

Nonperturbative QCD thermodynamics in the external magnetic field

M. A. Andreichikov^a , Yu. A. Simonov

State Research Center Institute of Theoretical and Experimental Physics, Moscow 117218, Russia

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Abstract The thermodynamics of quarks and gluons strongly depends on the vacuum colormagnetic field, which grows with the temperature T , so that spatial string tension $\sigma_s = \text{const } g^4(T)T^2$. We investigate below what happens when one imposes in addition constant magnetic field and discover remarkable structure of the resulting thermodynamic potential.

1 Introduction

The problem of quark gluon thermodynamics in magnetic field is of the high interest in the modern physics, since heavy-ion experiments produce important data on the properties of resulting hadron yields and hadron interactions, which might be influenced by the strong magnetic fields (MF) created during the collision process [1–3]. For a recent review on the effects of MF see [4].

On the theoretical side the problem of MF in the quark gluon plasma (qgp) was studied in different aspects, e.g. in the NJL-type models [5] and in the holographic approach [6, 7]. Within the nonperturbative QCD the theory of qgp in MF was developed in [8, 9, 11], where the general form of the thermodynamic potentials was found in MF with zero or nonzero baryon density, summing over all Landau levels including LLL.

In this approach the only nonperturbative interaction, which was taken into account, reduced to the inclusion of Polyakov lines in the resulting expression for the pressure.

The resulting expressions for magnetic susceptibilities $\hat{\chi}_q(T)$, obtained in [8], were used in [9, 11] to compare with the lattice data from [12, 13], and a reasonable agreement was found for $\hat{\chi}_q(T)$ with different $q = u, d, s$, in [9, 11] as well as for the sum [11], however somewhat renormalised values of effective quark masses were used.

Recently in [14, 15] a new step in the development of the np QCD thermodynamics was made, where the colormagnetic confinement (CC) was included in the dynamics of the qgp. This interaction with the spatial string tension σ_s grows with temperature, $\sigma_s \sim g^4 T^2$ and is important in the whole region $T_c < T < 10$ GeV. A concise form of the final expression was found in [14, 15] in the case of an oscillatory type CC and an approximate one in the realistic case of the linear CC. The resulting behavior both in the SU(3) case, found in [16, 17], and in the qgp case [14, 15] agrees well with the corresponding lattice data.

It is the purpose of the present paper to extend our previous analysis of the qgp thermodynamics in MF, done in [8, 9, 11], including the dynamics of CC with the explicit form of the magnetic screening mass m_D , generated by CC.

As will be seen, we propose a simple generalization of the results [8, 9, 11], where the CC produces the mass M_D , entering the final expressions in the combination $\sqrt{m_q^2 + m_D^2} \equiv \bar{M}$ instead of m_q . We check the limiting cases and compare the result with lattice data.

In the next section the general analysis of the MF effects in thermodynamics is explained, in Sect. 3 the magnetic susceptibility is defined, in Sect. 4 the results are compared to the lattice data.

2 General structure of the pressure with and without magnetic field

Field correlator method (FCM) [18, 19], based on the path integral formalism and Feynman–Fock–Schwinger proper time formalism (FSF) [20, 22], provides a relativistic- and gauge-invariant approach to QCD. Both perturbative and nonperturbative (np) gauge interactions between quarks could be written in a form of expansion in Wilson loops. A detailed review of the FCM technique and background field theory applied to the quark-gluon thermodynamics could be

^ae-mail: andreichicov@mail.ru

found in [21,23]. Here we emphasize that two basic interactions – colorelectric (CE) and colormagnetic (CM) are defined by two pairs of CE (D^E, D_1^E) and CM (D^H, D_1^H) gauge-invariant correlators obtained through the averaging over the stochastic background color field in FCM

$$\begin{aligned} & \frac{g^2}{N_c} \langle tr E_i(x) \Phi E_j(y) \Phi \rangle \\ &= \delta_{ij} \left(D^E(u) + D_1^E(u) + u_4^2 \frac{\partial D_1^E}{\partial u_4^2} \right) - u_i u_j \frac{\partial^2 D_1^E}{\partial u_4^2} \end{aligned} \tag{1}$$

$$\begin{aligned} & \frac{g^2}{N_c} \langle tr H_i(x) \Phi H_j(y) \Phi \rangle \\ &= \delta_{ij} \left(D^H(u) + D_1^H(u) + \mathbf{u}^2 \frac{\partial D_1^H}{\partial \mathbf{u}^2} \right) - u_i u_j \frac{\partial^2 D_1^H}{\partial \mathbf{u}^2} \end{aligned} \tag{2}$$

where $u = |x - y|$, the parallel transporter is $\Phi = e^{ig \int_x^y A_\mu dx_\mu}$, and its contour is parametrized with a straight line.

