




A special Higgs challenge: measuring the mass and production cross section with ultimate precision at FCC-ee

Paolo Azzurri¹, Gregorio Bernardi², Sylvie Braibant³, David d'Enterria⁴, Jan Eysermans⁵, Patrick Janot^{4,a} , Ang Li², Emmanuel Perez⁴

¹ INFN, Sezione di Pisa, Pisa, Italy

² CNRS/IN2P3, APC, Paris, France

³ INFN, Sezione di Bologna, Bologna University, Bologna, Italy

⁴ EP Department, CERN, Geneva, Switzerland

⁵ MIT, Cambridge, MA, USA

Received: 21 June 2021 / Accepted: 21 November 2021

© The Author(s) 2021, corrected publication 2022

Abstract The FCC-ee offers powerful opportunities to determine the Higgs boson parameters, exploiting over 10^6 $e^+e^- \rightarrow ZH$ events and almost 10^5 $WW \rightarrow H$ events at centre-of-mass energies around 240 and 365 GeV. This essay spotlights the important measurements of the ZH production cross section and of the Higgs boson mass. The measurement of the total ZH cross section is an essential input to the absolute determination of the HZZ coupling—a “standard candle” that can be used by all other measurements, including those made at hadron colliders—at the per-mil level. A combination of the measured cross sections at the two different centre-of-mass energies further provides the first evidence for the trilinear Higgs self-coupling, and possibly its first observation if the cross section measurement can be made accurate enough. The determination of the Higgs boson mass with a precision significantly better than the Higgs boson width (4.1 MeV in the standard model) is a prerequisite to either constrain or measure the electron Yukawa coupling via direct $e^+e^- \rightarrow H$ production at $\sqrt{s} = 125$ GeV. Approaching the statistical limit of 0.1% and $\mathcal{O}(1)$ MeV on the ZH cross section and the Higgs boson mass, respectively, sets highly demanding requirements on accelerator operation (ZH threshold scan, centre-of-mass energy measurement), detector design (lepton momentum resolution, hadronic final state reconstruction performance), theoretical calculations, and analysis techniques (efficiency and purity optimization with modern tools, constrained kinematic fits, control of systematic uncertainties). These challenges are examined in turn in this essay

1 Introduction: state of the art

In e^+e^- collisions at centre-of-mass energies (\sqrt{s}) from 200 to 400 GeV, the two main Higgs production mechanisms are the Higgsstrahlung process, $e^+e^- \rightarrow ZH$, and the WW fusion process, $e^+e^- \rightarrow H\nu_e\bar{\nu}_e$, with Feynman diagrams and cross sections shown in Fig. 1. In the baseline run plan [1] and with two interaction points (IPs), over one million ZH events

^ae-mail: patrick.janot@cern.ch (corresponding author)

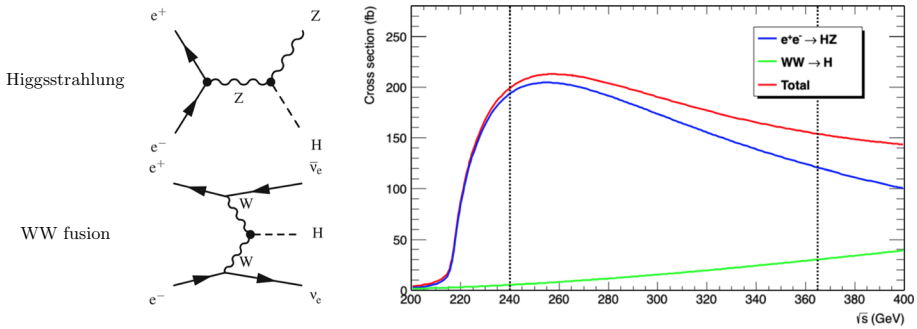


Fig. 1 (Left) Feynman diagrams for the Higgsstrahlung (top) and the WW fusion (bottom) processes. (Right) Improved-Born Higgs production cross sections (with initial state radiation included [2]), as predicted by HZHA [3] as a function of the centre-of-mass energy for $m_H = 125$ GeV. The small interference term between the two diagrams in the $H\nu_e\bar{\nu}_e$ final state is included in the WW fusion cross section. Vertical dashed lines indicate the \sqrt{s} values foreseen at FCC-ee

and almost 100,000 $WW \rightarrow H$ events Higgs bosons will be produced at FCC-ee with \sqrt{s} between 240 and 365 GeV. The design of a layout with four IPs is under study, and would increase these figures by a factor 1.7 in the same running time.

The total ZH cross section σ_{ZH} can thus be determined in principle with an ultimate statistical precision of 0.1%, should the ZH event selection be 100% efficient and pure. In addition, the selection can be made independent of the Higgs boson detailed properties by counting events with an identified Z boson, and for which the mass recoiling against the Z clusters around the Higgs boson mass (Sect. 2). An absolute measurement of g_{HZZ} , unique to e^+e^- colliders, is therefore at hand with a statistical precision of half a per mil at FCC-ee. The position of the recoil mass peak also provides in turn an accurate measurement of the Higgs boson mass. Once g_{HZZ} has been determined, the measurement of the cross sections for each exclusive Higgs boson decay, $H \rightarrow X\bar{X}$,

$$\sigma_{ZH} \times \mathcal{B}(H \rightarrow X\bar{X}) \propto \frac{g_{HZZ}^2 \times g_{HXX}^2}{\Gamma_H} \quad \text{and} \quad \sigma_{H\nu_e\bar{\nu}_e} \times \mathcal{B}(H \rightarrow X\bar{X}) \propto \frac{g_{HWW}^2 \times g_{HXX}^2}{\Gamma_H}, \quad (1)$$

gives access to all other couplings in a model-independent, absolute, way. For example, the ratio of the WW-fusion-to-Higgsstrahlung cross sections for the same Higgs boson decay, proportional to g_{HWW}^2/g_{HZZ}^2 , yields g_{HWW} , and the Higgsstrahlung rate with the $H \rightarrow ZZ$ decay, proportional to g_{HZZ}^4/Γ_H , provides a determination of the Higgs boson total decay width Γ_H . The measurement of g_{HZZ} , and thus of the total ZH cross section, is a cornerstone of the Higgs physics programme at FCC-ee. Conservative values for the statistical precision on inclusive and exclusive ZH cross sections, obtained from preliminary FCC-ee conceptual studies with realistic beam and detector parameters [4], are indicated in Table 1, and the resulting accuracy of Higgs couplings obtained from global fits to the FCC-ee measurements (the details of which are explained in Ref. [5]), are listed in Table 2.

