



On the β -decay of the accelerated proton and neutrino oscillations: a three-flavor description with CP violation

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Abstract The (inverse) β -decay of uniformly accelerated protons ($p \rightarrow n + e^+ + \nu_e$) has been recently analyzed in the context of two-flavor neutrino mixing and oscillations. It has been shown that the decay rates as measured by an inertial and comoving observer are in agreement, provided that: (i) the *thermal nature* of the accelerated vacuum (Unruh effect) is taken into account; (ii) the asymptotic behavior of neutrinos is described through *flavor* (rather than mass) *eigenstates*; (iii) the Unruh radiation is made up of *oscillating neutrinos*. Here we extend the above considerations to a more realistic scenario including three generations of Dirac neutrinos. By following the outlined recipe, we find that the equality between the two rates still holds true, confirming that mixing is perfectly consistent with the General Covariance of Quantum Field Theory. Notably, we prove that the analysis of CP violation in neutrino oscillations provides a further solid argument for flavor states as fundamental representation of asymptotic neutrino states. Our approach is finally discussed in comparison with the other treatments appeared in literature.

1 Introduction

It is well-known that physical laws for accelerated systems are far more subtle than the corresponding equations for inertially moving objects. In Classical Mechanics, for instance, one can hold onto Newton's law $\mathbf{F} = m\mathbf{a}$ for an observer trapped in a free-falling elevator or rotating on a merry-go-round, provided that some extra "fictitious" forces are introduced. In the same way, a charged particle undergoing an acceleration does radiate photons (whereas an inertial par-

tle does not), the rate of which is predicted by Classical Electrodynamics to be proportional to the square of the acceleration [1]. In this context, the question naturally arises as to whether similar inertial effects also come into play in a purely quantum realm.

Along this line, in 1976 Unruh found out that a uniformly accelerated (Rindler) observer experiences in the inertial (Minkowski) vacuum a thermal bath of particles at temperature [2]

$$T_U = \frac{a}{2\pi}, \quad (1)$$

where a is the magnitude of the proper acceleration. This confirmed previous results about the observer-dependence of the particle quantum concept even in the absence of gravity [3,4], providing a flat-counterpart of the best-known Hawking effect [5].

Notwithstanding the large number of theoretical applications and the experimental efforts made so far [6], direct evidences of Unruh radiation are still lacking, thereby opening up a lively debate on its actual existence [7–9], even through the study of analogue models [10–14]. Against the skepticism, however, a virtual confirmation of Unruh effect was elegantly proposed in the context of the inverse β -decay in Ref. [15], where it was shown that inertial and co-accelerated observers would draw incompatible conclusions about the stability of non-inertial protons if the vacuum radiation were not taken into account. In light of this, there is no question that the Unruh effect turns out to be mandatory for Quantum Field Theory (QFT) as well as fictitious forces are for Classical Mechanics, since both of them are required to preserve the internal consistency of successfully tested theories when investigated in accelerated frames.

The intimate connection between the Unruh effect and the inverse β -decay was first addressed in a toy model in Refs. [16,17], assuming all involved particles to be scalars, and then analyzed within a more rigorous framework with Dirac

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fields in Refs. [18, 19]. Surprisingly, only recently it was studied in connection with neutrino flavor mixing and oscillations [20–23], with conflicting results on the very nature of asymptotic neutrino states being reached. In these works, a preliminary description including only two flavors was considered.

Starting from the outlined scenario, in what follows we discuss the Unruh effect and revisit the inverse β -decay with mixed neutrinos in a more realistic three-flavor setting. By explicit calculation, we show that a covariant treatment consistent with the phenomena of mixing and oscillations unavoidably implies the choice of flavor (rather than mass) eigenstates for asymptotic neutrinos, as well as the occurrence of flavor oscillations even in the Unruh thermal bath. The obtained result is corroborated by very straightforward considerations on the necessity to allow for CP asymmetry in processes involving neutrino oscillations—a feature which mass eigenstates would fail to pinpoint. Based on these arguments, we also speculate on the possibility to have a non-trivial asymmetry between the Unruh baths experienced by the accelerated proton and antiproton, respectively.

The remainder of the work is organized as follows: in Sect. 2 we set the stage for the study of the inverse β -decay. Sect. 3 is devoted to the evaluation of the decay rate in the laboratory frame. The same calculation is independently performed from the point of view of a comoving observer in Sect. 4. We show that the two results are in full agreement, contrary to previous claims of Ref. [20]. Sect. 5 concerns a discussion on the incompatibility between the mass asymptotic representation and CP-violation effects. Closing remarks are contained in Sect. 6. Throughout the paper, we shall use the Minkowski metric with the timelike signature and natural units $k_B = \hbar = c = 1$.

2 Inverse β -decay and neutrino mixing: general considerations

Despite the common belief, the lifetime of a particle cannot be considered among its inherent and characteristic properties. The most eloquent example is provided by the proton, which is a *stable* bound state of quarks, at least according to the predictions of the Standard Model. In Refs. [16, 17], indeed, it was argued that the lifetime τ_p of the proton may significantly decrease if we expose it to a large acceleration a . This was rigorously shown in Refs. [18, 19], where the inverse β -decay

$$(i) \quad p \xrightarrow{a} n + e^+ + \nu_e \quad (2)$$

was analyzed in both the laboratory and comoving frames, obtaining non-vanishing (equal) results for the proton decay rate $\Gamma \sim \tau_p^{-1}$.

The study of the interaction (2) proceeds in a straightforward way if we regard the proton $|p\rangle$ and neutron $|n\rangle$ as unexcited and excited states of a two-level system, the nucleon, whose Hamiltonian obeys the relations

$$\hat{H}|p\rangle = m_p|p\rangle, \quad \hat{H}|n\rangle = m_n|n\rangle, \quad (3)$$

where $m_{p(n)}$ is the rest mass of the proton (neutron). Furthermore, we require that the momenta of both the positron $|e^+\rangle$ and neutrino $|\nu_e\rangle$ satisfy the condition $|\mathbf{k}_{e^+(\nu_e)}| \ll m_p, m_n$, so that the fermion emission does not change the four-velocity of the hadrons appreciably (*no-recoil approximation*). Within this semiclassical framework, it is reasonable to suppose that the nucleon system will move along a well-defined trajectory, the Rindler hyperbola, which indeed describes a uniformly accelerated motion. By assuming the acceleration to be directed along the z -axis, the associated current can be written as

$$\hat{J}_{h,\lambda} = \hat{q}(\tau)u_\lambda\delta(x)\delta(y)\delta(u - 1/a), \quad (4)$$

where $\hat{q}(\tau) = e^{i\hat{H}\tau}\hat{q}(0)e^{-i\hat{H}\tau}$ is the monopole operator and $G_F = |\langle n|\hat{q}(0)|p\rangle|$ is the Fermi constant. In the above expression, we have denoted by v and $\tau = v/a$ the Rindler time coordinate and proper time of the nucleon, respectively. The Dirac delta fixes the spatial coordinate u to the value $1/a$, which identifies the Rindler trajectory.¹ The nucleon four-velocity reads $u^\lambda = (a, 0, 0, 0)$ and $u^\lambda = (\sqrt{a^2t^2 + 1}, 0, 0, at)$ in Rindler and Minkowski coordinates.