In the confinement region, $T < T_c$, spatial CE string tension is defined by scalar D^E correlator $\sigma_E = \frac{1}{2} \int D^E(u) d^2u$. Since the D^E vanishes beyond the deconfinement transition region, the CE interaction is provided only by the D_1^E correlator for $T > T_c$, where the np part of D_1^E produces Polyakov loop $L = e^{-\frac{V_1(\infty, T)}{2T}}$, via the np saturating potential $V_1(r, T)$, and perturbative part yields color Coulomb potential. Despite the vanishing of the D^E , the scalar part of the CM interaction D^H still holds in the deconfinement region, yielding the spatial string tension $\sigma_S(T) = \frac{1}{2} \int D^H(z) d^2z$. As it was shown in [16,24], the magnetic string tension grows with temperature as $\sigma_S(T) = cg^4(T)T^2$, $c = 0.566$ [16] and therefore is important in the whole region $T > T_c$.

Using T -dependent path integral formalism, one can express quark and gluon pressures via the Wilson loop series

$$P = 2N_c \int_0^\infty \frac{ds}{s} \sum_{i=1,2,\dots} (-1)^{n+1} G^{(n)}(s), \tag{3}$$

where s is Schwinger proper time and $G^{(n)}(s)$ are the winding (Matsubara) path integrals

$$G^{(n)}(s) = \int (Dz) e^{-K - sm_q^2} \langle tr_a W(C_n), \rangle \tag{4}$$

with $K = \frac{1}{4} \int_0^s \left(\frac{dz_\mu}{d\tau} \right)^2 d\tau$ is a kinetic kernel in the Schwinger proper time method, m_q is a current quark mass, and $W(C_n)$ is the Wilson loop defined by the quark path C_n . It is convenient to make a redefinition of the kinetic kernel splitting it into spatial and temporal components $K = K_4 + K_{3d} = \frac{1}{4} \int_0^s \left(\frac{dz_4}{d\tau} \right)^2 d\tau + \frac{1}{4} \int_0^s \left(\frac{d\mathbf{z}}{d\tau} \right)^2 d\tau$. According to [21] CE and CM fields in $T > T_c$ region correlate very weakly, since the

field correlator $\langle E_i(x) \Phi H_k \Phi \rangle \simeq 0$ and therefore temporal and spatial Wilson loops can be factorized

$$\langle tr_a W(C_n) \rangle = L^{(n)} \langle W_3 \rangle, \tag{5}$$

where, after the applying FCM background averaging procedure, the temporal projection $L^{(n)}$ defines a Polyakov loop and the dynamics of the spatial projection $\langle W_3 \rangle$ is defined by the CM string tension σ_s . As a result, one can integrate out the z_4 part of the path integral in (4) and write the result as

$$\begin{aligned} G^{(n)}(s) &= G_4^{(n)}(s) S_3(s); \quad G^{(4)}(s) \\ &= \int (Dz_4) e^{-K_4 - sm_q^2} L^{(n)} \\ &= \frac{1}{2\sqrt{4\pi s}} e^{-\frac{n^2}{4T^2s} - sm_q^2} L^{(n)}. \end{aligned} \tag{6}$$

Here $S_3(s)$ is a 3d proper time Green’s function with a closed path, which can be expanded in a series of eigenstates of the quark-antiquark pair in FSF formalism with the CM confinement interaction between quarks [15].

$$\begin{aligned} S_3(s) &= \int (D\mathbf{z}) e^{-K_{3d}(s)} \langle W_3 \rangle \\ &= \frac{1}{\sqrt{\pi s}} \sum_{v=0,1,\dots} |\psi_v(0)|^2 e^{-m_v^2 s}; \quad m_v^2 = 0 \\ &= m_D^2 = 4\sigma_S(T), \end{aligned} \tag{7}$$

where the ground state energy for the 3d spatial motion is defined by a color-magnetic Debye mass m_D^2 . As it was argued in [15,16], in the case of linear potential for the CMC and in the absence of the external MF, one obtains for $S_3(s)$ an approximate form

$$S_3^{lin} \simeq \frac{1}{(4\pi s)^{3/2}} e^{-\frac{m_D^2}{4}s}, \tag{8}$$

and the pressure can be written as

$$\frac{P_q^f}{T^4} = \frac{N_c}{4\pi^2} \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^4} L^{(n)} \Phi_n(T), \tag{9}$$

where

$$\Phi_n(T) = \frac{8n^2 \bar{M}^2}{T^2} K_2 \left(\frac{\bar{M}' n}{T} \right); \quad \bar{M}' = \sqrt{m_q^2 + \frac{m_D^2}{4}} \tag{10}$$

