}$ can be made arbitrarily small and is chosen so that its nonzero elements are only in positions where the corresponding elements of $\mathbf{g}_1$ are nonzero:

$$e_\ell \neq 0 \Longleftrightarrow \ell \in F_p. \tag{50}$$

Let this perturbation be orthogonal with $\mathbf{g}_1$, that is, $\mathbf{e}^H\mathbf{g}_1 = 0$. Then, we have

$$\sum_{\ell \in F_p} |g_{\ell 1}'|^2 = \sum_{\ell \in F_p} |g_{\ell 1}|^2 + \sum_{\ell \in F_p} |e_\ell|^2. \tag{51}$$

We now define, as ε_ℓ, the difference between the squared magnitudes of $g_{\ell 1}'$ and $g_{\ell 1}$, that is,

$$|g_{\ell 1}'|^2 = |g_{\ell 1}|^2 + \varepsilon_\ell, \quad \varepsilon_\ell \in R,\ \varepsilon_\ell \neq 0 \Longleftrightarrow \ell \in F_p, \tag{52}$$

where

$$\sum_{\ell \in F_p} \varepsilon_\ell = \sum_{\ell \in F_p} |e_\ell|^2. \tag{53}$$

We assume that

$$(g_{R,\ell 1}')^2 = g_{R,\ell 1}^2 + \frac{\varepsilon_\ell}{2}, \qquad (g_{I,\ell 1}')^2 = g_{I,\ell 1}^2 + \frac{\varepsilon_\ell}{2}. \tag{54}$$

By evaluating $\mathcal{J}(\mathbf{g}_1')$, we find

$$\begin{aligned}
\mathcal{J}(\mathbf{g}_1') = E^2[a_R^2]\Bigg[&3\Big(\sum_{\ell \in F_p} |g_{\ell 1}|^2 + \varepsilon_\ell\Big)^2 - \sum_{\ell \in F_p}\Big(|g_{\ell 1}|^2 + \varepsilon_\ell\Big)^2 \\
&- 4\sum_{\ell \in F_p}\Big(|g_{\ell 1}|^2 + \varepsilon_\ell\Big) - 4\sum_{\ell \in F_p}\Big(g_{R,\ell 1}^2 + \frac{\varepsilon_\ell}{2}\Big) \\
&\times \Big(g_{I,\ell 1}^2 + \frac{\varepsilon_\ell}{2}\Big)\Bigg] + R^2, \\
\mathcal{J}(\mathbf{g}_1') = E^2[a_R^2]\Bigg[&3\Big(\sum_{\ell \in F_p} |g_{\ell 1}|^2\Big)^2 - \sum_{\ell \in F_p} |g_{\ell 1}|^4 \\
&- 4\sum_{\ell \in F_p} |g_{\ell 1}|^2 - 4\sum_{\ell \in F_p} g_{R,\ell 1}^2 g_{I,\ell 1}^2\Bigg] + R^2 \\
+ E^2[a_R^2]\Bigg[&6\sum_{\ell \in F_p} |g_{\ell 1}|^2 \sum_{\ell \in F_p} \varepsilon_\ell - 4\sum_{\ell \in F_p} |g_{\ell 1}|^2 \varepsilon_\ell \\
&- 4\sum_{\ell \in F_p} \varepsilon_\ell + 3\Big(\sum_{\ell \in F_p} \varepsilon_\ell\Big)^2 - 2\sum_{\ell \in F_p} \varepsilon_\ell^2\Bigg].
\end{aligned} \tag{55}$$

Using (46) in (55), and after some simplifications, we get

$$\mathcal{J}(\mathbf{g}_1') = \mathcal{J}(\mathbf{g}_1) + E^2[a_R^2]\Bigg[3\Big(\sum_{\ell \in F_p} \varepsilon_\ell\Big)^2 - 2\sum_{\ell \in F_p} \varepsilon_\ell^2\Bigg]. \tag{56}$$

It exists $\varepsilon_\ell \in \mathbb{R}, (\ell \in F_p)$ so that

$$3\Big(\sum_{\ell \in F_p} \varepsilon_\ell\Big)^2 - 2\sum_{\ell \in F_p} \varepsilon_\ell^2 < 0. \tag{57}$$

Then, $\exists \varepsilon_\ell \in \mathbb{R}, (\ell \in F_p)$ so that

$$\mathcal{J}(\mathbf{g}_1') < \mathcal{J}(\mathbf{g}_1). \tag{58}$$

Hence, $\mathcal{J}(\mathbf{g}_1)$ cannot be a local minimum.