In turn, leptons are treated as Dirac quantum fields with a current given by

$$\hat{J}_l^\lambda = \sum_{\ell=e,\mu,\tau} \left(\hat{\Psi}_{\nu_\ell} \gamma^\lambda \hat{\Psi}_\ell + \hat{\bar{\Psi}}_\ell \gamma^\lambda \hat{\Psi}_{\nu_\ell} \right), \quad (5)$$

where $\hat{\Psi}_\ell$ ($\hat{\Psi}_{\nu_\ell}$) is the charged lepton (neutrino) Dirac field and e, μ, τ label the three lepton flavors. Rigorously speaking, we should consider a current including also the axial term. As shown in Ref. [15], however, the above oversimplification does not affect the overall validity of our analysis.

By resorting to Eqs. (4) and (5), the Fermi-like effective action describing the interaction takes the form

$$\hat{S}_I = \int d^4x \sqrt{-g} \hat{J}_{h,\lambda} \hat{J}_l^\lambda, \quad (6)$$

where $g = \det(g_{\mu\nu})$ and γ^λ are the gamma matrices in the Dirac representation (see Ref. [24]).

In the simplest extended version of the Standard Model, it is well-known that neutrinos weakly interact with charged leptons in flavor eigenstates $|\nu_\ell\rangle$ [25], which are superpositions of mass states $|\nu_j\rangle$ ($j = 1, 2, 3$) determined by the transformation

¹ Note that the Rindler coordinates (v, x', y', u) are related to the corresponding Minkowski coordinates (t, x, y, z) by $t = u \sinh v$, $x' = x$, $y' = y$, $z = u \cosh v$.

$$\begin{pmatrix} | \nu_e \rangle \\ | \nu_\mu \rangle \\ | \nu_\tau \rangle \end{pmatrix} = U \begin{pmatrix} | \nu_1 \rangle \\ | \nu_2 \rangle \\ | \nu_3 \rangle \end{pmatrix}, \tag{7}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [26] matrix in the standard parameterization [27]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{8}$$

Here $c_{jk} = \cos \theta_{jk}$, $s_{jk} = \sin \theta_{jk}$, θ_{jk} is the ν_j - ν_k mixing angle and δ is the CP-violating phase.

By using the S -matrix framework, in the next Section we evaluate the transition probability for the process (2) by assuming that the states of neutrino far before and after the interaction are those with definite flavor [28–33]. Calculations are performed at tree level both in the laboratory and comoving frames, with emphasis on the mandatory rôle of the Unruh effect for the consistency of the two approaches.

3 Inverse β -decay in the laboratory frame

The inverse β -decay as seen by a Minkowski (inertial) observer is given by Eq. (2) (see also Fig. 1). In order to evaluate the transition rate, we quantize the fermion fields in the standard way [21]

$$\hat{\Psi}(t, \mathbf{x}) = \sum_{\sigma=\pm} \int d^3k \left[\hat{b}_{\mathbf{k}\sigma} \psi_{\mathbf{k}\sigma}^{(+\omega)} + \hat{d}_{\mathbf{k}\sigma}^\dagger \psi_{-\mathbf{k}-\sigma}^{(-\omega)} \right], \tag{9}$$

where $\mathbf{x} \equiv (x, y, z)$ (for simplicity, we have omitted the spacetime dependence of the spinors in the r.h.s. of the expansion). We have denoted by $\hat{b}_{\mathbf{k}\sigma}$ ($\hat{d}_{\mathbf{k}\sigma}$) the canonical annihilation operators of particles (antiparticles) with momentum $\mathbf{k} \equiv (k^x, k^y, k^z)$, polarization $\sigma = \pm$, frequency

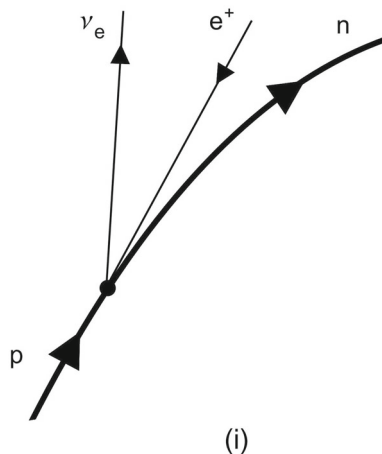


Fig. 1 Inverse β -decay in the laboratory frame in the absence of flavor oscillations (time flows in the vertical direction)

$\omega = \sqrt{\mathbf{k}^2 + m^2}$ and mass m . The modes $\psi_{\mathbf{k}\sigma}^{(\pm\omega)}$ are positive and negative energy solutions of the Dirac equation in Minkowski spacetime. They are given by

$$\psi_{\mathbf{k}\sigma}^{(\pm\omega)}(t, \mathbf{x}) = \frac{e^{i(\mp\omega t + \mathbf{k}\cdot\mathbf{x})}}{2^2\pi^{\frac{3}{2}}} u_{\sigma}^{(\pm\omega)}(\mathbf{k}), \tag{10}$$

where

$$u_{+}^{(\pm\omega)}(\mathbf{k}) = \frac{1}{\sqrt{\omega(\omega \pm m)}} \begin{pmatrix} m \pm \omega \\ 0 \\ k^z \\ k^x + ik^y \end{pmatrix}, \tag{11}$$

$$u_{-}^{(\pm\omega)}(\mathbf{k}) = \frac{1}{\sqrt{\omega(\omega \pm m)}} \begin{pmatrix} 0 \\ m \pm \omega \\ k^x - ik^y \\ -k^z \end{pmatrix}. \tag{12}$$

With the above definition, one can easily prove that the modes $\psi_{\mathbf{k}\sigma}^{(\pm\omega)}$ are orthonormal with respect to the inner product [34]

$$\begin{aligned} \langle \psi_{\mathbf{k}\sigma}^{(\pm\omega)}, \psi_{\mathbf{k}'\sigma'}^{(\pm\omega')} \rangle &\equiv \int_{\Sigma} d\Sigma_{\lambda} \bar{\psi}_{\mathbf{k}\sigma}^{(\pm\omega)} \gamma^{\lambda} \psi_{\mathbf{k}'\sigma'}^{(\pm\omega')} \\ &= \delta_{\sigma\sigma'} \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\pm\omega\pm\omega'}, \end{aligned} \tag{13}$$

where $\bar{\psi} = \psi^\dagger \gamma^0$, $d\Sigma_{\lambda} = n_{\lambda} d\Sigma$ and n_{λ} is a unit vector orthogonal to the hypersurface Σ of constant t .