The precise measurement of the ZH cross section can also give access to the Higgs boson self-coupling g_{HHH} via loop diagrams (shown in the left panel of Fig. 2) as was realized for the first time in Ref. [6]. Indeed, the contribution of these diagrams to the ZH cross section amounts to $\sim 2\%$ at 240 GeV and $\sim 0.5\%$ at 365 GeV [7], similar to or significantly larger than the experimental precision expected at FCC-ee. The dependence of the ZH cross section on the centre-of-mass energy allows in addition the g_{HZZ} and g_{HHH} couplings to be determined separately in a robust and model-independent manner [7,8], with a

Table 1 From Ref. [4]: Relative uncertainty (in %) on $\sigma_{ZH} \times \mathcal{B}(H \rightarrow \overline{X}X)$ and $\sigma_{\nu_e \bar{\nu}_e H} \times \mathcal{B}(H \rightarrow \overline{X}X)$, as expected from the FCC-ee data at 240 and 365 GeV

\sqrt{s}	240 GeV		365 GeV	
Integrated luminosity	5 ab ⁻¹		1.5 ab ⁻¹	
Channel	ZH	$\nu_e \bar{\nu}_e H$	ZH	$\nu_e \bar{\nu}_e H$
H → any	±0.5		±0.9	
H → b \bar{b}	±0.3	±3.1	±0.5	±0.9
H → c \bar{c}	±2.2		±6.5	±10
H → gg	±1.9		±3.5	±4.5
H → W ⁺ W ⁻	±1.2		±2.6	±3.0
H → ZZ	±4.4		±12	±10
H → $\tau^+ \tau^-$	±0.9		±1.8	±8
H → $\gamma\gamma$	±9.0		±18	±22
H → $\mu^+ \mu^-$	±19		±40	
H → invisible	< 0.3		< 0.6	

Table 2 From Ref. [5]: Precision on a few Higgs couplings g_{HXX} and on the total width Γ_H at FCC-ee, in the κ framework and in a global Effective Field Theory fit

Coupling	Precision (%) (κ framework / EFT)
g_{HZZ}	0.17 / 0.26
g_{HWW}	0.41 / 0.27
g_{Hbb}	0.64 / 0.56
g_{Hcc}	1.3 / 1.2
g_{Hgg}	0.89 / 0.82
$g_{H\tau\tau}$	0.66 / 0.57
$g_{H\mu\mu}$	3.9 / 3.8
$g_{H\gamma\gamma}$	1.3 / 1.2
$g_{HZ\gamma}$	10. / 9.3
g_{Htt}	3.1 / 3.1
Γ_H	1.1

relative precision illustrated in the right panel of Fig. 2. The corresponding contribution to the WW fusion cross section is almost independent of \sqrt{s} [8, 9], which offers a complementary constraint on g_{HHH} . No meaningful constraint can be obtained with one single centre-of-mass energy. Altogether, with the σ_{ZH} accuracy given in Table 1, a precision of ±33% can be achieved at FCC-ee on the Higgs self-coupling, reduced to ±24% with four interaction points (and therefore, four detectors) instead of two [7, 10]. The first 3-to-4 σ evidence of the existence of the Higgs self-coupling is therefore within reach in 15 years at FCC-ee, a unique feature among all low-energy e⁺e⁻ EW and Higgs factories currently contemplated.

It is important to note here that the σ_{ZH} accuracy of 0.5% given in Table 1 for $\sqrt{s} = 240$ GeV is significantly worse than the ultimate statistical precision of 0.1% (0.2%) that could be naively hoped for with one million (200,000) ZH events at $\sqrt{s} = 240$ GeV (365 GeV).

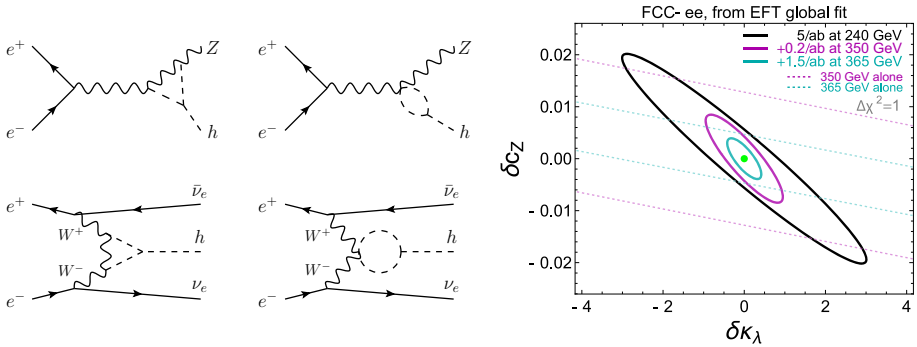


Fig. 2 From Ref. [8]. Left: Sample next-to-leading-order Feynman diagrams for single Higgs production involving the Higgs self-coupling. Right: Relative precision in the simultaneous determination of the Higgs self-coupling (here denoted κ_λ) and the HZZ/HWW coupling (here denoted c_Z) at FCC-ee, with 240 GeV (black ellipse), 350 GeV (purple dashed), and 365 GeV (green dashed) data, and by combining data at 240 and 350 GeV (purple ellipse), and at 240, 350, and 365 GeV (green ellipse)

The opportunities and challenges to improve the experimental accuracy are examined in the following sections. The physics motivations to do so are numerous. First, the measurement of the ZH cross section provides an absolute determination of g_{HZZ} , which in turn fixes all the other Higgs boson couplings (and its width), be they measured at FCC-ee (Eq. 1) or at a hadron collider (LHC, FCC-hh) where only coupling ratios can be inferred without theoretical assumptions. A more accurate σ_{ZH} measurement therefore reduces the corresponding parametric uncertainty on all Higgs boson properties. Second, the quantum corrections to Higgs couplings are at the level of a few % in the standard model (SM). The quantum nature of the Higgs boson can therefore only be tested if the measurement of its properties is pushed well below this level of precision, to a few per mil or better. For example, an improvement by a factor two of the ZH cross section measurement would turn automatically to a twice better Higgs self-coupling determination, and would enable the first discovery of this long-sought coupling. Lastly, interactions between the Higgs boson and other new particles at a higher energy scale Λ typically modify the Higgs boson couplings to SM particles by less than 5% for $\Lambda = 1$ TeV, with a $1/\Lambda^2$ dependence. A sub-per-mil accuracy on a given coupling measurement, e.g., g_{HZZ} , would be needed to access the $\Lambda = 10$ TeV energy scale. An analysis of the deviation pattern among all couplings would shed light on the underlying new physics.