Now we consider another perturbation which takes the form

$$g_{\ell 1}' = \begin{cases} \sqrt{1+\xi}\, g_{\ell 1} & \text{if } \ell \in F_p, \\ 0 & \text{otherwise,} \end{cases} \tag{59}$$

where ξ is a small positive constant.

By evaluating $\mathcal{J}(\mathbf{g}_1')$, we obtain

$$\begin{aligned}
\mathcal{J}(\mathbf{g}_1') = E^2[a_R^2]\Bigg[&3\Big(\sum_{\ell \in F_p} (1+\xi)|g_{\ell 1}|^2\Big)^2 \\
&- \sum_{\ell \in F_p} (1+\xi)^2 |g_{\ell 1}|^4 - 4\sum_{\ell \in F_p} (1+\xi)|g_{\ell 1}|^2 \\
&- 4\sum_{\ell \in F_p} (1+\xi)^2 g_{R,\ell 1}^2 g_{I,\ell 1}^2\Bigg] + R^2.
\end{aligned} \tag{60}$$

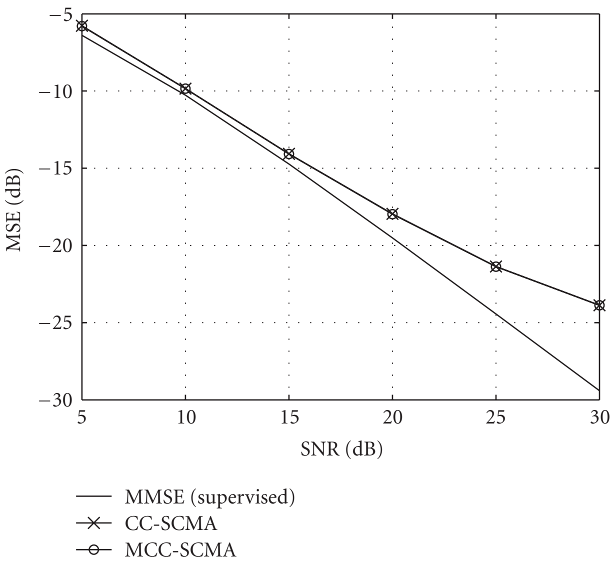

FIGURE 5: *MSE* versus *SNR* of CC-SCMA, MCC-SCMA, and supervised MMSE.

Using (36) and after some simplifications, we get

$$\mathcal{J}(\mathbf{g}_1') = \mathcal{J}(\mathbf{g}_1) + \frac{4\xi^2 p}{3p-2} E^2[a_R^2]. \quad (61)$$

Therefore, we always have

$$\mathcal{J}(\mathbf{g}_1') > \mathcal{J}(\mathbf{g}_1), \quad \forall p \in N^{+*}. \quad (62)$$

Hence, $\mathbf{g}_1$ cannot be a local maximum.

Then, on the basis of (58) and (62), $\mathbf{g}_1$ is a saddle point for $p \geq 2$.

Therefore, the only stable minima correspond to $p = 1$.

We conclude that the only stable minima take the form $\mathbf{g}_1 = [0, \ldots, 0, c_\ell e^{j\phi_\ell}, 0, \ldots, 0]^T$, which ensure the extraction of only one source signal and cancel the other ones. For the remainder of the analysis, we proceed exactly as we did for QAM signals.

Finally, in order to conclude this section, we can say that the minimization of the cost function in (6) ensures the recovery of all source signals in the case of source signals drawn from QAM or PSK constellations.

