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Dark Gauge $U(1)$ symmetry for an alternative left–right model

Corey Kownacki, Ernest Ma[a], Nicholas Pollard, Oleg Popov, Mohammadreza Zakeri

Department of Physics and Astronomy, University of California, Riverside, CA 92521, USA

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Abstract An alternative left–right model of quarks and leptons, where the $SU(2)_R$ lepton doublet $(\nu, l)_R$ is replaced with $(n, l)_R$ so that n_R is not the Dirac mass partner of ν_L, has been known since 1987. Previous versions assumed a global $U(1)_S$ symmetry to allow n to be identified as a dark-matter fermion. We propose here a gauge extension by the addition of extra fermions to render the model free of gauge anomalies, and just one singlet scalar to break $U(1)_S$. This results in two layers of dark matter, one hidden behind the other.

1 Introduction

The alternative left–right model [1] of 1987 was inspired by the E_6 decomposition to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry through an $SU(2)_R$, which does not have the conventional assignments of quarks and leptons. Instead of $(u, d)_R$ and $(\nu, l)_R$ as doublets under $SU(2)_R$, a new quark h and a new lepton n per family are added so that $(u, h)_R$ and $(n, e)_R$ are the $SU(2)_R$ doublets, and h_L, d_R, n_L, ν_R are singlets.

This structure allows for the absence of tree-level flavor-changing neutral currents (unavoidable in the conventional model), as well as the existence of dark matter. The key new ingredient is a $U(1)_S$ symmetry, which breaks together with $SU(2)_R$, such that a residual global S' symmetry remains for the stabilization of dark matter. Previously [2–4], this $U(1)_S$ was assumed to be global. We show in this paper how it may be promoted to a gauge symmetry. To accomplish this, new fermions are added to render the model free of gauge anomalies. The resulting theory has an automatic discrete Z_2 symmetry which is unbroken as well as the global S', which is now broken to Z_3. Hence dark matter has two components [5]. They are identified as one Dirac fermion (nontrivial under both Z_2 and Z_3) and one complex scalar (nontrivial under Z_3).

In Sect. 2 we make a digression to the historical perspective which motivated this study. In Sect. 3 our model is described, with a complete list of its particle content. In Sect. 4 the gauge sector is shown in detail. In Sect. 5 the fermions are discussed with details of how they obtain masses. In Sect. 6 we deal with the scalars and show how the desirable pattern of symetry breaking is obtained. In Sect. 7 we discuss the present phenomenological constraints on the new Z' bosons and would-be dark-matter candidates. In Sect. 8 we show an example of two viable dark-matter candidates, both in terms of relic abundance and direct detection. In Sect. 9 we conclude.

2 Motivation and historical perspective

This section is for those who are unfamiliar with, but interested in the historical perspective which motivated our study. In the beginning, the idea of an $SU(2)_L \times SU(2)_R$ electroweak extension of the standard model (SM), which is based only on $SU(2)_L$, was very attractive, because it restores left–right symmetry to the interactions of the quarks and leptons. In the conventional approach, $(u, d)_{iL}$ are $SU(2)_L$ doublets and $(u, d)_{jR}$ are $SU(2)_R$ doublets. To allow them to have masses, a scalar bidoublet

$$\Delta = \begin{pmatrix} \delta_1^0 & \delta_2^+ \\ \delta_1^- & \delta_2^0 \end{pmatrix}$$

is needed, so that $\bar{u}_{iL} u_{jR}$ couple to δ_1^0 and $\bar{d}_{iL} d_{jR}$ couple to δ_2^0, thereby obtaining masses from the vacuum expectation values of the two neutral scalars. However, because of the peculiarity of $SU(2)$ doublets, the bidoublet

$$\tilde{\Delta} = \sigma_2 \Delta^* \sigma_2 = \begin{pmatrix} \bar{\delta}_2^0 & -\delta_1^+ \\ -\delta_2^- & \bar{\delta}_1^0 \end{pmatrix}$$

[a] e-mail: ma@phyun8.ucr.edu

transforms identically as Δ. Hence δ_2 contributes to the u mass matrix, and δ_1 contributes to the d mass matrix. In other words, each quark sector gets its masses from two different Higgs particles. This means that flavor changing neutral currents (FCNC) are unavoidable at tree level through neutral Higgs exchange. This is a very strong constraint on the masses of these particles, of order 10–100 TeV. As such they are not likely to be observable at the Large Hadron Collider (LHC). On the general issue of FCNC, they are, of course, present in the SM, but only at the loop level, and they are known to be small and consistent with experimental data. In any extension of the SM, they may occur at tree level, and if so the scalar particles in question are required to be very heavy and out of reach of the LHC. It is thus a valid question to ask whether a model beyond the SM may be constructed with the absence of tree-level FCNC, so that it may have new scalars which are light enough to be discovered in addition to the SM Higgs boson of 125 GeV.