The measurement of the Higgs boson mass, m_H , with uncertainties much below 0.1% has not been considered a priority so far in the current experimental and theoretical landscape: on the one hand, SM precision electroweak observables depend, via radiative corrections, only logarithmically on this quantity; and on the other hand, the current LHC experimental precision on Higgs coupling and width measurements is not particularly demanding in this respect for a sound theoretical interpretation. At FCC-ee, however, a measurement of m_H with the current ± 125 MeV precision would translate into a parametric uncertainty of, e.g., 0.7% on $\sigma_{ZH} \times \text{BR}(H \rightarrow b\bar{b})$, which is more than twice as large as the expected statistical precision on this observable (Table 1). It is estimated [5] that a 0.01% precision on m_H , i.e., $\mathcal{O}(10)$ MeV, would be enough to predict Higgs boson absolute production cross sections and decay branching fractions with an accuracy sufficiently smaller than their corresponding expected statistical precision. This requirement is almost met by the precision of 20 MeV ultimately reachable at HL-LHC [11]. One notable exception is the determination of the

electron Yukawa coupling via resonant $e^+e^- \rightarrow H$ production at $\sqrt{s} = 125$ GeV, for which a precision smaller than the SM Higgs total width of 4.1 MeV is required [12].

The opportunities and challenges to achieve the relevant precisions on the Higgs boson mass and production cross section at FCC-ee are now examined.

2 Opportunities and challenges: The “recoil mass” method

The precise determination of the Higgs boson coupling to the Z boson and of the Higgs boson mass at an e^+e^- Higgs factory is initially optimized as follows.

1. The centre-of-mass energy is chosen so as to maximize the number of ZH events. At FCC-ee, the luminosity steeply increases as the centre-of-mass energy decreases, so that the centre-of-mass energy was fixed to 240 GeV, approximately 15 GeV below the value that maximizes the theoretical ZH cross section [13].
2. In an initial approach, only the leptonic decays of the Z boson ($Z \rightarrow \ell^+\ell^-$, with $\ell = e$ or μ) are used for the cross section measurement, as they allow the ZH events to be inclusively and efficiently selected independently of the Higgs boson decay mode. This choice is therefore effective towards an almost fully model-independent determination of the HZZ coupling, but the small Z dielectron and dimuon branching ratios are expensive in terms of statistical precision (Table 1).
3. The mass m_{recoil} recoiling against the lepton pair is determined from total energy-momentum conservation as $m_{\text{recoil}}^2 = s + m_{\ell\ell}^2 - 2\sqrt{s}(E_{\ell^+} + E_{\ell^-})$, where $m_{\ell\ell}$ is the lepton pair invariant mass, and E_{ℓ^+} , E_{ℓ^-} are the two lepton energies. In absence of initial state radiation and beam-energy spread, and with a perfect determination of the lepton pair kinematics, m_{recoil} coincides exactly with the Higgs boson mass. In practice, the Higgs boson mass and the ZH total cross section are fitted from the actual experimental m_{recoil} distribution.

Candidate ZH events where the Z boson decays to $\mu^+\mu^-$ are selected by identifying two muons with an invariant mass close to m_Z and a total momentum transverse to the beam axis typically between 15 and 70 GeV, while using as little information as possible from the rest of the event. The resulting m_{recoil} distribution, obtained with a DELPHES simulation [14] of the IDEA detector concept [4], in particular its drift chamber [15], is displayed in the left panel of Fig. 3 for an integrated luminosity of 5 ab^{-1} simulated at $\sqrt{s} = 240$ GeV and with a nominal Higgs boson mass of $m_H = 125$ GeV. The background processes include the dominant diboson production $e^+e^- \rightarrow WW$ and ZZ (where “Z” can be a Z or a virtual photon), the single boson production $e^+e^- \rightarrow Ze^+e^-$, as well as the (radiative) dilepton events $e^+e^- \rightarrow (\gamma)\ell^+\ell^-$. The dilepton and diboson background processes were simulated with Pythia [16], while WHIZARD was used for the other background processes and the signal [17].

In the right panel of Fig. 3, the recoil mass distribution is fitted around m_H with a double-sided Crystal Ball function for the signal and a 2nd-order polynomial for the background. To minimize the biases and the need for a-posteriori corrections arising from this choice of specific functional forms, the Higgs boson mass and the ZH cross section can also be adjusted from template distributions obtained from simulation, and calibrated with control processes with data.

In both cases, and even in this first simple approach, the large signal-to-background ratio on the one hand, and the excellent drift-chamber muon momentum resolution on the other, offer the possibility to determine the inclusive ZH cross section and the Higgs boson mass with