5. MODIFIED CROSS-CORRELATION TERM

In the previous section, we have seen that, in the case of QAM constellation, the signals are recovered with modulo $\pi/2$ phase rotation. By taking advantage of this result, we suggest to use, instead of cross-correlation term in (6), the following term:

$$\sum_{m=1}^{\ell-1} E^2[z_{R,\ell}(n) z_{R,m}(n)] + E^2[z_{R,\ell}(n) z_{I,m}(n)], \quad \ell = 1, \ldots, M. \quad (63)$$

Using (63) in (6), instead of the classical cross-correlation term, the criterion becomes

$$\mathcal{J}_\ell(n) = E\big[\big(z_{R,\ell}^2(n) - R\big)^2\big] + \alpha \sum_{m=1}^{\ell-1} \big(E^2[z_{R,\ell}(n) z_{R,m}(n)] + E^2[z_{R,\ell}(n) z_{I,m}(n)]\big), \quad \ell = 1, \ldots, M. \quad (64)$$

Please note that in (64) the multiplications in cross-correlation terms are not complexes, as opposed to (6) which reduces complexity.

The cost function in (64) is named modified cross-correlation SCMA (MCC-SCMA).

Remark 1. The new cross-correlation term could be also used by the MMA algorithm, because it recovers QAM signals with modulo $\pi/2$ phase rotation.

In the following section, the complexity of the modified cross-correlation term will be discussed and compared with the classical cross-correlation term.

6. IMPLEMENTATION AND COMPUTATIONAL COMPLEXITY

6.1. *Implementation*

In order to implement (6) and (64), we suggest to use the classical stochastic gradient algorithm (SGA) [25]. The general form of the SGA is given by

$$\mathbf{W}(n+1) = \mathbf{W}(n) - \frac{1}{2}\mu \nabla_{\mathbf{W}}(\mathcal{J}), \quad (65)$$

where $\nabla_{\mathbf{W}}(\mathcal{J})$ is the gradient of $\mathcal{J}$ with respect to $\mathbf{W}$.

6.1.1. *For CC-SCMA*

The equalizer update equation at the nth iteration is written as

$$\mathbf{w}_\ell(n+1) = \mathbf{w}_\ell(n) - \mu e_\ell(n) \mathbf{y}^*(n), \quad \ell = 1, \ldots, M, \quad (66)$$

where the constant which arises from the differentiation of (6) is absorbed within the step size μ. $e_\ell(n)$ is the instantaneous error $e_\ell(n)$ for the ℓth equalizer given by

$$e_\ell(n) = \big(z_{R,\ell}^2(n) - R\big) z_{R,\ell}(n) + \frac{\alpha}{2} \sum_{m=1}^{\ell-1} \hat{r}_{\ell m}(n) z_m(n), \quad (67)$$

where the scalar quantity $\hat{r}_{\ell m}$ represents the estimate of $r_{\ell m}$, it can be recursively computed as [25]

$$\hat{r}_{\ell m}(n+1) = \lambda \hat{r}_{\ell m}(n) + (1-\lambda) z_\ell(n) z_m^*(n), \quad (68)$$

where $\lambda \in [0, 1]$ is a parameter that controls the length of the effective data window in the estimation.

Please note that since $E[\hat{r}_{\ell m}(n)] = E[z_\ell(n) z_m^*(n)]$, then the estimator $\hat{r}_{\ell m}(n)$ is unbiased.

Table 1: Comparison of the algorithms complexity against weight update.