To distinguish $\tilde{\Delta}$ from Δ, an extra symmetry is needed. This is what happens in supersymmetry, but then the u quark mass matrix must be proportional to the d quark mass matrix, which disagrees with data. The solution to this conundrum was pointed out 30 years ago [1]. It was discovered in the context of superstring-inspired E_6 models, but applicable to the $SU(2)_L \times SU(2)_R$ case [2,3]. The idea is to add another quark h to each family which has the same charge as d, i.e. $-1/3$. Both h_L and h_R are singlets in the SM, but they are distinguished from d_L and d_R in their $SU(2)_R$ assignments, i.e.

$$(u, d)_L \sim (2, 1), \quad (u, h)_R \sim (1, 2),$$
$$d_R \sim (1, 1), \quad h_L \sim (1, 1).$$

To forbid the term $\bar{h}_L d_R$, a global $U(1)_S$ symmetry is added which also distinguishes Δ from $\tilde{\Delta}$. In this way, the d mass comes from an $SU(2)_L$ Higgs doublet, the h mass comes from an $SU(2)_R$ Higgs doublet, and the u mass comes from only δ_1^0 whereas δ_2^0 has no vacuum expectation value. Thus the model is guaranteed the absence of tree-level FCNC. It was realized a few years ago [2,3] that this extra $U(1)_S$ also serves the purpose of a dark symmetry, because even though it is broken, the combination $T_{3R} + S$ or $T_{3R} - S$ may remain unbroken and protects the condition $\langle \delta_2^0 \rangle = 0$. In other words, the symmetry which allows us to solve the FCNC conundrum has now been connected to that of dark matter. Contrast this with most models of dark matter, where the existence of the dark symmetry is completely ad hoc, and unrelated to any other symmetry of the original model. This we believe is a good motivation for studying alternative left–right models. The logical next step is to ask the question whether it is possible for this $U(1)_S$ to be gauged. What follows is a simple example of how it can be done and the resulting consequences.

Table 1 Particle content of proposed model of dark gauge $U(1)$ symmetry

Particles	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_X$	$U(1)_S$
$(u, d)_L$	3	2	1	1/6	0
$(u, h)_R$	3	1	2	1/6	$-1/2$
d_R	3	1	1	$-1/3$	0
h_L	3	1	1	$-1/3$	-1
$(\nu, l)_L$	1	2	1	$-1/2$	0
$(n, l)_R$	1	1	2	$-1/2$	1/2
ν_R	1	1	1	0	0
n_L	1	1	1	0	1
(ϕ_L^+, ϕ_L^0)	1	2	1	1/2	0
(ϕ_R^+, ϕ_R^0)	1	1	2	1/2	1/2
η	1	2	2	0	$-1/2$
ζ	1	1	1	0	1
$(\psi_1^0, \psi_1^-)_R$	1	1	2	$-1/2$	2
$(\psi_2^+, \psi_2^0)_R$	1	1	2	1/2	1
χ_R^+	1	1	1	1	$-3/2$
χ_R^-	1	1	1	-1	$-3/2$
χ_{1R}^0	1	1	1	0	$-1/2$
χ_{2R}^0	1	1	1	0	$-5/2$
σ	1	1	1	0	3

3 Model

The particle content of our model is given in Table 1.

Without $U(1)_S$ as a gauge symmetry, the model is free of anomalies without the addition of the ψ and χ fermions. In the presence of gauge $U(1)_S$, the additional anomaly-free conditions are all satisfied by the addition of the ψ and χ fermions. The $[SU(3)_C]^2 U(1)_S$ anomaly is canceled between $(u, h)_R$ and h_L; the $[SU(2)_L]^2 U(1)_S$ anomaly is zero because $(u, d)_L$ and $(\nu, l)_L$ do not transform under $U(1)_S$; the $[SU(2)_R]^2 U(1)_S$ and $[SU(2)_R]^2 U(1)_X$ anomalies are both canceled by summing over $(u, h)_R$, $(n, l)_R$, $(\psi_1^0, \psi_1^-)_R$, and $(\psi_2^+, \psi_2^0)_R$; the addition of $\chi_R^\pm$ renders the $[U(1)_X]^2 U(1)_S$, $U(1)_X [U(1)_S]^2$, $[U(1)_X]^3$, and $U(1)_X$ anomalies zero; and the further addition of χ_{1R}^0 and χ_{2R}^0 kills both the $[U(1)_S]^3$ and the $U(1)_S$ anomalies, i.e.