One can see that the Debye mass m_D growing with T provides an effective damping of the quark pressure. Due to this effect the high- T limit of P_q never tends to Stefan–Boltzmann value. The same behaviour was observed in lattice calculations [10] where the difference of $\frac{P}{T^4}$ with Stefan–Boltzmann limit is $\sim 25\%$ at $T \geq 500$ MeV. In the presence of the external magnetic field (MF) $\mathbf{B}||z$, the discrete spectrum of the spatial Green’s function (7) modifies [8] as

$$m_v^2 \rightarrow (\varepsilon_{n\perp}^{\bar{\sigma}})^2 = |e_q B|(2n_{\perp} + 1 - \bar{\sigma}) + \frac{m_D^2}{4}; \quad \bar{\sigma} = \frac{e_q}{|e_q|} \sigma_z, \quad (11)$$

and the summation over eigenstates ν is replaced by the summation over Landau levels and spin projections $\sum_{\nu} \rightarrow \sum_{n_{\perp}, \sigma}$. An influence of the MF in $|eB| \geq m_D^2 \simeq 0.3 \text{ GeV}^2$ (Fig. 1) region dominates over the CMC interaction for quarks. It is sufficient to assume that the quark spectrum comprises Landau levels and self-energy quark mass m_D^2 . However, the detailed behaviour in $0 < |eB| < m_D^2 \sim g^2(T)T^2$ region in $B - T$ plane requires to solve the spectral problem in (7) with both CM and MF interactions. One should also take into account an additional multiplier proportional to eB and s due to the $|\psi_{\nu}|^2$ normalization in the presence of the MF during the phase space modification $V_3 \frac{d^3 p}{(2\pi)^3} \rightarrow \frac{dp_z}{2\pi} \frac{|e_q B|}{2\pi} V^2$. As a result, one can rewrite (9) as

$$\frac{P_q^f}{T^4} = \frac{N_c |e_q B| T}{\pi^2} \sum_{n_{\perp}, \sigma} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \times L^{(n)} \sqrt{(\varepsilon_{n\perp}^{\bar{\sigma}})^2 + m_q^2} K_1 \left(\frac{n}{T} \sqrt{(\varepsilon_{n\perp}^{\bar{\sigma}})^2 + m_q^2} \right) \quad (12)$$

The total quark pressure is defined as a sum over flavors $P_q = \sum_f P_q^f$. We have neglected above the influence of MF on the gluonic component of pressure, since in the deconfined region it is of perturbative origin only. As was shown in [8], the form (12) can be summed up over N_{\perp} and σ to obtain the following result

$$P_q^{(f)}(B) = \frac{N_c |e_q B| T}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} L^n \left\{ \bar{M} K_1 \left(\frac{n \bar{M}}{T} \right) + \frac{2T}{n} \frac{|e_q B| + \bar{M}^2}{|e_q B|} K_2 \left(\frac{n}{T} \sqrt{|e_q B| + \bar{M}^2} \right) - \frac{n |e_q B|}{12T} K_0 \left(\frac{n}{T} \sqrt{\bar{M}^2 + |e_q B|} \right) \right\}, \quad (13)$$

where $\bar{M}^2 = m_q^2 + \frac{m_D^2}{4} = m_q^2 + c\sigma_s$ and c in the definition of the Debye screening mass [15, 25] is a constant, $c \simeq 1$ for $T \rightarrow \infty$.

Note, that the first term in (13) is provided by the lowest Landau levels (LLL). For these levels it is known from analysis in [26], that the asymptotic quark energy values do not depend on eB and are equal to $\sqrt{\sigma}$ for small quark mass. This agrees with our values $\bar{M} = \sqrt{m_q^2 + \sigma_s} \approx \sqrt{4\sigma_s}$ and supports our expression (13) at least in the high $e_q B$ limit, $e_q B \gg \bar{M}$, when the second and the third term in (13) tend to zero. Hence one obtains in the limit $|e_q B| \gg \bar{M}, T$

$$P_q^{(f)}(B)|_{|e_q B| \rightarrow \infty} = \frac{N_c |e_q B| T}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} L^n \bar{M} K_1 \left(\frac{n \bar{M}}{T} \right). \quad (14)$$

Note, that the factor $|e_q B|$ appears due to the phase space factor in MF, discussed above..