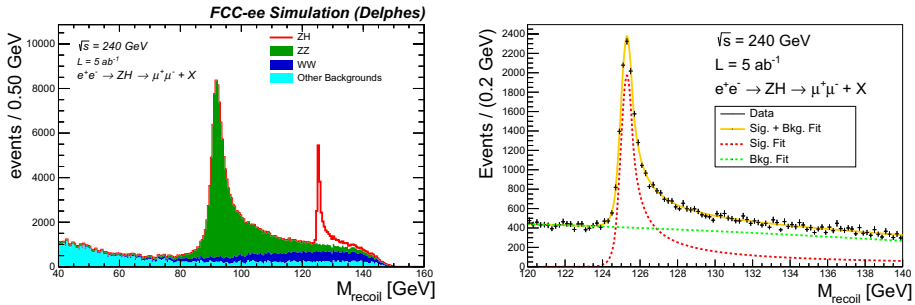


Fig. 3 Left: Inclusive m_{recoil} distribution for events with a Z decaying to $\mu^+\mu^-$, between 40 and 160 GeV displaying the Z peak from the ZZ background and the H peak from the ZH signal. Right: Expanded scale showing the m_{recoil} distribution in the region around m_H . The ZH signal is fitted to a double-sided Crystal Ball function [18, 19], and the simulated background to a second-order polynomial

a statistical precision of $\sim 1\%$ and ~ 6 MeV, respectively. The muon momentum resolution achieved with the CLD Silicon tracker [20] is affected by its larger amount of material, and therefore of multiple scattering, than in the IDEA drift chamber. The effect on the ZH cross section determination is marginal (given the large signal-to-background ratio), but it directly translates to a degraded statistical precision of 9 MeV on the Higgs boson mass. This observation will need to be included in the requirements on the detector design, if a precision of $\mathcal{O}(1)$ MeV is to be achieved on m_H .

Experimental systematic effects are also expected to be relatively larger for the mass than for the cross section, when compared to the corresponding statistical precision—a fraction of a per cent for σ_{ZH} and a few 10^{-5} for m_H . Methods to tackle and calibrate these effects will therefore need to be carefully designed. First and foremost, the centre-of-mass energy (and its spread)—which enters directly in the calculation of the recoil mass—must be determined with a similar or better accuracy. The requirements on the detector design to achieve such a precision on \sqrt{s} , regarding in particular the lepton and jet angular resolution, as well as systematic detector acceptance and possible hadronic effects, can be studied with a consolidated analysis of the $e^+e^- \rightarrow Z(\gamma)$ process ($Z \rightarrow \ell^+\ell^-, q\bar{q}$) at $\sqrt{s} = 240$ GeV, as proposed in Ref. [21], with realistic FCC-ee collision parameters. The feasibility of a calibration of the method, to reduce systematic uncertainties of various origins, with $e^+e^- \rightarrow Z(\gamma)$ events recorded at the WW threshold—where the centre-of-mass energy can be determined with resonant depolarization with a few 100 keV accuracy as well—will need to be ascertained. The centre-of-mass energy spread can be inferred and monitored with an analysis of dimuon events as explained in Ref. [21]. The absolute muon momentum scale—and its stability—is the second essential input to the determination of m_H . The need of calibration data around the Z pole ($\sqrt{s} \simeq 91.2$ GeV), recorded with a regular frequency, has to be estimated in this respect, complemented by the (radiative) dimuon final state and the $e^+e^- \rightarrow ZZ \rightarrow \ell^+\ell^-X$ process at $\sqrt{s} = 240$ GeV. The latter can also be exploited to check the shape of the ZZ background and tune the Monte Carlo generators accordingly.

Several avenues should be explored to improve the m_H precision to the desired level. The possibility to increase the experiment magnetic field from 2 to 3 T, which directly improves the momentum resolution by 30%, will be evaluated. This study includes checking that a reasonable luminosity can be preserved in this configuration for the Z calibration data, in spite of a significant beam emittance blow-up at the interaction point. The $Z \rightarrow e^+e^-$ decays might boost the precision to almost 4 MeV with the IDEA drift chamber, but additional work

is in order in this channel with a heavier tracker, for which a dedicated Bremsstrahlung photon recovery will be needed to preserve the recoil mass resolution. Experience with a full simulation of the CMS detector [22], with a much heavier tracker than the CLD concept, showed that a performance with electrons similar to that with muons can ultimately be achieved. As also demonstrated in Ref. [22], most of the exclusive final states, with specific Z decays ($\ell^+\ell^-$, $\tau^+\tau^-$, $\nu\bar{\nu}$, $q\bar{q}$) and H decays ($b\bar{b}$, $\tau^+\tau^-$, $\gamma\gamma$, $\mu^+\mu^-$) provide an estimate of the Higgs boson mass on an event by event basis, either from the total energy and momentum conservation constraints, or from the direct mass resolution in the case of the $H \rightarrow \gamma\gamma$ and $\mu^+\mu^-$ decays. With the CMS detector, a 30% statistical improvement on the mass precision was shown to be possible by combining these channels to the $Z \rightarrow \ell^+\ell^-$ decays. Taken at face value, this might bring the precision down to 2.5 MeV with the baseline run plan, and maybe 2 MeV with a larger magnetic field. The description of the challenges, systematic studies, and detector requirements associated to these multiple final states is beyond the scope of this short essay, and will be part of the scientific outcome of the forthcoming FCC feasibility study. The comprehensive work and intellectual input required to bring these studies to completion will be instrumental for the training of young physicists (the future leaders of the field when FCC-ee is in operation mode) to e^+e^- collider physics and techniques.

To improve the precision of the ZH cross section measurement, the inclusion of the Z decays to e^+e^- is the first obvious step. A first challenge will then be to optimize the selection towards the best precision on σ_{ZH} , or alternatively, on the Higgs self-coupling, and perform the fit parameter extraction with cutting-edge analysis methods, in order to approach the previously reported and typically 30% more precise projections [23] used in Table 1. This optimization includes the choice of the centre-of-mass energy, which affects the total number of events [13], the lepton momenta and their resolution, and the relative enhancement of the ZH cross section from the Higgs self-coupling (Fig. 4), all favouring a \sqrt{s} value slightly below 240 GeV. The inclusion of the hadronic Z decay in the recoil mass method is even tougher a challenge. On the bright side, the Z hadronic branching fraction is about ten times larger than its dielectron and dimuon counterpart, which increases significantly the statistical power of the method. On the other hand, the clean identification of a $Z \rightarrow q\bar{q}$ decay independently of the Higgs boson decay—necessary for a model-independent measurement of the cross section—is complicated by the ambiguities in jet clustering algorithms, the multiple possibilities to associate jets to the Z or the Higgs decays (especially when the Higgs boson decays hadronically too, which happens most often), the selection efficiency dependence on the Higgs boson decay channel, and the hadronic mass intrinsic detector resolution, which is usually significantly worse than lepton momentum resolution. An excellent hadronic mass resolution, however, would significantly reduce the other ambiguities: this observation will define the performance of the whole detector in terms of particle-flow reconstruction. (The value of the experiment magnetic field is relevant here as well.) Several attempts have been made in the past to apply the recoil mass method to hadronic Z decays, the most thorough of which can be found in Ref. [24]. A fresh and systematic look is now in order, to bring in new ideas, to test cutting-edge analyses methods, and to develop innovative detector concepts, motivated by the first potential discovery of the Higgs self-coupling at FCC-ee.

