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No scalar hair behaviors of static massive scalar fields with nodes

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Abstract We study no scalar field hair behavior for spherically symmetric objects in the scalar-Gauss–Bonnet gravity. In this work, we focus on static massive scalar fields with nodes. We analytically obtain a bound on the coupling parameter. Below the bound, the static massive scalar field with nodes cannot exist outside the object. In particular, our conclusion is independent of surface boundary conditions.

1 Introduction

One famous property of classical black holes is the no hair theorem, which states that asymptotically flat spherically symmetric black holes cannot support external static scalar fields, see references [1–8] and reviews [9,10]. Interestingly, no hair behavior also appears for horizonless reflecting stars [11–23]. Recently, it was found that scalar field hairs can exist outside black holes and reflecting stars when considering the non-minimal coupling between scalar fields and the Gauss–Bonnet invariant [24–30]. This scalar-Gauss–Bonnet gravity attracted lots of attentions and other models were constructed [31–45].

The mostly studied scalar configurations are the cases with no nodes. Theoretically, scalar configurations can possess nodes. It is well known that, in the general case, scalar configurations with nodes are usually unstable [46–49]. So it is natural to conjecture that scalar configurations with nodes may finally evolve into the more stable nodeless solutions. However, it is still meaningful to study the solution with nodes. Firstly, the solution with nodes may be sufficiently stable, which means that the perturbation growth time is extremely large [50]. In this case, the unstable node solution stays for a long time and can be observed from physical aspects. And secondly, in some gravity models, it is very surprising that the solution with nodes seems to be the endpoint of the tachyonic instability [51].

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In particular, for massless scalar fields non-minimally coupled to the Gauss–Bonnet invariant in the background of black holes, an interesting relation $\Delta_n = \sqrt{\bar{\eta}_{n+1}} - \sqrt{\bar{\eta}_n} = \frac{\sqrt{3}}{2}\pi$ for $n \to \infty$ was obtained through WKB approach in [52] and this relation is also precisely supported by numerical data in [27], where n is the number of nodes. For massive scalar fields in black hole spacetimes, with both analytical and numerical methods, we demonstrated that this relation still holds in the large node number limit [53]. On the other side, some quantum-gravity theories suggest that quantum effects may prevent the formation of classical black hole absorbing horizons and the horizonless compact star may serve as an alternative [54–61]. So it is also interesting to search for properties independent of surface boundary conditions. In this work, we plan to study the existence (or non-existence) of scalar fields with nodes outside general spherically symmetric objects.

In the following, we shall consider a gravity system with a scalar field coupled to the Gauss–Bonnet invariant in the background of general spherically symmetric objects. With analytical methods, we obtain a bound on the scalar-Gauss–Bonnet coupling parameter, below which there is no hair behavior for static massive scalar fields with nodes. We summarize main results in the last section.

2 Bounds on the scalar-Gauss–Bonnet coupling parameter

We consider a gravity with a scalar field Ψ coupled to the Gauss–Bonnet invariant $\mathcal{R}^2_{GB}$. The general Lagrange density can be written as [26–31]

$$\mathcal{L} = R - |\nabla_\mu \Psi|^2 - m^2\Psi^2 + f(\Psi)\mathcal{R}^2_{GB}. \quad (1)$$

Here m is the scalar field mass and the Gauss–Bonnet invariant is defined as $\mathcal{R}^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$. In the probe limit, there is $\mathcal{R}^2_{GB} = \frac{48M^2}{r^6}$. The function $f(\Psi)$

describes the nontrivial coupling between scalar fields and the Gauss–Bonnet invariant. In the linearized regime, one can generally put the coupling function in the quadratic form $f(\Psi)=\eta\Psi^2$ with η as the model parameter [28–30].

The exterior metric solution of spherically symmetric objects is [29]

$$ds^2=-g(r)dt^2+\frac{dr^2}{g(r)}+r^2(d\theta^2+sin^2\theta d\phi^2). \tag{2}$$

In the probe limit, the metric is given by the function $g(r)=1-\frac{2M}{r}$, where M is the mass of the objects. We define $r_s\geq 2M$ as the object surface radii. When $r_s=2M$, the metric is a black hole.