In the S -matrix formalism, the transition amplitude for the process (2) reads² [21]

$$\begin{aligned} \mathcal{A}^{(i)} &\equiv \langle n | \otimes \langle e^+, \nu_e | \hat{S}_I | 0 \rangle \otimes | p \rangle \\ &= \frac{G_F}{2^4\pi^3} \sum_{j=1}^3 |U_{ej}|^2 \mathcal{I}_{\sigma\nu\sigma_e}(\omega_{\nu_j}, \omega_e), \end{aligned} \tag{14}$$

where the latin number in the superscript of the l.h.s. labels the process under consideration and $U_{\ell j}$ ($\ell = e, \mu, \tau$) is the generic element of the PMNS matrix (8). The function $\mathcal{I}_{\sigma\nu\sigma_e}$ is defined as

$$\begin{aligned} \mathcal{I}_{\sigma\nu\sigma_e}(\omega_{\nu_j}, \omega_e) &= \int_{-\infty}^{+\infty} d\tau u_{\lambda} \left[\bar{u}_{\sigma\nu}^{(+\omega_{\nu_j})} \gamma^{\lambda} u_{-\sigma_e}^{(-\omega_e)} \right] \\ &\times e^{i[\Delta m\tau + a^{-1}(\omega_{\nu_j} + \omega_e) \sinh a\tau - a^{-1}(k_{\nu}^z + k_e^z) \cosh a\tau]}. \end{aligned} \tag{15}$$

Note that, in the above calculation, we have implemented the PMNS transformation on both neutrino state and field, the latter being transformed as

² The consistency of the S -matrix formalism with flavor asymptotic states has been questioned several times in literature [23, 35, 36]. In spite of this, one can prove that such an approach is well-posed both physically (since its predictions are in agreement with the ones of the Standard Model) and mathematically (as the asymptotic $t \rightarrow \pm\infty$ limits do not entail any technical problem in the calculation of transition amplitudes) [29].

$$\psi_{\nu_e}(x) = \sum_{j=1}^3 U_{ej}^* \psi_{\nu_j}(x), \tag{16}$$

and similarly for ψ_{ν_μ} and ψ_{ν_τ} .³ Moreover, we have assumed equal momenta and polarizations for neutrino states with definite mass.

Now, by plugging the amplitude (14) in the following expression of the scalar transition probability per proper time T ,

$$\Gamma^{(i)} \equiv \frac{1}{T} \sum_{\sigma_e, \sigma_\nu} \int d^3k_\nu \int d^3k_e |\mathcal{A}^{(i)}|^2, \tag{17}$$

we get

$$\begin{aligned} \Gamma^{(i)} &= |U_{e1}|^4 \Gamma_1 + |U_{e2}|^4 \Gamma_2 + |U_{e3}|^4 \Gamma_3 \\ &+ \left(|U_{e1}|^2 |U_{e2}|^2 \Gamma_{12} + |U_{e1}|^2 |U_{e3}|^2 \Gamma_{13} \right. \\ &\left. + |U_{e2}|^2 |U_{e3}|^2 \Gamma_{23} + \text{c.c.} \right). \end{aligned} \tag{18}$$

Here, the contribution

$$\begin{aligned} \Gamma_j &\equiv \frac{1}{T} \frac{G_F^2}{2^8 \pi^6} \sum_{\sigma_\nu, \sigma_e} \int d^3k_\nu \int d^3k_e |\mathcal{S}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e)|^2 \\ &= \frac{G_F^2}{a \pi^6 e^{\pi \Delta m/a}} \int d^3k_\nu \int d^3k_e \left\{ K_{2i \Delta m/a} \left(\frac{2(\omega_{\nu_j} + \omega_e)}{a} \right) \right. \\ &\left. + \frac{m_{\nu_j} m_e}{\omega_{\nu_j} \omega_e} \text{Re} \left[K_{2i \Delta m/a+2} \left(\frac{2(\omega_{\nu_j} + \omega_e)}{a} \right) \right] \right\} \end{aligned} \tag{19}$$

represents the decay rate we would obtain by using $|\nu_j\rangle$ as asymptotic neutrino state, $K_{i\nu}(x)$ is the modified Bessel function of second kind, and the interference term

$$\begin{aligned} \Gamma_{jk} &\equiv \frac{1}{T} \frac{G_F^2}{2^8 \pi^6} \\ &\times \sum_{\sigma_\nu, \sigma_e} \int d^3k_\nu \int d^3k_e \mathcal{S}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e) \mathcal{S}_{\sigma_\nu \sigma_e}^*(\omega_{\nu_k}, \omega_e) \end{aligned} \tag{20}$$

arises from the coherent superpositions of neutrino states with different masses. The explicit expression of Γ_{jk} is rather awkward to exhibit. We remand to Ref. [21] for a more detailed treatment of this term.

Some comments are in order here: first, we observe that the decay rate (18) is given by the coherent sum of the Γ_j -components of the inverse β -decay amplitudes. By contrast, there is no summation over j in the corresponding result (48)

³ In Ref. [28] it has been shown that the structure of the field mixing relation (16) is that of a Bogoliubov transformation nested into the standard quantum mechanical rotation. As a result, the definition (7) of mixed states turns out to be inconsistent with Eq. (16). Nevertheless, for relativistic neutrinos and in the approximation of small mass differences we are employing here, one can prove that Pontecorvo states well-approximate the exact field theoretical states [32,33], thus validating the simultaneous use of Eqs. (7) and (16).

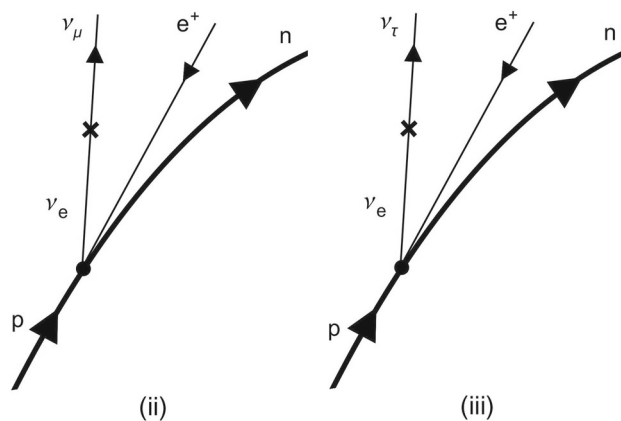


Fig. 2 Inverse β -decay in the laboratory frame in the presence of flavor oscillations (time flows in the vertical direction). The crosses denote the occurrence of the oscillation

of Ref. [23], where asymptotic neutrinos are assumed to be mass eigenstates. Such a controversy is addressed more specifically in Refs. [22,29,32,33]. Furthermore, for $\theta_{jk} \rightarrow 0$, we recover the result of Ref. [19], where the inverse β -decay is analyzed in the absence of mixing. Similar considerations hold in the approximation of small mass differences, since both Γ_i ($i = 2, 3$) and Γ_{jk} reduce to Γ_1 and

$$\begin{aligned} &|U_{e1}|^4 + |U_{e2}|^4 + |U_{e3}|^4 \\ &+ 2 \left(|U_{e1}|^2 |U_{e2}|^2 + |U_{e1}|^2 |U_{e3}|^2 + |U_{e2}|^2 |U_{e3}|^2 \right) = 1. \end{aligned} \tag{21}$$