Algorithm	Multiplications	Additions
CC-CMA	$2M(3M+4N+1)-2$	$4M(M+2N-1)$
CC-SCMA	$6M(M+N)-2$	$M(4M+6N-5)+1$
MCC-SCMA	$4M(M+N)-1$	$2M(M+2N-1)$

6.1.2. For MCC-SCMA

We have exactly the same equation as (66), but the instantaneous error signal of the ℓth equalizer is given by

$$e_\ell(n) = (z_{R,\ell}^2(n) - R)z_{R,\ell}(n) + \frac{\alpha}{2}\sum_{m=1}^{\ell-1}[\hat{r}_{RR,\ell m}(n)z_{R,m}(n) + \hat{r}_{RI,\ell m}(n)z_{I,m}(n)], \quad (69)$$

where

$$\hat{r}_{RR,\ell m}(n+1) = \lambda\hat{r}_{RR,\ell m}(n) + (1-\lambda)z_{R,\ell}(n)z_{R,m}^*(n),$$
$$\hat{r}_{RI,\ell m}(n+1) = \lambda\hat{r}_{RI,\ell m}(n) + (1-\lambda)z_{R,\ell}(n)z_{I,m}^*(n). \quad (70)$$

6.2. Complexity

We consider the computational complexity of (66) for one iteration and for all equalizer outputs. With

(i) for CC-SCMA,

$$e_\ell(n) = (z_{R,\ell}(n)^2 - R)z_{R,\ell}(n) + \frac{\alpha}{2}\sum_{m=1}^{\ell-1}\hat{r}_{\ell m}(n)z_m(n); \quad (71)$$

(ii) for MCC-SCMA (modified cross-correlation SCMA),

$$e_\ell(n) = (z_{R,\ell}(n)^2 - R)z_{R,\ell}(n) + \frac{\alpha}{2}\sum_{m=1}^{\ell-1}[\hat{r}_{RR,\ell m}(n)z_{R,m}(n) + \hat{r}_{RI,\ell m}(n)z_{I,m}(n)]; \quad (72)$$

(iii) for CC-CMA (cross-correlation CMA)

$$e_\ell(n) = (|z_\ell(n)|^2 - R)z_\ell(n) + \frac{\alpha}{2}\sum_{m=1}^{\ell-1}\hat{r}_{\ell m}(n)z_m(n). \quad (73)$$

According to Table 1, the CC-SCMA presents a low complexity compared to that of CC-CMA. On a more interesting note, the results in Table 1 show that the use of modified cross-correlation term reduce significantly the complexity. Please note that the number of operations is per iteration.

7. NUMERICAL RESULTS

Some numerical results are now presented in order to confirm the theoretical analysis derived in the previously sections. For that purpose, we use the signal to interference and noise ratio (SINR) defined as

$$\text{SINR}_k = \frac{|g_{kk}|^2}{\sum_{\ell,\ell\neq k}|g_{\ell k}|^2 + \mathbf{w}_k^T\mathbf{R_b}\mathbf{w}_k^*},$$
$$\text{SINR} = \frac{1}{M}\sum_{k=1}^{M}\text{SINR}_k, \quad (74)$$

where SINR_k is the signal-to-interference and noise ratio at the kth output. $g_{ij} = \mathbf{h}_i^T\mathbf{w}_j$, where $\mathbf{w}_j$ and $\mathbf{h}_i$ are the jth and ith column vector of matrices $\mathbf{W}$ and $\mathbf{H}$, respectively. $\mathbf{R_b} = E[\mathbf{b}\mathbf{b}^H] = \sigma_b^2\mathbf{I}_N$ is the noise covariance matrix. The source signals are assumed to be of unit variance.

The SINR is estimated via the average of 1000 independent trials. Each estimation is based on the following model. The system inputs are independent, uniformly distributed and drawn from 16-QAM, 4-PSK, and 8-PSK constellations. We considered M transmit and N receive spatially decorrelated antennas. The channel matrix $\mathbf{H}$ is modeled by an $(N\times M)$ matrix with independent and identically distributed (i.i.d.), complex, zero-mean, Gaussian entries. We considered $\alpha = 1$ (this value satisfy the theorem condition) and $\lambda = 0.97$. The variance of noise is determined according to the desired Signal-to-Noise Ratio (SNR).

Figures 1, 2, and 3 show the constellations of the source signals, the received signals, and the receiver outputs (after convergence) using the proposed algorithm for 16-QAM, 8-PSK, and 4-PSK constellations, respectively. We have considered that SNR = 30 dB, $M = 2$, $N = 2$, and that $\mu = 5\times10^{-3}$. Please note that the constellations on Figures 1, 2, and 3 are given before the phase ambiguity is removed (this ambiguity can be solved easily by using differential decoding).