The scalar field differential equation is

$$\nabla^\nu\nabla_\nu\Psi-m^2\Psi+\frac{f'_\Psi\mathcal{R}^2_{GB}}{2}=0. \tag{3}$$

We choose to study a static massive scalar field in the form

$$\Psi(t,r,\theta,\phi)=\sum_{lm}e^{im\phi}S_{lm}(\theta)R_{lm}(r). \tag{4}$$

Here l is the spherical harmonic index and $l(l+1)$ is the characteristic eigenvalue of the angular scalar eigenfunction $S_{lm}(\theta)$ [27,62,63]. For simplicity, we label $R_{lm}(r)$ as $\psi(r)$. One can obtain the scalar field equation

$$\psi''+\left(\frac{2}{r}+\frac{g'}{g}\right)\psi'+\left(\frac{\eta\mathcal{R}^2_{GB}}{g}-\frac{l(l+1)}{r^2g}-\frac{m^2}{g}\right)\psi=0, \tag{5}$$

where $g=1-\frac{2M}{r}$ and $\mathcal{R}^2_{GB}=\frac{48M^2}{r^6}$.

In this work, we focus on scalar fields with nodes. That is to say, there is at least a point r_0 satisfying $\psi(r_0)=0$. The bound-state massive static scalar fields satisfy asymptotically decaying behaviors $\psi(r\to\infty)\sim\frac{1}{r}e^{-mr}$. In the range (r_0,∞), the scalar field boundary conditions are

$$\psi(r_0)=0,\quad \psi(\infty)=0. \tag{6}$$

According to boundary conditions (6), the scalar field $\psi(r)$ must possess one extremum point $r=r_{peak}$ between the vanishing point $r=r_0$ and the infinity. It may be a positive maximum extremum point satisfying

$$\psi(r_{peak})>0,\quad \psi'(r_{peak})=0,\quad \psi''(r_{peak})\leq 0, \tag{7}$$

otherwise it will be a negative minimum extremum point with

$$\psi(r_{peak})<0,\quad \psi'(r_{peak})=0,\quad \psi''(r_{peak})\geq 0. \tag{8}$$

Relations (7) and (8) yield that

$$\{\psi\neq 0,\quad \psi'=0\ \ and\ \ \psi\psi''\leq 0\}\ \ for\ \ r=r_{peak}. \tag{9}$$

Multiplying both sides of (5) with ψ, we obtain the new equation

$$\psi\psi''+\left(\frac{2}{r}+\frac{g'}{g}\right)\psi\psi' +\left(\frac{\eta\mathcal{R}^2_{GB}}{g}-\frac{l(l+1)}{r^2g}-\frac{m^2}{g}\right)\psi^2=0. \tag{10}$$

At the extremum point, relations (9) and (10) yield the inequality

$$\frac{\eta\mathcal{R}^2_{GB}}{g}-\frac{l(l+1)}{r^2g}-\frac{m^2}{g}\geq 0\ \ for\ \ r=r_{peak}. \tag{11}$$

The extremum point is outside the gravitational radius $r_{peak}>r_s\geq 2M$, which yields

$$g(r)=1-\frac{2M}{r}>0\ \ for\ \ r=r_{peak}. \tag{12}$$

With relations (11) and (12), we obtain the inequality

$$\eta\mathcal{R}^2_{GB}-\frac{l(l+1)}{r^2}-m^2\geq 0\ \ for\ \ r=r_{peak}. \tag{13}$$

From relation (13), one deduces that

$$\eta\mathcal{R}^2_{GB}-m^2\geq 0\quad for\ \ r=r_{peak}. \tag{14}$$

The inequality (14) is equal to

$$\eta\geq\frac{m^2}{\mathcal{R}^2_{GB}}=\frac{m^2r^6_{peak}}{48M^2}. \tag{15}$$

The extremum point is outside the object surface $r_{peak}>r_s\geq 2M$. It yields the relation

$$\eta\geq\frac{m^2r^6_{peak}}{48M^2}>\frac{m^2r^6_s}{48M^2}\geq\frac{m^2(2M)^6}{48M^2}=\frac{4m^2M^4}{3}. \tag{16}$$

For scalar fields with nodes, we obtain a lower bound on the coupling parameter. Below this bound, the static scalar field with nodes cannot exist outside the object surface. It implies that static massive scalar fields with nodes usually cannot exist outside objects of large mass. Here we obtain a no hair behavior for scalar fields with nodes in the region

$$\eta\leq\frac{4}{3}m^2M^4. \tag{17}$$

In particular, for $\eta=0$, (17) always holds. It means minimally coupled massive scalar fields always cannot exist outside general spherically symmetric objects, such as black holes and horizonless stars.

3 Conclusions

We studied no scalar field hair behavior for spherically symmetric objects in the scalar-Gauss–Bonnet gravity. We considered a static massive scalar field with nodes. Through analytical methods, we obtained a bound on the scalar-Gauss–Bonnet coupling parameter as $\eta \leq \frac{4}{3}m^2M^4$, where η is the coupling parameter, m is the scalar field mass and M is the mass of objects. Below the lower bound, static massive scalar fields with nodes cannot exist outside the objects. In particular, our analysis doesn't depend on the surface condition. So this no hair behavior for scalar fields with nodes is a very general property.

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