It is now worth noting that, due to the asymptotic effects of neutrino oscillations [22], the total rate for the inverse β -decay also gets non-vanishing contributions from the two flavor-violating processes (see Fig. 2)

$$\text{(ii) } p \rightarrow n + e^+ + \nu_\mu, \tag{22}$$

$$\text{(iii) } p \rightarrow n + e^+ + \nu_\tau. \tag{23}$$

By using Eq. (7), one can show that the transition amplitudes for these channels take the form

$$\begin{aligned} \mathcal{A}^{(ii)} &\equiv \langle n | \otimes \langle e^+, \nu_\mu | \hat{S}_I | 0 \rangle \otimes | p \rangle \\ &= \frac{G_F}{2^4 \pi^3} \sum_{j=1}^3 U_{\mu j}^* U_{ej} \mathcal{S}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e), \end{aligned} \tag{24}$$

$$\begin{aligned} \mathcal{A}^{(iii)} &\equiv \langle n | \otimes \langle e^+, \nu_\tau | \hat{S}_I | 0 \rangle \otimes | p \rangle \\ &= \frac{G_F}{2^4 \pi^3} \sum_{j=1}^3 U_{\tau j}^* U_{ej} \mathcal{S}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e), \end{aligned} \tag{25}$$

which lead to the following expressions for the transition probabilities per proper time

$$\Gamma^{(ii)} = |U_{\mu 1}|^2 |U_{e 1}|^2 \Gamma_1 + |U_{\mu 2}|^2 |U_{e 2}|^2 \Gamma_2 + |U_{\mu 3}|^2 |U_{e 3}|^2 \Gamma_3 + \left(U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* \Gamma_{12} + U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \Gamma_{13} + U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^* \Gamma_{23} + \text{c.c.} \right), \tag{26}$$

$$\Gamma^{(iii)} = |U_{\tau 1}|^2 |U_{e 1}|^2 \Gamma_1 + |U_{\tau 2}|^2 |U_{e 2}|^2 \Gamma_2 + |U_{\tau 3}|^2 |U_{e 3}|^2 \Gamma_3 + \left(U_{\tau 1}^* U_{e 1} U_{\tau 2} U_{e 2}^* \Gamma_{12} + U_{\tau 1}^* U_{e 1} U_{\tau 3} U_{e 3}^* \Gamma_{13} + U_{\tau 2}^* U_{e 2} U_{\tau 3} U_{e 3}^* \Gamma_{23} + \text{c.c.} \right). \tag{27}$$

Unlike $\Gamma^{(i)}$ in Eq. (18), both $\Gamma^{(ii)}$ and $\Gamma^{(iii)}$ vanish for $\theta_{jk} \rightarrow 0$ and/or for small mass differences, since

$$|U_{\chi 1}|^2 |U_{e 1}|^2 + |U_{\chi 2}|^2 |U_{e 2}|^2 + |U_{\chi 3}|^2 |U_{e 3}|^2 + \left(U_{\chi 1}^* U_{e 1} U_{\chi 2} U_{e 2}^* + U_{\chi 1}^* U_{e 1} U_{\chi 3} U_{e 3}^* + U_{\chi 2}^* U_{e 2} U_{\chi 3} U_{e 3}^* + \text{c.c.} \right) = 0, \quad \chi = \mu, \tau. \tag{28}$$

This shows that $\Gamma^{(ii)}$ and $\Gamma^{(iii)}$ are pure interference terms, whose origin is intimately related to the non-trivial nature of neutrino mixing and oscillations.

Finally, by summing up the contributions in Eqs. (18), (26) and (27), the total decay rate becomes

$$\Gamma^{\text{lab}} \equiv \sum_{s=i}^{\text{iii}} \Gamma^{(s)} = |U_{e 1}|^2 \Gamma_1 + |U_{e 2}|^2 \Gamma_2 + |U_{e 3}|^2 \Gamma_3. \tag{29}$$

From Eq. (29) it arises that the sum over flavors of the inverse β -decay rates amounts to the weighted average over masses, with weights given by the square modulus of the projections of $|v_e\rangle$ on $|v_j\rangle$ ($j = 1, 2, 3$). The meaning of this result can be illustrated as follows: let us consider the lepton charges for mixed neutrinos as derived from Noether’s theorem. If we denote by $Q_j = \int d^3x \Psi_{v_j}^\dagger(x) \Psi_{v_j}(x)$ the conserved charge for the neutrino field with mass m_j and by $Q_\ell(t) = \int d^3x \Psi_{v_\ell}^\dagger(x) \Psi_{v_\ell}(x)$ the (time-dependent) flavor charge for the field with definite flavor ℓ , we simply have $Q = \sum_{j=1}^3 Q_j = \sum_{\ell=e,\mu,\tau} Q_\ell(t)$, with Q being the total charge of the system [37]. Beyond the pure mathematical

equality, the physical interpretation of this relation is non-trivial, as it states that the total lepton number is a conserved quantity both in the presence and in the absence of flavor mixing. On the left side (i.e. when mixing is not taken into account), such quantity is given by the sum of three separately conserved family lepton numbers; conversely, on the right side (i.e. when mixing is included) it is obtained by summing up three non-conserved flavor charges, which are indeed associated to the phenomenon of neutrino oscillations.

4 Inverse β -decay in the comoving frame

From the viewpoint of an observer comoving with the proton, the process (2) is clearly forbidden by energy conservation. According to such an observer, however, the Minkowski vacuum appears as a thermal bath of virtual particles with which the proton can interact. Consequently, the following new channels become accessible (see Fig. 3):

$$\text{(iv)} \quad p + e^- \rightarrow n + \nu_e, \tag{30a}$$

$$\text{(v)} \quad p + \bar{\nu}_e \rightarrow n + e^+, \tag{30b}$$

$$\text{(vi)} \quad p + e^- + \bar{\nu}_e \rightarrow n, \tag{30c}$$

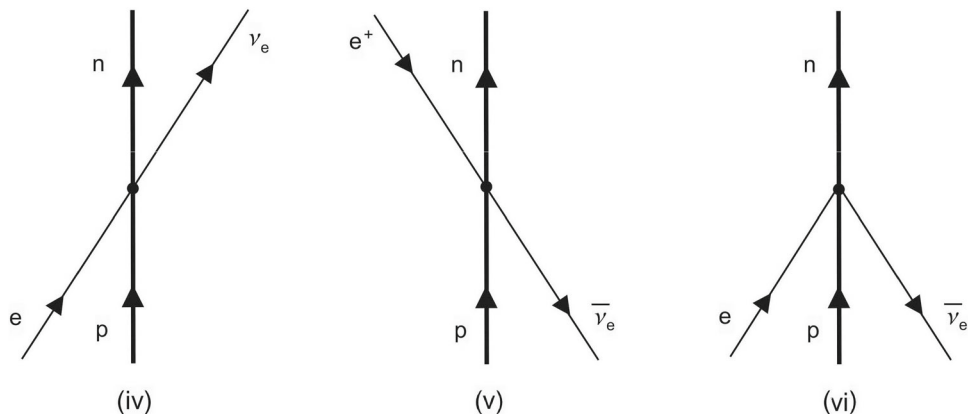
i.e. the proton at rest is allowed to decay due to the absorption of an electron (Eq. (30a)), an antineutrino (Eq. (30b)) and both an electron and an antineutrino (Eq. (30c)) from the thermal bath.