In Figure 1, we see that the algorithm recovers the 16-QAM source signals, but up to a modulo $\pi/2$ phase rotation which may be different for each output. Figure 2 shows that the 8-PSK signals are recovered with an arbitrary phase rotation. In Figure 3, the 4-PSK signals are recovered with a $(2k+1)(\pi/4)$ phase rotation and an amplitude of $\sqrt{2}$. Theses results are in accordance with the theoretical analysis given in Section 4.

In order to compare the performances of CC-SCMA and CC-CMA, the same implementation is considered for both algorithms (see Section 6). We have considered $M = 2$, $N = 3$, SNR = 25 dB, and the step sizes were chosen so that the algorithms have sensibly the same steady-state performances. We have also used the supervised least-mean square algorithm (LMS) as a reference.

Figure 4 represents the SINR performance plots for the proposed approach and the CC-CMA algorithm. We observe that the speed of convergence of the proposed approach is very close to that of the CC-CMA. Hence, it represents a good compromise between performance and complexity.

Figure 5 represents the mean-square error (MSE) versus SNR. In order to verify the effectiveness of the modified cross-correlation term, we have considered $M = 2$, $N = 3$, 16-QAM, and $\mu = 0.02$ for both algorithms. In this figure, the supervised minimum mean square (MMSE) receiver serves as reference. We observe that CC-SCMA and MCC-SCMA have almost the same behavior. So, in the case of QAM signals, it is preferable to use a modified cross-correlation term because of its low complexity and of its similar performances compared to the one of the classical cross-correlation term.

8. CONCLUSION

In this paper, we have proposed a new globally convergent algorithm for the multiple-input multiple-output (MIMO) adaptive blind separation of QAM and PSK signals. The criterion is based on one dimension (either real or imaginary) and consists in penalizing the deviation of the real (or the imaginary) part from a constant. It was demonstrated that the proposed approach is globally convergent to a setting that recovers perfectly, in the absence of noise, all the source signals. A modification for the cross-correlation constraint in the case of QAM constellation has been suggested. Our algorithm has shown a low computational complexity compared to that of CMA, especially when the modified cross-correlation constraint is used, which makes it attractive for implementation in practical applications. Simulation results have shown that the suggested algorithm has a good performance despite its lower complexity.

APPENDICES

A. QAM CASE

From (11),

$$\mathcal{J}(\mathbf{g}_1) = E[z_{R,1}^4] - 2RE[z_{R,1}^2] + R^2. \tag{A.1}$$

We have

$$z_1(n) = \mathbf{g}_1^T \mathbf{a}(n), \tag{A.2}$$

where

$$\begin{aligned} \mathbf{g}_1 &= [g_{11}, \dots, g_{M1}]^T, \quad g_{k1} = g_{R,k1} + jg_{I,k1}, \\ \mathbf{a} &= [a_1, \dots, a_M]^T, \quad a_k = a_{R,k} + ja_{I,k}. \end{aligned} \tag{A.3}$$

Then

$$z_{R,1}(n) = \sum_{k=1}^{M} (g_{R,k1}a_{R,k} - g_{I,k1}a_{I,k}). \tag{A.4}$$

For i.i.d. and mutually independent source signals that drawn from square QAM constellation, we have

$$\begin{aligned} E[a_R] &= E[a_I] = 0, \\ E[a_R^m] &= E[a_I^m], \quad \forall m, \\ E[a_{R,k}^m a_{I,\ell}^n] &= E[a_{R,k}^m]E[a_{I,\ell}^n], \quad \forall k, \ell, m, n, \\ E[a_{R,k}^m a_{R,\ell}^n] &= E[a_{I,k}^m a_{I,\ell}^n] = \begin{cases} E[a_R^{m+n}], & \text{if } k = \ell, \\ E[a_R^m]E[a_R^n], & \text{otherwise.} \end{cases} \end{aligned} \tag{A.5}$$