3 Opportunities and challenges: scan of the ZH threshold

In analogy to the W boson mass and width determination from the measurement of the W-pair threshold cross section lineshape [25–27], the cross section lineshape for the $e^+e^- \rightarrow ZH$

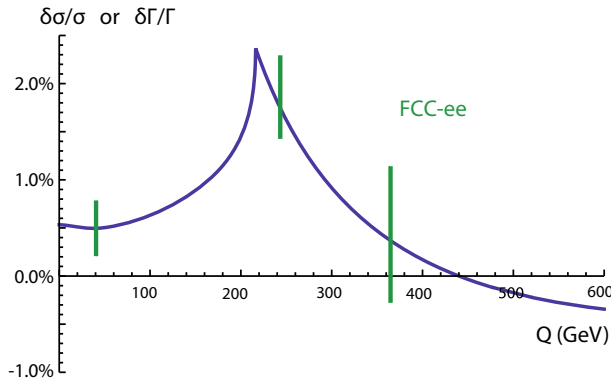


Fig. 4 Adapted from Fig. 9.11 of Ref. [7]: Relative change in the SM $e^+e^- \rightarrow Z^* \rightarrow ZH$ Born cross section, or in the $H \rightarrow WW^*$ partial width, as a function of Q (blue curve), where Q^2 is the four-momentum squared of the off-shell vector boson, Z^* or W^* . The change is caused by the one-loop diagrams involving the triple Higgs coupling shown in Fig. 2. The quantity Q equals $\sqrt{s} = 240$ or 365 GeV for the ZH production cross section. For the $H \rightarrow WW^*$ decay width, Q is set to its maximum value $m_H - m_W \simeq 40$ GeV. The vertical lines show the uncertainties expected from FCC-ee single measurements of these quantities. The relative enhancement is largest at the $e^+e^- \rightarrow ZH$ production threshold, $\sqrt{s} \simeq 216$ GeV

process at the production threshold ($\sqrt{s} \approx m_H + m_Z \simeq 216$ GeV) can be exploited to determine the Higgs boson mass (and width). This collision energy point is currently not foreseen in the baseline FCC-ee operation model [4], but no stone should be left unturned. A side motivation for examining this new opportunity is illustrated in Fig. 4: the relative enhancement of the ZH cross section due to the Higgs self-coupling, via the loop diagrams shown in Fig. 2, is maximal at the ZH production threshold and could be best constrained at this energy. For this threshold measurement to be useful, the Z boson mass and width need to be known with a precision significantly better than the target precision on the Higgs boson mass. The FCC-ee run at the Z pole, with its superior control of the centre-of-mass energy [21,28], satisfies such a requirement. The accuracy of the centre-of-mass energy at the ZH threshold (with $e^+e^- \rightarrow Z(\gamma)$ events, already alluded to in Sect. 2) must also be smaller than the target m_H precision.

To determine the optimal centre-of-mass energy for the Higgs boson mass determination, the ZH cross section is determined here with MADGRAPH5_AMC@NLO (v2.6.7) [29] as a function of the centre-of-mass energy, the Higgs boson mass and width, and the Z boson mass and width. The ZH cross section is displayed in the left panel of Fig. 5 as a function of \sqrt{s} from 200 to 250 GeV, with $m_Z = 91$ GeV, $\Gamma_Z = 2.5$ GeV, $m_H = 125$ GeV and $\Gamma_H = 4.1$ MeV. For illustration, the ZH cross section lineshapes are also shown with large (1 GeV) increases of each mass and width. For \sqrt{s} around 216 GeV, a large sensitivity to the Higgs boson mass is observed, with a variation of the cross section by a factor two. At this centre-of-mass energy, the sensitivity to the Z and Higgs boson widths is minimal.

The statistical sensitivity to the Higgs boson mass of a σ_{ZH} measurement at a single collision energy point is given by

$$\Delta m_H(\text{stat}) = \left(\frac{d\sigma_{ZH}}{dm_H} \right)^{-1} \sqrt{\frac{\sigma_{ZH}}{\mathcal{L}\epsilon p}} = \left(\frac{d\sigma_{ZH}}{dm_H} \right)^{-1} \sqrt{\frac{\sigma_{ZH}}{\epsilon \mathcal{L}}} \sqrt{1 + \frac{\sigma_B}{\epsilon \sigma_{ZH}}} \quad (2)$$

where \mathcal{L} is the integrated luminosity collected at this energy, ϵ is the selection efficiency, and p the selection purity, alternatively expressed in terms of σ_B , the total selected background

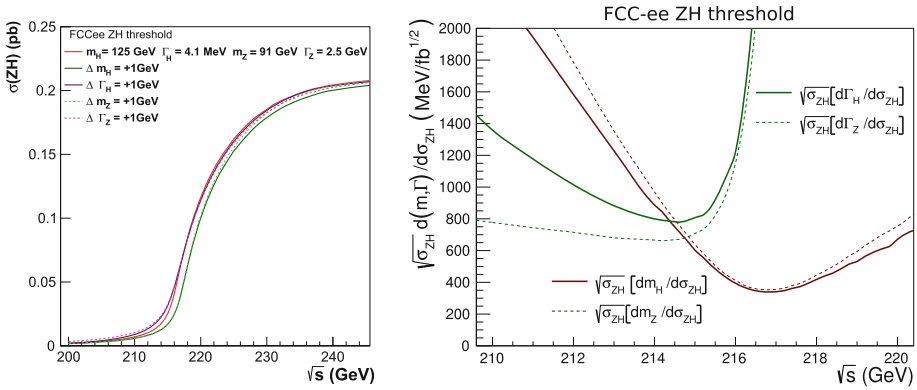


Fig. 5 Left: ZH production cross section as a function of \sqrt{s} around the production threshold. The full red curve is obtained with the default Z and H masses and widths and the full green curve with a Higgs boson mass increased by 1 GeV. The other three curves (full purple, dashed green, and dashed pink) correspond to 1 GeV increases of the Higgs width, the Z mass, and the Z width respectively. Right: ZH cross section differential functions with respect to the Z and Higgs boson masses and widths, for an integrated luminosity of 1 fb^{-1} and a fully efficient and pure ZH event selection