In order to compute the total transition probability, let us remind that the proper way to quantize fields for a uniformly accelerated observer is the Rindler–Fulling scheme, according to which [21]

$$\hat{\psi}(v, \mathbf{x}) = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \int d^2k \left[\hat{b}_{\mathbf{w}\sigma} \psi_{\mathbf{w}\sigma}^{(+\omega)} + \hat{d}_{\mathbf{w}\sigma}^\dagger \psi_{\mathbf{w}-\sigma}^{(-\omega)} \right], \tag{31}$$

where $\mathbf{x} \equiv (x, y, u)$, $\mathbf{w} \equiv (\omega, k^x, k^y)$. Here, we have denoted by $\hat{b}_{\mathbf{w}\sigma}$ ($\hat{d}_{\mathbf{w}\sigma}$) the canonical annihilation operators of Rindler particles (antiparticles) with transverse momentum

Fig. 3 Inverse β -decay in the comoving frame in the absence of flavor oscillations (time flows in the vertical direction). The three processes are allowed due to the absorption (emission) of particles from (to) the Unruh thermal bath



$k \equiv (k^x, k^y)$, polarization $\sigma = \pm$ and frequency $\omega > 0$. Note that, contrary to the Minkowski case, this frequency is independent of the mass of Rindler quanta, since it does not satisfy any dispersion relation. The positive/negative energy solutions of the Dirac equation in Rindler spacetime take the form

$$\psi_{\mathbf{w}\sigma}^{(\omega)}(v, \mathbf{x}) = \frac{e^{i(-\omega v/a + k_\alpha x^\alpha)}}{(2\pi)^{\frac{3}{2}}} u_\sigma^{(\omega)}(u, \mathbf{w}), \quad \alpha = 1, 2, \tag{32}$$

where

$$u_+^{(\omega)}(u, \mathbf{w}) = N \begin{pmatrix} ilK_{i\omega/a-1/2}(ul) + mK_{i\omega/a+1/2}(ul) \\ -(k^x + ik^y)K_{i\omega/a+1/2}(ul) \\ ilK_{i\omega/a-1/2}(ul) - mK_{i\omega/a+1/2}(ul) \\ -(k^x + ik^y)K_{i\omega/a+1/2}(ul) \end{pmatrix}, \tag{33}$$

$$u_-^{(\omega)}(u, \mathbf{w}) = N \begin{pmatrix} (k^x - ik^y)K_{i\omega/a+1/2}(ul) \\ ilK_{i\omega/a-1/2}(ul) + mK_{i\omega/a+1/2}(ul) \\ -(k^x - ik^y)K_{i\omega/a+1/2}(ul) \\ -ilK_{i\omega/a-1/2}(ul) + mK_{i\omega/a+1/2}(ul) \end{pmatrix} \tag{34}$$

with $N \equiv \sqrt{\frac{a \cosh(\pi\omega/a)}{\pi l}}$ and $l \equiv \sqrt{(k^x)^2 + (k^y)^2 + m^2}$.

Let us now sketch the procedure to evaluate the decay rate for the process (iv) in Eq. (30a); similar considerations can be straightforwardly generalized to the channels (v) and (vi). First, by using the field expansion (31) and rotating the neutrino state and field according to Eqs. (7) and (16), the transition amplitude can be expressed as [21]

$$\begin{aligned} \mathcal{A}^{(iv)} &\equiv \langle n | \otimes \langle \nu_e | \hat{S}_I | e^- \rangle \otimes | p \rangle \\ &= \frac{G_F}{(2\pi)^2} \sum_{j=1}^3 |U_{ej}|^2 \mathcal{J}_{\sigma_\nu \sigma_e}^{(j)}(\omega_\nu, \omega_e) \end{aligned} \tag{35}$$

where

$$\mathcal{J}_{\sigma_\nu \sigma_e}^{(j)}(\omega_\nu, \omega_e) = \delta(\omega_e - \omega_\nu - \Delta m) \bar{u}_{\sigma_\nu}^{(\omega_\nu)} \gamma^0 u_{\sigma_e}^{(\omega_e)}, \tag{36}$$

and we have assumed equal frequencies, transverse momenta and polarizations for neutrino states with definite mass. As a next step, it should be considered that, due to the Unruh effect, the probability that the proton absorbs a lepton of frequency ω from the thermal bath is given by the Fermi-Dirac distribution

$$n_F(\omega) = (e^{\omega/T_U} + 1)^{-1}, \tag{37}$$

(similarly, the probability to emit a particle to the bath reads $\tilde{n}_F(\omega) = 1 - n_F(\omega)$), where the temperature T_U is defined as in Eq. (1). Calculations are finalized by multiplying the above thermal factors by the squared modulus of the amplitude (35), then integrating over the Rindler momentum-space volume $dV_{k,R} = d\omega_\nu d\omega_e d^2k_\nu d^2k_e$ and summing over the leptons' polarizations σ_ν, σ_e (see Ref. [21] for explicit calculations).

If we now follow the above recipe for all three processes in Eqs. (30) and add up the resulting expressions for the decay rates, we obtain

$$\begin{aligned} \Gamma^{(iv)} + \Gamma^{(v)} + \Gamma^{(vi)} &= |U_{e1}|^4 \tilde{\Gamma}_1 + |U_{e2}|^4 \tilde{\Gamma}_2 + |U_{e3}|^4 \tilde{\Gamma}_3 \\ &+ \left(|U_{e1}|^2 |U_{e2}|^2 \tilde{\Gamma}_{12} + |U_{e1}|^2 |U_{e3}|^2 \tilde{\Gamma}_{13} \right. \\ &\left. + |U_{e2}|^2 |U_{e3}|^2 \tilde{\Gamma}_{23} + \text{c.c.} \right), \end{aligned} \tag{38}$$

where

$$\begin{aligned} \tilde{\Gamma}_j &\equiv \frac{2G_F^2}{a^2 \pi^7 e^{\pi \Delta m/a}} \int d\omega \left\{ \int d^2k_\nu l_{\nu j} \left| K_{i(\omega-\Delta m)/a+1/2} \left(\frac{l_{\nu j}}{a} \right) \right|^2 \right. \\ &\times \int d^2k_e l_e \left| K_{i\omega/a+1/2} \left(\frac{l_e}{a} \right) \right|^2 \\ &+ m_{\nu_j} m_e \text{Re} \left[\int d^2k_\nu K_{i(\omega-\Delta m)/a-1/2}^2 \left(\frac{l_{\nu j}}{a} \right) \right. \\ &\left. \times \int d^2k_e K_{i\omega/a+1/2}^2 \left(\frac{l_e}{a} \right) \right] \left. \right\}, \end{aligned} \tag{39}$$

$$\begin{aligned} \tilde{\Gamma}_{jk} &= \frac{G_F^2}{a^2 \pi^7 e^{\pi \Delta m/a}} \int d\omega \left\{ \int d^2k_e l_e \left| K_{i\omega/a+1/2} \left(\frac{l_e}{a} \right) \right|^2 \right. \\ &\times \int \frac{d^2k_\nu}{\sqrt{l_{\nu j} l_{\nu k}}} \left((k_\nu^x)^2 + (k_\nu^y)^2 + m_{\nu_j} m_{\nu_k} + l_{\nu_j} l_{\nu_k} \right) \\ &\times \left[K_{i(\omega-\Delta m)/a+1/2} \left(\frac{l_{\nu j}}{a} \right) K_{i(\omega-\Delta m)/a-1/2} \left(\frac{l_{\nu k}}{a} \right) \right] \\ &+ \frac{m_e}{\sqrt{l_{\nu j} l_{\nu k}}} \int d^2k_e \int d^2k_\nu (l_{\nu j} m_{\nu_k} + l_{\nu k} m_{\nu_j}) \\ &\times \left[K_{i\omega/a+1/2}^2 \left(\frac{l_e}{a} \right) K_{i(\omega-\Delta m)/a-1/2}^2 \left(\frac{l_{\nu j}}{a} \right) \right. \\ &\left. \times K_{i(\omega-\Delta m)/a-1/2} \left(\frac{l_{\nu k}}{a} \right) \right] \left. \right\}. \end{aligned} \tag{40}$$

Again, for vanishing mixing angles and/or small mass differences, we recover the result of Ref. [19] [see the discussion before Eq. (21)].