We have

$$\begin{aligned} E[z_{R,1}^2] &= E\left\{\left[\sum_{k=1}^{M} (g_{R,k1}a_{R,k} - g_{I,k1}a_{I,k})\right]^2\right\} \\ &= \sum_{k=1}^{M}\sum_{\ell=1}^{M} E[(g_{R,k1}a_{R,k} - g_{I,k1}a_{I,k})(g_{R,\ell 1}a_{R,\ell} - g_{I,\ell 1}a_{I,\ell})] \\ &= \sum_{k=1}^{M}\sum_{\ell=1}^{M} (g_{R,k1}g_{R,\ell 1}E[a_{R,k}a_{R,\ell}] - g_{R,k1}g_{I,\ell 1}E[a_{R,k}a_{I,\ell}] \\ &\quad - g_{I,k1}g_{R,\ell 1}E[a_{I,k}a_{R,\ell}]) + g_{I,k1}g_{I,\ell 1}E[a_{I,k}a_{I,\ell}]. \end{aligned} \tag{A.6}$$

Using (A.5), we obtain

$$E[z_{R,1}^2] = E[a_R^2]\sum_{k=1}^{M} |g_{k1}|^2. \tag{A.7}$$

Similarly

$$\begin{aligned} E[z_{R,1}^4] &= E\left\{\left[\sum_{k=1}^{M} (g_{R,k1}a_{R,k} - g_{I,k1}a_{I,k})\right]^4\right\} \\ &= \sum_{k=1}^{M}\sum_{\ell=1}^{M}\sum_{m=1}^{M}\sum_{n=1}^{M} E[(g_{R,k1}a_{R,k} - g_{I,k1}a_{I,k}) \\ &\quad \times (g_{R,\ell 1}a_{R,\ell} - g_{I,\ell 1}a_{I,\ell}) \\ &\quad \times (g_{R,m1}a_{R,m} - g_{I,m1}a_{I,m}) \\ &\quad \times (g_{R,n1}a_{R,n} - g_{I,n1}a_{I,n})]. \end{aligned} \tag{A.8}$$

Developing (A.8) and using (A.5), we have the following three cases:

(i) $k = \ell = m = n$:

$$E[z_{R,1}^4] = E[a_R^4]\sum_{k=1}^{M} (g_{R,k1}^4 + g_{I,k1}^4) + 6E^2[a_R^2]\sum_{k=1}^{M} (g_{R,k1}^2 g_{I,k1}^2); \tag{A.9}$$

(ii) $k = \ell \neq m = n$:

$$E[z_{R,1}^4] = 3E^2[a_R^2]\sum_{k=1}^{M}\sum_{\ell=1,\ell\neq k}^{M} |g_{k1}|^2 |g_{\ell 1}|^2; \tag{A.10}$$

(iii) otherwise: (A.8) is equal to zero.

From (A.9) and (A.10), (A.8) becomes

$$\begin{aligned} E[z_{R,1}^4] &= E[a_R^4]\sum_{k=1}^{M} (g_{R,k1}^4 + g_{I,k1}^4) + 6E^2[a_R^2]\sum_{k=1}^{M} (g_{R,k1}^2 g_{I,k1}^2) \\ &\quad + 3E^2[a_R^2]\sum_{k=1}^{M}\sum_{\ell=1,\ell\neq k}^{M} |g_{k1}|^2 |g_{\ell 1}|^2. \end{aligned} \tag{A.11}$$

Then

$$E[z_{R,1}^4] = E[a_R^4]\sum_{k=1}^{M}|g_{k1}|^4 - 2E[a_R^4]\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2 + 6E^2[a_R^2]\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2 + 3E^2[a_R^2]\sum_{k=1}^{M}\sum_{\ell=1,\ell\neq k}^{M}|g_{k1}|^2|g_{\ell 1}|^2. \tag{A.12}$$

On the other hand, we have

$$\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] = \sum_{k=1}^{M}\sum_{\ell=1,\ell\neq k}^{M}|g_{k1}|^2|g_{\ell 1}|^2. \tag{A.13}$$

Substituting (A.13) into (A.12), we get

$$E[z_{R,1}^4] = E[a_R^4]\sum_{k=1}^{M}|g_{k1}|^4 + 3E^2[a_R^2]\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] + 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2, \tag{A.14}$$

where $\beta = 3E^2[a_R^2] - E[a_R^4]$.