cross section. If several final states are combined, each with its efficiency ϵ_i and purity p_i , the selection quality factor $Q = \sqrt{\epsilon p}$ is to be replaced by $Q = \sqrt{\sum \epsilon_i p_i}$. This purely statistical differential factor is shown in Fig. 5 (right), for an efficiency and a purity of 100%, and an integrated luminosity of 1 fb^{-1} . The statistical accuracy on m_H reaches a minimum at $\sqrt{s} \simeq m_Z + m_H + 0.6 \text{ GeV} \simeq 217 \text{ GeV}$, and at this point, it amounts to

$$\left(\sqrt{\sigma_{\text{ZH}}} \frac{dm_H}{d\sigma_{\text{ZH}}} \right)_{\min} \simeq 350 \text{ MeV} \sqrt{\text{fb}^{-1}} \simeq 10 \text{ MeV} \sqrt{\text{ab}^{-1}}, \tag{3}$$

i.e., just about a factor 10 larger than the $1 \text{ MeV} \sqrt{\text{ab}^{-1}}$ accuracy predicted for the W mass precision at the WW production threshold. The difference arises from both the overall smaller ZH cross section and its slower rise with collision energy. It is also interesting to note that, not unlike at the WW threshold [27], the small sensitivity of the cross section to the H and Z widths vanishes at the point of maximal sensitivity to the masses.

Taking this formula at face value, an integrated luminosity of 5 ab^{-1} at $\sqrt{s} \simeq 217 \text{ GeV}$ would turn into a measurement of m_H with a statistical precision of about 5 MeV. More realistically, and still optimistically assuming a selection quality factor of 0.3 (efficiency and purity values of 90% with the leptonic recoil and 25% with the hadronic recoil are typically achieved with Monte Carlo studies), the statistical precision would already degrade to 9 MeV. The hadronic recoil analysis might prove very challenging at this centre-of-mass energy, with the Higgs and Z bosons produced at rest, and it is safer to assume a precision of 10 MeV with the $Z \rightarrow \ell^+ \ell^-$ channels alone. Propagated systematic uncertainties from the centre-of-mass energy determination (between 1 and 2 MeV), from the Z mass and width knowledge (negligible), from the knowledge of the residual background (supposedly well predictable from control processes), from integrated luminosity (2 MeV if measured and predicted with a per-mil accuracy), and from theory (again 2 MeV if the ZH cross section can be predicted with a per-mil accuracy), seem to be manageable but need to be estimated accurately. Of course, any new physics modifying, e.g., the Higgs boson coupling to the Z boson would lead to unpredictable effects.

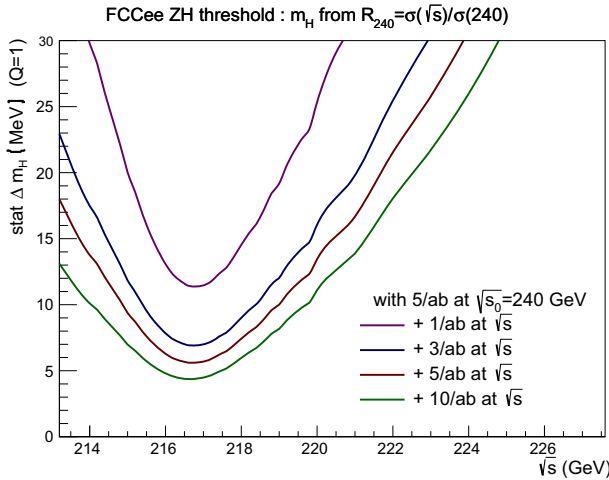


Fig. 6 Expected ideal statistical uncertainties on the Higgs boson mass from the threshold cross section ratio R (Eq. 4) assuming an integrated luminosity of 5 ab^{-1} at $\sqrt{s} = 240 \text{ GeV}$ and different integrated luminosities accumulated at lower centre-of-mass energies around the ZH production threshold

A significant fraction of these statistical and systematic setbacks in the determination of m_H can be alleviated by using exclusive ZH channels, rather than using the recoil method, to measure $\sigma_{ZH} \times \mathcal{B}(Z \rightarrow f\bar{f}) \times \mathcal{B}(H \rightarrow X\bar{X})$ at $\sqrt{s} = 217 \text{ GeV}$. On the one hand, most of the ZH events can be included with a much better purity in each specific channel, which moves the quality factor much closer to unity; and on the other, each measurement can be divided by the corresponding measurement at $\sqrt{s} = 240 \text{ GeV}$, to obtain a ratio

$$R = \frac{\sigma_{ZH} \times \mathcal{B}(Z \rightarrow f\bar{f}) \times \mathcal{B}(H \rightarrow X\bar{X})|_{\sqrt{s}=217 \text{ GeV}}}{\sigma_{ZH} \times \mathcal{B}(Z \rightarrow f\bar{f}) \times \mathcal{B}(H \rightarrow X\bar{X})|_{\sqrt{s}=240 \text{ GeV}}} = \frac{\sigma_{ZH}(\sqrt{s} = 217 \text{ GeV})}{\sigma_{ZH}(\sqrt{s} = 240 \text{ GeV})}, \quad (4)$$

strictly independent of the Higgs boson branching fractions and, at least at tree level, independent of the Higgs boson coupling to the Z. Other systematic experimental and theoretical uncertainties may also delicately cancel in the ratio. In addition, the sensitivity of R to the Higgs boson mass is only slightly smaller than that of the ZH cross section itself, because $\sigma_{ZH}(\sqrt{s} = 240 \text{ GeV})$ depends only mildly on m_H .