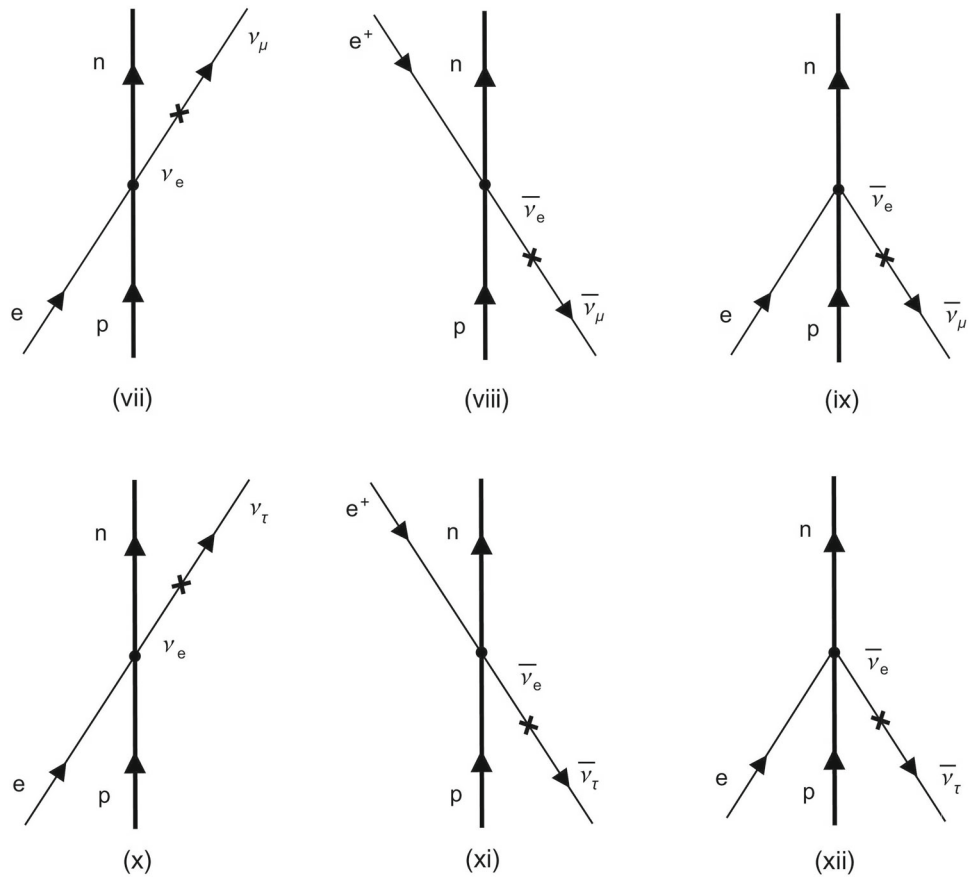
In Sect. 3 we have seen that, in addition to the flavor-conserving process (2), the decay channels (22) and (23) also have a non-vanishing probability because of the asymptotic occurrence of neutrino oscillations. Guided by the principle of General Covariance of QFT, we thus search for the corresponding processes to be considered in the comoving frame. To this aim, we propose the following interactions as candidates for the non-inertial counterparts of (ii) and (iii) (see Fig. 4)

$$\text{(vii)} \quad p + e^- \rightarrow n + \nu_\mu \tag{41a}$$

$$\text{(viii)} \quad p + \bar{\nu}_\mu \rightarrow n + e^+ \tag{41b}$$

$$\text{(ix)} \quad p + e^- + \bar{\nu}_\mu \rightarrow n, \tag{41c}$$

Fig. 4 Inverse β -decay in the comoving frame in the presence of flavor oscillations (time flows in the vertical direction). The crosses denote the occurrence of the oscillation



and

$$(x) \quad p + e^- \rightarrow n + \nu_\tau, \tag{42a}$$

$$(xi) \quad p + \bar{\nu}_\tau \rightarrow n + e^+, \tag{42b}$$

$$(xii) \quad p + e^- + \bar{\nu}_\tau \rightarrow n. \tag{42c}$$

Note that, whilst the processes (vii) and (x) are of the same type as (ii) and (iii) in Eqs. (22) and (23), since they only provide for the oscillation of the emitted (electron) neutrino, the remaining channels in Eqs. (41) and (42) bring new physics into play with respect to (v) and (vi) in (30), as they require that a muon- or tau-antineutrino in the Unruh thermal bath oscillates into an electron-antineutrino before being absorbed by the proton (we remind that, at tree-level, the lepton charge must be conserved in the interaction vertices).

For the above processes, the decay rates can be evaluated in the same way as in Eq. (35), yielding

$$\begin{aligned} &\Gamma^{(vii)} + \Gamma^{(viii)} + \Gamma^{(ix)} \\ &= |U_{\mu 1}|^2 |U_{e 1}|^2 \tilde{\Gamma}_1 + |U_{\mu 2}|^2 |U_{e 2}|^2 \tilde{\Gamma}_2 \\ &\quad + |U_{\mu 3}|^2 |U_{e 3}|^2 \tilde{\Gamma}_3 + \left(U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* \tilde{\Gamma}_{12} \right. \\ &\quad \left. + U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \tilde{\Gamma}_{13} + U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^* \tilde{\Gamma}_{23} + \text{c.c.} \right), \end{aligned} \tag{43}$$

$$\begin{aligned} &\Gamma^{(x)} + \Gamma^{(xi)} + \Gamma^{(xii)} \\ &= |U_{\tau 1}|^2 |U_{e 1}|^2 \tilde{\Gamma}_1 + |U_{\tau 2}|^2 |U_{e 2}|^2 \tilde{\Gamma}_2 \\ &\quad + |U_{\tau 3}|^2 |U_{e 3}|^2 \tilde{\Gamma}_3 + \left(U_{\tau 1}^* U_{e 1} U_{\tau 2} U_{e 2}^* \tilde{\Gamma}_{12} \right. \\ &\quad \left. + U_{\tau 1}^* U_{e 1} U_{\tau 3} U_{e 3}^* \tilde{\Gamma}_{13} + U_{\tau 2}^* U_{e 2} U_{\tau 3} U_{e 3}^* \tilde{\Gamma}_{23} + \text{c.c.} \right), \end{aligned} \tag{44}$$

where $\tilde{\Gamma}_j$ and $\tilde{\Gamma}_{jk}$ have been defined in Eqs. (39) and (40), respectively. As a consequence, the total decay rate in the comoving system takes the form

$$\Gamma^{\text{com}} \equiv \sum_{s=\text{iv}}^{\text{xii}} \Gamma^{(s)} = |U_{e 1}|^2 \tilde{\Gamma}_1 + |U_{e 2}|^2 \tilde{\Gamma}_2 + |U_{e 3}|^2 \tilde{\Gamma}_3. \tag{45}$$

Now one can prove that the following equalities hold:

$$\Gamma_j = \tilde{\Gamma}_j, \tag{46}$$

and

$$\Gamma_{jk} = \tilde{\Gamma}_{jk}, \tag{47}$$

with Eq. (47) being valid at least in the approximation of small mass differences.⁴ Hence, by use of the above relations,

⁴ Formally, one can prove each of the three equalities in Eq. (47) by employing the same assumptions as in Ref. [22].