Using (A.6) and (A.14) in (A.1), we obtain

$$\begin{aligned}\mathcal{J}(\mathbf{g}_1) &= E[a_R^4]\sum_{k=1}^{M}|g_{k1}|^4 + 3E^2[a_R^2]\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] \\ &\quad + 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2 - 2E[a_R^2]\sum_{k=1}^{M}|g_{k1}|^2 + R^2 \\ &= E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 \\ &\quad + E[a_R^4]\sum_{k=1}^{M}|g_{k1}|^4 + 3E^2[a_R^2]\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] \\ &\quad + 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2 - 2E[a_R^2]\sum_{k=1}^{M}|g_{k1}|^2 + R^2 + E[a_R^4] - E[a_R^4].\end{aligned} \tag{A.15}$$

Rearranging terms, we get

$$\mathcal{J}(\mathbf{g}_1) = E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2 - 1\right)^2 + \beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] + 2\beta\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2 - E[a_R^4] + R^2. \tag{A.16}$$

Finally, we get (12).

B. P-PSK CASE ($P \geq 8$)

In the case of P-PSK ($P \geq 8$), (A.5) hold and moreover we have

$$E[a_{R,k}^2 a_{I,\ell}^2] = \begin{cases}\dfrac{1}{3}E[a_R^4] & \text{if } k = \ell, \\ E^2[a_R^2] & \text{otherwise.}\end{cases} \tag{B.17}$$

Using (A.5) in (A.6),

$$E[z_{R,1}^2] = E[a_R^2]\sum_{k=1}^{M}|g_{k1}|^2. \tag{B.18}$$

Using (A.5), and (B.17) in (A.8), we find

(i) $k = \ell = m = n$:

$$E[z_{R,1}^4] = E[a_R^4]\sum_{k=1}^{M}|g_{k1}|^4; \tag{B.19}$$

(ii) $k = \ell \neq m = n$:

$$E[z_{R,1}^4] = 3E^2[a_R^2]\sum_{k=1}^{M}\sum_{\ell=1,\ell\neq k}^{M}|g_{k1}|^2|g_{\ell 1}|^2; \tag{B.20}$$

(iii) otherwise: (A.8) is equal to zero.
Then

$$E[z_{R,1}^4] = E[a_R^4]\sum_{k=1}^{M}(g_{R,k1}^4 + g_{I,k1}^4) + 3E^2[a_R^2]\sum_{k=1}^{M}\sum_{\ell=1,\ell\neq k}^{M}|g_{k1}|^2|g_{\ell 1}|^2. \tag{B.21}$$

By developing the above equation as in the QAM case, we get

$$\mathcal{J}(\mathbf{g}_1) = E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2 - 1\right)^2 + \beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4\right] - E[a_R^4] + R^2. \tag{B.22}$$

C. 4-PSK CASE

For 4-PSK signals,

$$E[a_{R,k}^2 a_{I,\ell}^2] = \begin{cases}0 & \text{if } k = \ell, \\ E^2[a_R^2] & \text{otherwise.}\end{cases} \tag{C.23}$$

Considering (A.5), and (C.23), and proceeding in the same way as in the case of P-PSK signals ($P \geq 8$), we can easily find

$$\mathcal{J}(\mathbf{g}_1) = E^2[a_R^2]\left[3\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2 - \sum_{k=1}^{M}|g_{k1}|^4 - 4\sum_{k=1}^{M}|g_{k1}|^2 - 4\sum_{k=1}^{M} g_{R,k1}^2 g_{I,k1}^2\right] + R^2. \tag{C.24}$$

ACKNOWLEDGMENT

The authors are grateful to the anonymous referees for their constructive critics and valuable comments.

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