Figure 6 shows the statistical uncertainty on m_H from the measurement of the ratio R with ideal event selections in all ZH final states ($Q = \sqrt{\epsilon p} = 1$), assuming an integrated luminosity of 5 ab^{-1} at $\sqrt{s} = 240 \text{ GeV}$ and 1, 2, 5 or 10 ab^{-1} collected at lower centre-of-mass energies. A precision very close to 5 MeV can be obtained with 5 ab^{-1} at $\sqrt{s} = 217 \text{ GeV}$. The impact from the Q factor is expected to be much milder than with the recoil method, but needs to be estimated and optimized for each of the exclusive ZH final states. This precision can then be combined with the direct m_H reconstruction in each of the exclusive ZH final states at 217 GeV, and with that obtained at 240 GeV, which may allow the figure of 1 MeV to be ultimately approached. Whether collecting 5 ab^{-1} at 217 and 240 GeV is better than collecting 10 ab^{-1} at 240 GeV only (or whatever the optimal energy turns out to be for the Higgs self-coupling determination) remains to be demonstrated.

4 Conclusions

The large luminosities provided by FCC-ee at centre-of-mass energies above 200 GeV, and the possibility to simultaneously operate different detectors at various interaction points (IPs), offer multiple opportunities and techniques to measure the Higgs boson mass and production cross section with ambitious accuracies and precisions. In the baseline FCC-ee operation model, the integrated luminosities expected to be delivered and shared between two IPs, amount to 5, 0.2, and 1.5 ab^{-1} at $\sqrt{s} = 240, 340\text{--}350,$ and 365 GeV. Exploiting these integrated luminosities, and the muon momentum resolution of the IDEA drift chamber concept, a preliminary analysis of the $\mu^+\mu^-X$ final state with the “recoil mass” method leads to statistical precisions of 1% on σ_{ZH} and 6 MeV on m_H from the data collected at $\sqrt{s} = 240 \text{ GeV}$ alone.

The inclusion of all Z and H decay channels with constrained kinematic fits of the final states can improve the m_H precision to 2–3 MeV, for which the centre-of-mass energy calibration with the $e^+e^- \rightarrow Z(\gamma)$ process needs a solid systematic uncertainty appraisal to reach or exceed this level of precision (achievable with data recorded at the WW production threshold, where beam energy calibration with resonant depolarization is available). The absolute momentum (energy) scale determination for leptons (jets) will benefit from a regular detector calibration with data produced at the Z pole. Such a precision on the Higgs boson mass would already be sufficient to make good use of a dedicated run at $\sqrt{s} = m_H$, aiming at the determination of the electron Yukawa coupling, for which there will be much bigger hurdles on the way [12,30].

The “recoil mass” method, in which the Z boson is tagged via its decay products (e^+e^- , $\mu^+\mu^-$, or $q\bar{q}$) without using the information from the rest of the ZH event, offers a way to determine the ZH production cross section independently of the Higgs decay branching fractions. An absolute and model-independent determination of the Higgs coupling to the Z boson g_{HZZ} can therefore be accomplished from the σ_{ZH} measurement at $\sqrt{s} = 240 \text{ GeV}$, which can be used in turn as a standard candle for the measurement of all other Higgs properties. The combination of the σ_{ZH} measurements at $\sqrt{s} = 240$ and 365 GeV opens up a simultaneous and absolute determination of the Higgs self-coupling, g_{HHH} , with a 3σ significance in the baseline run plan. Many avenues can be explored to increase this significance, either indirectly by improving the m_H precision, thereby diminishing the correlations with σ_{ZH} , or by directly reducing the σ_{ZH} uncertainty. These avenues include the refined optimization of the choice of centre-of-mass energies, and the corresponding integrated luminosity sharing; an increase of the detector magnetic field to improve the lepton and jet resolutions; the use of modern analysis methods to better separate signal from backgrounds; the development of innovative detector designs towards a more accurate global event reconstruction; the increase of the integrated luminosities, either by extending the FCC-ee high-energy runs beyond the baseline run plan, or by operating detectors at four IPs instead of two (or both); and even an additional run at the ZH threshold ($\sqrt{s} = 217 \text{ GeV}$) that would deliver an additional independent m_H measurement. Matching the experimental and theoretical accuracies to the statistical accuracy needed to reach the first 5σ discovery of the Higgs self-coupling at FCC-ee will be a fascinating journey in the coming years.

Funding This project is co-funded from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 951754.

Data availability Raw data were generated at CERN. Derived data supporting the findings of this study are available from the corresponding author upon request