it follows that the decay rates for each neutrino flavor in the laboratory and comoving frames are in agreement with each other, i.e.

$$\Gamma^{(i)} = \Gamma^{(iv)} + \Gamma^{(v)} + \Gamma^{(vi)}, \tag{48}$$

$$\Gamma^{(ii)} = \Gamma^{(vii)} + \Gamma^{(viii)} + \Gamma^{(ix)}, \tag{49}$$

$$\Gamma^{(iii)} = \Gamma^{(x)} + \Gamma^{(xi)} + \Gamma^{(xii)}. \tag{50}$$

Equations (48)–(50) naturally imply that

$$\Gamma^{\text{lab}} = \Gamma^{\text{com}}. \tag{51}$$

This substantiates the result that neutrino mixing is consistent with the General Covariance of QFT, since the (scalar) decay rate is independent of the reference frame. In passing, we mention that the opposite outcome is exhibited in Ref. [20], where the equality between the two rates is claimed to be spoiled when taking into account flavor mixing. From comparison with our framework, it is clear that such a contradiction originates from the fact that the authors of Ref. [20] assume flavor neutrinos as fundamental objects in the laboratory frame, while they choose the mass representation in the comoving system in order not to spoil the Kubo-Martin-Schwinger condition (KMS) for the vacuum. They also argue that picking flavor states in both frames would solve the contradiction, as explicitly shown above.

5 CP violation in neutrino oscillations in Unruh radiation

It is well-known that neutrino oscillations in the three-flavor description can exhibit non-trivial CP-violation effects [38–44]. Quantitatively speaking, the size of these effects is controlled by the Jarlskog invariant J , which is a phase-convention-independent measure of CP violation in the Standard Model. Despite being originally introduced in the context of quark mixing [45], the definition of the Jarlskog invariant can be straightforwardly rephrased in terms of the PMNS matrix (8) for neutrinos as follows

$$\text{Im} \left[U_{\delta i} U_{\gamma i}^* U_{\delta j}^* U_{\gamma j} \right] \equiv J \sum_{\lambda, k} \varepsilon_{\delta\gamma\lambda} \varepsilon_{ijk}, \tag{52}$$

where $\delta, \gamma, \lambda = \{e, \mu, \tau\}$ and $i, j, k = \{1, 2, 3\}$. By using the parameterization (8) for the PMNS matrix, J can be explicitly written as

$$J = c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13} \sin \delta \\ = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \cos \theta_{13} \sin 2\theta_{13} \sin \delta. \tag{53}$$

Clearly, since physical quantities cannot depend on the choice of the parameterization of the PMNS matrix, all CP-violating observables must depend on the invariant J solely [45]. In passing, we mention that a useful way of representing

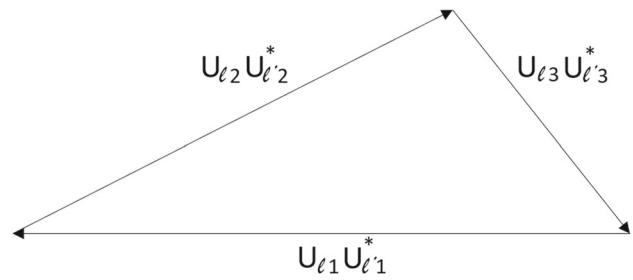


Fig. 5 Schematic representation of a unitarity triangle

CP violation are the unitarity triangles (see Fig. 5). These are constructed exploiting the unitarity of the matrix (8), which implies that different rows or columns are orthogonal to each other. For instance, we have

$$\sum_{j=1}^3 U_{\ell j} U_{\ell' j}^* = 0, \quad \ell \neq \ell'. \tag{54}$$

The above relation can be represented as a unitarity triangle in the complex plane by drawing arrows corresponding to the numbers $U_{\ell j} U_{\ell' j}^*$ etc., and arranging the tip of each arrow in such a way it coincides with the base of another (the orientation of these triangles has no physical meaning since, under rephasing transformations, they simply rotate in the complex plane). One can construct different unitarity triangles, depending on which row or column is considered. In spite of this, their area is invariant, being one-half the Jarlskog invariant introduced in Eq. (52). Hence, it provides a measure of CP violation. It goes without saying that, if all the elements of the PMNS matrix are real (i.e. if there is not CP asymmetry), the unitarity triangles collapse in a line of vanishing area, as expected from the condition $J = 0$.

Starting from the above considerations, let S_{weak} be the scattering matrix of a given charged-current weak interaction. In order to study CP-violation effects in a neutrino flavor-changing process, we assume that a neutrino of a certain flavor (e.g. an electron neutrino) is produced in the final state, so that S_{weak} will depend on Dirac bilinears containing the (electron) neutrino field ψ_{ν_e} (we may refer, for example, to the inverse β -decay discussed above, as well as to any other similar process involving a neutrino in the final state). By describing asymptotic neutrinos by means of flavor states, the probability that, after being emitted, the neutrino is detected with a different flavor (for instance, as a muon neutrino) can be derived from the following transition amplitude:

$$\mathcal{A}_{\nu_e, \nu_\mu} =_{out} \langle \nu_\mu, \dots | S_{weak} (\bar{\psi}_{\nu_e} \dots) | \dots \rangle_{in}, \tag{55}$$

where the first (second) subscript in the l.h.s. refers to the neutrino field (state) appearing in the S -matrix in the r.h.s., and the dots must be filled with the appropriate fields and particles involved in the considered interaction.

If we now implement the mixing transformations (7) and (16) on neutrino state and field, Eq. (55) can be cast in the form

$$\mathcal{A}_{\nu_e, \nu_\mu} = \sum_{j=1}^3 U_{\mu j}^* U_{ej} \mathcal{A}_j, \tag{56}$$

where we have used the shorthand notation

$$\mathcal{A}_j \equiv \text{out} \langle \nu_j, \dots | S_{\text{weak}}(\bar{\psi}_{\nu_j} \dots) | \dots \rangle_{\text{in}}. \tag{57}$$

Thus, by exploiting the definition (17) and squaring both sides of Eq. (56), we obtain

$$\begin{aligned} \Gamma_{\nu_e, \nu_\mu} &\sim |\mathcal{A}_{\nu_e, \nu_\mu}|^2 \\ &= |U_{\mu 1}|^2 |U_{e 1}|^2 |\mathcal{A}_1|^2 + |U_{\mu 2}|^2 |U_{e 2}|^2 |\mathcal{A}_2|^2 + |U_{\mu 3}|^2 |U_{e 3}|^2 |\mathcal{A}_3|^2 \\ &\quad + (U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* \mathcal{A}_1 \mathcal{A}_2^* + U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \mathcal{A}_1 \mathcal{A}_3^* \\ &\quad + U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^* \mathcal{A}_2 \mathcal{A}_3^* + \text{c.c.}). \end{aligned} \tag{58}$$

where, in order not to burden the notation, we have omitted the sum over polarizations and the integration over momenta of the emitted particles in the r.h.s.