Now we turn to the limit of small MF, $|e_q B| \ll \bar{M}, T$. One obtains from (13) the contribution of the second term only

$$P_q^{(f)}(B \rightarrow 0) = \frac{2N_c T^2 \bar{M}^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} L^n K_2 \left(\frac{n \bar{M}}{T} \right) + O((e_q B)^2). \quad (15)$$

One can compare (14) with (9), obtained in the case of zero MF, and insertion of (10) in (9) yields the same answer as in (15).

3 Magnetic susceptibility of the quark matter

Using general expression for the quark pressure (13), one can define a more convenient quantity, the magnetic susceptibilities $\hat{\chi}_q^{(n)}, \hat{\chi}_q^{(2)} \equiv \hat{\chi}_q$,

$$P_q^f(B, T) - P_q^f(0, T) = \frac{\hat{\chi}_q}{2} (e_q B)^2 + O((e_q B)^4). \quad (16)$$

To this end one expands the Mc Donald functions $K_n(\sqrt{n^2 + b^2})$ entering in (13) in powers of b , following [8], and one obtains

$$P_q^f(B, T) - P_q^f(0, T) = \frac{(e_q B)^2 N_c}{2\pi^2} \sum_{n=1}^{\infty} (-1)^{n+1} L^n f_n \quad (17)$$

$$f_n = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{n e_q B}{2T \bar{M}} \right)^k K_k \left(\frac{n \bar{M}}{T} \right) \times \left[\frac{1}{(k+1)(k+2)} - \frac{1}{6} \right]. \quad (18)$$

As a consequence, one has for $\hat{\chi}_q$

$$\hat{\chi}_q(T) = \frac{N_c}{3\pi^2} \sum_{n=1}^{\infty} (-1)^{n+1} L^n K_0 \left(\frac{n \bar{M}}{T} \right). \quad (19)$$

It is possible to sum up the series over n in (17), when one exploits the representation

$$K_0 \left(\frac{n \bar{M}}{T} \right) = \frac{1}{2} \int_0^{\infty} \frac{dx}{x} e^{-n \left(\frac{1}{x} + \frac{\bar{M}^2 x}{4T^2} \right)}. \quad (20)$$

As a result one obtains for the quadratic magnetic susceptibility (ms)

$$\hat{\chi}_q(T) = \frac{N_c}{3\pi^2} I_q, \quad I_q = \frac{1}{2} \int_0^{\infty} \frac{dx}{x} \frac{L e^{-\left(\frac{1}{x} + \frac{\bar{M}^2 x}{4T^2} \right)}}{1 + L e^{-\left(\frac{1}{x} + \frac{\bar{M}^2 x}{4T^2} \right)}}. \quad (21)$$

The ms in (21) is defined for a given quark flavor q , and the total m.s. for the quark ensemble, e.g. for 2 + 1 species of quarks can be written as

$$\hat{\chi}_q(T) = \sum_{q=u,d,s} \chi_q(T) \left(\frac{e_q}{e}\right)^2. \quad (22)$$

4 Results and discussions

We have presented above the thermodynamic theory of quarks in the magnetic field, when quarks are affected also by Polyakov line interaction and the CMC interaction, generalizing in this way our old results of [8,9,11], where the CMC part was absent.

We have included the CMC interaction in the energy eigenvalues $\varepsilon_{n_\perp}^\sigma$, Eq. (7) tentatively via the replacement (11). This substitute can be corroborated in the case of lowest Landau levels with $\bar{\sigma} = 1$, $n_\perp = 0$, where magnetic field does not enter, and the effective quark mass in subject to the CMC interaction only. In the general case one can expect possible interference of eB and CC terms, which can spoil the suggested replacement.

To make this first analysis more realistic, we have checked the limits of small and large values of eB . In the first case we have shown the correct correspondence with the $eB = 0$ result of [14,15], and in the second case of large eB , the leading linear in eB term is just LLL term, which is not influenced by magnetic fields, except for the phase space redefinition. These results enable us to proceed with the analysis and comparisons of obtained equations with lattice data.