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

1. A. Blondel, P. Janot, FCC-ee overview: new opportunities create new challenges, in *A Future Higgs and Electroweak Factory (FCC): Challenges Towards Discovery, EPJ+ Special Issue, Focus on FCC-ee*. (2021). [arXiv:2106.13885](https://arxiv.org/abs/2106.13885) [hep-ex]
2. F.A. Berends, R. Kleiss, Initial state radiation at LEP energies and the corrections to Higgs Boson production. *Nucl. Phys. B* **260**, 32–60 (1985)
3. G. Ganis, P. Janot, The HZHA generator, in *Physics at LEP2: CERN, Geneva, Switzerland 2 - 3 Nov 1995. Workshop on Physics at LEP2*, ed. by G. Altarelli, T. Sjöstrand, F. Zwirner, CERN, Geneva (1996)
4. A. Abada et al., FCC-ee: The Lepton Collider FCC CDR Vol. 2. *Eur. Phys. J. ST* **228**(2), 261–623 (2019). <https://doi.org/10.1140/epjst/e2019-900045-4>
5. J. de Blas et al., Higgs Boson studies at future particle colliders. *JHEP* **01**, 139 (2020). [arXiv:1905.03764](https://arxiv.org/abs/1905.03764) [hep-ph]
6. M. McCullough, An indirect model-dependent probe of the Higgs self-coupling, *Phys. Rev.* **D90** no. 1 (2014) 015001, [arXiv:1312.3322](https://arxiv.org/abs/1312.3322) [hep-ph]. [Erratum: *Phys. Rev.* **D92**, no. 3, 039903 (2015)]
7. J. Alison et al., Higgs boson potential at colliders: status and perspectives. *Rev. Phys.* **5**, 100045 (2020). [arXiv:1910.00012](https://arxiv.org/abs/1910.00012) [hep-ph]
8. S. Di Vita, G. Durieux, C. Grojean, J. Gu, Z. Liu, G. Panico, M. Riemann, T. Vantalón, A global view on the Higgs self-coupling at lepton colliders. *JHEP* **02**, 178 (2018). [arXiv:1711.03978](https://arxiv.org/abs/1711.03978) [hep-ph]
9. F. Maltoni, D. Pagani, X. Zhao, Constraining the Higgs self couplings at e^+e^- colliders (2018). [arXiv:1802.07616](https://arxiv.org/abs/1802.07616) [hep-ph]
10. A. Blondel, P. Janot, Future strategies for the discovery and the precise measurement of the Higgs self coupling (2018). [arXiv:1809.10041](https://arxiv.org/abs/1809.10041) [hep-ph]
11. M. Cepeda et al., Report from working group 2: Higgs physics at the HL-LHC and HE-LHC. *CERN Yellow Rep. Monogr.* **7**, 221–584 (2019). [arXiv:1902.00134](https://arxiv.org/abs/1902.00134) [hep-ph]
12. D. d'Enterria, A. Poldaru, G. Wojcik, Measuring the electron Yukawa coupling via resonant s-channel Higgs production at FCC-ee in a future Higgs and electroweak factory (FCC): Challenges towards discovery, EPJ+ special issue, Focus on FCC-ee (2021). [arXiv:2107.02686](https://arxiv.org/abs/2107.02686) [hep-ex]
13. The TLEP Design Study Working Group, M. Bicer, et al., First Look at the Physics Case of TLEP, *JHEP* **01** (2014) 164. [arXiv:1308.6176](https://arxiv.org/abs/1308.6176) [hep-ex]
14. M. Selvaggi, DELPHES 3: A modular framework for fast-simulation of generic collider experiments. *J. Phys. Conf. Ser.* **523**, 012033 (2014)
15. IDEA COLL., G. F. Tassielli, A proposal of a drift chamber for the IDEA experiment for a future e^+e^- collider, *PoS ICHEP2020* (2021) 877
16. T. Sjöstrand, S. Mrenna, P. Z. Skands, A Brief Introduction to PYTHIA 8.1, *Comput. Phys. Commun.* **178** (2008) 852–867. [arXiv:0710.3820](https://arxiv.org/abs/0710.3820) [hep-ph]
17. W. Kilian, T. Ohl, J. Reuter, WHIZARD: simulating multi-particle processes at LHC and ILC. *Eur. Phys. J. C* **71**, 1742 (2011). [arXiv:0708.4233](https://arxiv.org/abs/0708.4233) [hep-ph]
18. A double-sided Crystal Ball function, named after the Crystal Ball Collaboration, is composed of a Gaussian distribution at the core, connected with two power-law distributions describing the lower and upper tails. Both the function and its first derivative are continuous. See Appendix E of the next reference for an early example.
19. T. Skwarnicki, A study of the radiative CASCADE transitions between the Upsilon-Prime and Upsilon resonances, PhD Thesis, Cracow, INP, DESY-F31-86-02. <http://inspirehep.net/record/230779/>, 1986
20. N. Bacchetta et al., CLD – A Detector Concept for the FCC-ee (2019). [arXiv:1911.12230](https://arxiv.org/abs/1911.12230) [physics.ins-det]
21. A. Blondel, P. Janot, J. Wenninger, et al., Polarization and Centre-of-mass Energy Calibration at FCC-ee. [arXiv:1909.12245](https://arxiv.org/abs/1909.12245) [physics.acc-ph]
22. P. Azzi, C. Bernet, C. Botta, P. Janot, M. Klute, P. Lenzi, L. Malgeri, M. Zanetti, Prospective Studies for LEP3 with the CMS Detector (2012). [arXiv:1208.1662](https://arxiv.org/abs/1208.1662) [hep-ex]

23. T. Barklow, K. Fujii, S. Jung, R. Karl, J. List, T. Ogawa, M.E. Peskin, J. Tian, Improved formalism for precision Higgs coupling fits. *Phys. Rev. D* **97**(5), 053003 (2018). [arXiv:1708.08912](https://arxiv.org/abs/1708.08912) [hep-ph]
24. M. Thomson, Model-independent measurement of the $e^+ e^- \rightarrow HZ$ cross section at a future $e^+ e^-$ linear collider using hadronic Z decays. *Eur. Phys. J. C* **76**(2), 72 (2016). [arXiv:1509.02853](https://arxiv.org/abs/1509.02853) [hep-ex]
25. P. Azzurri et al., Physics behind precision (2017). [arXiv:1703.01626](https://arxiv.org/abs/1703.01626) [hep-ph]
26. M. Beguin, P. Azzurri, E. Locci, Reconstruction of the W mass and width at and above WW threshold at FCC-ee, *PoSSEPS-HEP2019* (2020) 653
27. P. Azzurri, The W mass and width measurement challenge at FCC-ee in *A future Higgs and Electroweak factory (FCC): Challenges towards discovery, EPJ+ special issue, Focus on FCC-ee* (2021). [arXiv:2107.04444](https://arxiv.org/abs/2107.04444) [hep-ex]
28. A. Blondel and E. Gianfelice, The challenges of beam polarization and keV-scale centre-of-mass energy calibration at the FCC-ee. *Eur. Phys. J. Plus* **136**(11), 1103 (2021). <https://doi.org/10.1140/epjp/s13360-021-02038-y>
29. J. Alwall et al., The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations. *JHEP* **07**, 079 (2014). [arXiv:1405.0301](https://arxiv.org/abs/1405.0301) [hep-ph]
30. A. Faus-Golfe, M. A. Valdivia Garcia, and F. Zimmermann, *The challenge of monochromatization: Direct s-channel Higgs production in A future Higgs and Electroweak factory (FCC): Challenges towards discovery, EPJ+ special issue, Focus on FCC-ee*. <https://doi.org/10.1140/epjp/s13360-021-02151-y>