To quantify CP asymmetry in the above interaction, we now assume that particles swap places with their antiparticles while viewed in a mirror. By computing the decay rate $\Gamma_{\bar{\nu}_e, \bar{\nu}_\mu}$ for the ensuing process, we finally arrive at

$$\begin{aligned} A_{CP}^{(e, \mu)} &\equiv \Gamma_{\nu_e, \nu_\mu} - \Gamma_{\bar{\nu}_e, \bar{\nu}_\mu} \\ &= 4J \left\{ -\text{Im} [\mathcal{A}_1 \mathcal{A}_2^*] + \text{Im} [\mathcal{A}_1 \mathcal{A}_3^*] - \text{Im} [\mathcal{A}_2 \mathcal{A}_3^*] \right\}, \end{aligned} \tag{59}$$

which is indeed non-vanishing and does not depend on the specific parameterization of the mixing matrix, as it should be. It is worth noting that, if we calculate the same quantity for the transition between the electron- and tau-neutrino flavors, from Eq. (52) we obtain

$$\begin{aligned} A_{CP}^{(e, \tau)} &\equiv \Gamma_{\nu_e, \nu_\tau} - \Gamma_{\bar{\nu}_e, \bar{\nu}_\tau} \\ &= 4J \left\{ \text{Im} [\mathcal{A}_1 \mathcal{A}_2^*] - \text{Im} [\mathcal{A}_1 \mathcal{A}_3^*] + \text{Im} [\mathcal{A}_2 \mathcal{A}_3^*] \right\}. \end{aligned} \tag{60}$$

By adding up $A_{CP}^{(e, \mu)}$ and $A_{CP}^{(e, \tau)}$, it follows that

$$A_{CP}^{(e, e)} + A_{CP}^{(e, \mu)} + A_{CP}^{(e, \tau)} = 0, \tag{61}$$

where we have exploited the fact that a CP asymmetry can be measured only in transitions between different flavors, i.e. $A_{CP}^{(e, e)} = 0$.

On the other hand, if one adopts the point of view of Ref. [23] and assumes the mass representation as the fundamental one, the above CP-violation feature does not emerge at all. Indeed, by straightforward calculations, one has

$$\begin{aligned} A_{CP}^{(e, j)} &= \Gamma_{\nu_e, \nu_j} - \Gamma_{\bar{\nu}_e, \bar{\nu}_j} \\ &= |U_{ej}|^2 |\mathcal{A}_j|^2 - |U_{ej}^*|^2 |\mathcal{A}_j|^2 = 0, \quad j = 1, 2, 3, \end{aligned} \tag{62}$$

which clearly shows that mass states are inconsistent with the picture of CP violation in the neutrino sector. At the same time, however, it is immediate to see that

$$\sum_{\ell=e, \mu, \tau} A_{CP}^{(e, \ell)} = \sum_{j=1}^3 A_{CP}^{(e, j)} = 0, \tag{63}$$

as it might be expected from Eq. (29) and the related discussion.

Finally, with reference to the inverse β -decay analyzed above and, in particular, to the processes (viii), (ix), (xi) and (xii) in the comoving frame, we note that the occurrence of CP violation manifests itself in an asymmetry between the thermal baths experienced by the accelerated proton and antiproton, respectively: this originates from the different oscillating behavior between neutrinos and anti-neutrinos in the Unruh radiation. Clearly, this is a novel feature of the Unruh effect which does not appear in the previous literature on the inverse β -decay with mixed neutrinos [21–23], since it is peculiar of the three-flavor description.

6 Discussion and conclusions

The inverse β -decay of uniformly accelerated protons has been investigated in the context of three-flavor neutrino mixing and oscillations. By assuming neutrinos to be Dirac particles and working within the S -matrix framework, we have shown that the decay rates in the laboratory and comoving frames agree with each other, provided that the asymptotic behavior of neutrinos is described by means of flavor (rather than mass) eigenstates. It is worth noting that such an analysis would be a rather straightforward generalization of the two-flavor treatment of Ref. [22], if it were not for the presence of the Dirac phase in the PMNS matrix. As well known, such phase induces non-trivial CP-violation effects in neutrino oscillations, which determine an asymmetry between the Unruh radiation detected by the accelerated proton and antiproton, respectively. In this connection, we have proved that the mass representation is inconsistent with the picture of CP asymmetry in the neutrino sector, as it leads to an identically vanishing expression for the quantity A_{CP} [see Eq. (62)]. On the other hand, by adopting flavor asymptotic states, A_{CP} turns out to be proportional to the Jarlskog rephasing-invariant, as one would expect for any physical observable which quantifies CP-violation.

Despite the obtained equality between the inertial and comoving decay rates, we emphasize that the analysis carried out here may not represent the end of the story, since it holds

in the approximation of small neutrino mass differences [see the discussion after Eq. (47)]. The question inevitably arises as to how accommodate next-to-leading order corrections without affecting the internal consistency of the formalism. In this regard, we envisage that some new features might come into play, as for example the necessity of a full-fledged QFT treatment of neutrino mixing instead of the Pontecorvo quantum mechanical one [28], or the possibility to violate the thermality of the Unruh effect when considering mixed fields [46–48]. Note that similar non-thermal distortions of the Unruh-Hawking spectrum have been recently highlighted also in other contexts, such as the emission on the background of a quantum collapsing null shell [49, 50], the polymer (loop) quantization for the calculation of the two-point function along Rindler trajectories [51], the Casimir effect between uniformly accelerated atoms [52] and the Generalized Uncertainty Principle framework [53–55].

As remarked above, in our study we have considered the case of Dirac neutrinos. However, the question about the very nature of neutrinos—Dirac or Majorana—is still open. As it is well-known [56, 57], oscillation experiments do not allow us to discriminate between these two alternatives, the only feasible test being the neutrinoless double β -decay [58–61]. Also from the point of view of the Unruh effect, it has been shown that there is no difference between the employment of Dirac and Majorana fermion fields in the computation of the (accelerated) thermal distribution [62]. Thus, in light of the above considerations, we expect the overall validity of our analysis to be unaffected by the nature of neutrinos, although some formal differences may arise when working with Majorana fields, due to the presence of two additional phases in the mixing matrix. In particular, concerning General Covariance, we envisage that the equality between the decay rates in the two frames must hold true, owing to the fact that the mixing matrix is still unitary. Likewise, one can repeat the same reasoning on CP violation as in Sect. 5 and come up with the same conclusion, since the Majorana phases do not contribute to the Jarlskog invariant [36].

Apart from its intrinsic theoretical interest, we remark that a deeper understanding of the very nature of asymptotic neutrino states may also be relevant from the experimental point of view. In Ref. [29], indeed, it has been shown that the spectrum of the Tritium β -decay near the end point energy is sensitive to whether neutrinos interact as massive or flavor eigenstates. Similar considerations are valid for the neutrino capture by Tritium too. In light of this, it is reasonable to expect that accurate measurements from such current experiments as KATRIN (which aims to appoint an upper limit to the electron antineutrino mass by examining the spectrum of electrons emitted from the Tritium β -decay) [63] and PTOLEMY (that is projected to detect the cosmic neutrino background) [64] might provide important pieces of information in the problem at hand.

Finally, we highlight that the above study is closely related to the issue of non-inertial/gravitational effects on the oscillation probability. A preliminary investigation of this problem has been proposed for the case of accelerated systems [65, 66], in curved spacetime [67–71], in astrophysical and cosmological regimes [72–78], in extended theories of gravity [79, 80] and in stochastic model for spacetime foam [81–83]. Worthy of attention may be also quantum-gravity decoherence effects in oscillations [84, 85] and the entanglement among neutrinos and the other particles [86, 87] in decay processes. All of these issues are currently under active considerations.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.]

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