We present below an analysis of the MF influence on the quark thermodynamics. It is interesting, that the basic expression for the pressure $P_q^{(f)}$ in (14) has the property, that at small $eB < (eB)_{crit}$, the dependence of $\Delta P(B, T)$ on eB is quadratic with a good accuracy, according to Eq. (17). At larger eB , $eB > (eB)_{crit}$, one has the linear dependence of $\Delta P(B, T)$ on eB , given in (15). Figure 1 illustrates this behaviour for three fixed temperatures $T = 0.15, 0.2, 0.4$ GeV. One can see from (14) and Fig. 1 that $(eB)_{crit} > \bar{M}$, and actually is around 0.5 GeV^2 for $T \simeq 0.2 \text{ GeV}$. We have computed analytically the difference $\Delta P(T) = P(T, eB) - P(T, 0)$ using Eq. (14) and compare with the lattice data from [27] for averaged u, d, s quark ensemble. One can see in Fig. 2 the normalized pressure $\Delta P(B, T)$ for $T > 0.135 \text{ GeV}$ and $eB = 0.2, 0.4 \text{ GeV}^2$ for the averaged quark ensemble of u, d, s quarks. At this point one should notice, that the calculations in FCM formalism were performed for the quark-gluon plasma in the infinite volume, which yields to the isotropic (thermodynamic) pressure distribution in the presence of the external MF. The effect of the pressure anisotropy observed on the lattice [27] is provided by the finite volume effects caused by the surface currents on the volume boundary, which are induced by the bulk magnetization of the quark-gluon plasma, as was discussed in [29,30]. Neglecting these effects we are comparing

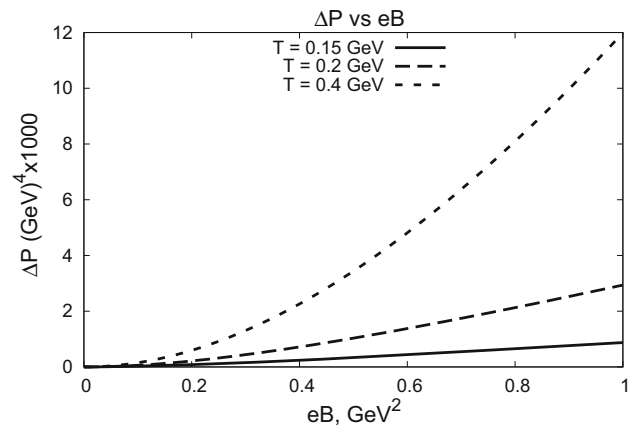


Fig. 1 The dependence of $\Delta P(T, B)$ on eB for fixed $T = 0.15, 0.2, 0.4 \text{ GeV}$. ΔP demonstrates quadratic behaviour $\sim (eB)^2$ for $eB < 0.5 \text{ GeV}^2$ and linear behaviour $\sim (eB)$ for $> 0.5 \text{ GeV}^2$ according to (13)

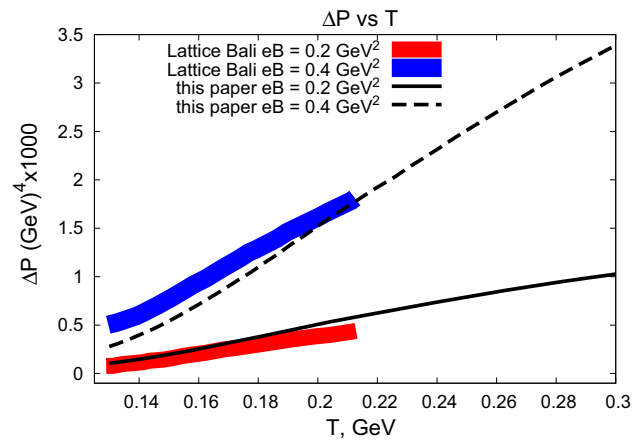


Fig. 2 The dependence of $\Delta P(T, B)$ on T for fixed values of $eB = 0.2, 0.4 \text{ GeV}^2$ in comparison with lattice data from [27]. For lattice results, the line thickness corresponds to the estimated error of the calculation

our results with the lattice scheme-independent longitudinal pressure from [27]. In Fig. 2 are shown our calculations of $\Delta P(B, T)$ for $eB = 0.2, 0.4 \text{ GeV}^2$ and $m_D^2 = 3\sigma_s$ using (14) are compared with the lattice data from [27]. We have used in (14) the Polyakov line $L(T)$ obtained in [28]. One can see a reasonable agreement within the accuracy of the lattice data, which supports the main structure of the theory used in the paper. Detailed analysis for larger intervals of eB and T and for specific quark flavours is possible within the approach and is planned for next publications.

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