$$0 = 3\left[6(-1/2)^3 - 3(-1)^3 + 2(1/2)^3 - (1)^3\right] + 2(2)^3 + 2(1)^3 + 2(-3/2)^3 + (-1/2)^3 + (-5/2)^3, \tag{1}$$

$$0 = 3\left[6(-1/2) - 3(-1) + 2(1/2) - (1)\right] + 2(2) + 2(1) + 2(-3/2) + (-1/2) + (-5/2). \tag{2}$$

The scalar $SU(2)_L \times SU(2)_R$ bidoublet is given by

$$\eta = \begin{pmatrix} \eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0 \end{pmatrix}, \tag{3}$$

with $SU(2)_L$ transforming vertically and $SU(2)_R$ horizontally. Under $T_{3R}+S$, the neutral scalars ϕ_R^0 and η_2^0 are zero, so that their vacuum expectation values do not break $T_{3R}+S$, which remains as a global symmetry. However, $\langle\sigma\rangle \neq 0$ does break $T_{3R}+S$ and gives masses to $\psi_{1R}^0\psi_{2R}^0 - \psi_{1R}^-\psi_{2R}^+$, $\chi_R^+\chi_R^-$, and $\chi_{1R}^0\chi_{2R}^0$. These exotic fermions all have half-integral charges [6] under $T_{3R}+S$ and only communicate with the others with integral charges through $W_R^\pm$, $\sqrt{2}Re(\phi_R^0)$, ζ, and the two extra neutral gauge bosons beyond the Z. Some explicit Yukawa terms are

$$(\psi_{1R}^0\phi_R^- + \psi_{1R}^-\bar{\phi}_R^0)\chi_R^+, \quad (\psi_{2R}^+\phi_R^0 - \psi_{2R}^0\phi_R^+)\chi_R^-, \tag{4}$$

$$(\psi_{1R}^0\phi_R^0 - \psi_{1R}^-\phi_R^+)\chi_{2R}^0, \quad (\psi_{2R}^+\phi_R^- + \psi_{2R}^0\bar{\phi}_R^0)\chi_{1R}^0. \tag{5}$$

This dichotomy of particle content results in an additional unbroken symmetry of the Lagrangian, i.e. discrete Z_2 under which the exotic fermions are odd. Hence dark matter has two layers: those with nonzero $T_{3R}+S$ and even Z_2, i.e. $n, h, W_R^\pm, \phi_R^\pm, \eta_1^\pm, \eta_1^0, \bar{\eta}_1^0, \zeta$, and the underlying exotic fermions with odd Z_2. Without ζ, a global S' symmetry remains. With ζ, because of the $\zeta^3\sigma^*$ and $\chi_{1R}^0\chi_{1R}^0\zeta$ terms, the S' symmetry breaks to Z_3.

Let

$$\langle\phi_L^0\rangle = v_1, \quad \langle\eta_2^0\rangle = v_2, \quad \langle\phi_R^0\rangle = v_R, \quad \langle\sigma\rangle = v_S, \tag{6}$$

then the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_S$ gauge symmetry is broken to $SU(3)_C \times U(1)_Q$ with S', which becomes Z_3, as shown in Table 2 with $\omega^3 = 1$. The discrete Z_2 symmetry is unbroken. Note that the global S' assignments for the exotic fermions are not $T_{3R}+S$ because of v_S, which breaks the gauge $U(1)_S$ by 3 units.

4 Gauge sector

Consider now the masses of the gauge bosons. The charged ones, $W_L^\pm$ and $W_R^\pm$, do not mix because of $S'(Z_3)$, as in the original alternative left–right models. Their masses are given by

$$M_{W_L}^2 = \frac{1}{2}g_L^2\left(v_1^2+v_2^2\right), \quad M_{W_R}^2 = \frac{1}{2}g_R^2\left(v_R^2+v_2^2\right). \tag{7}$$

Since $Q = I_{3L} + I_{3R} + X$, the photon is given by

$$A = \frac{e}{g_L}W_{3L} + \frac{e}{g_R}W_{3R} + \frac{e}{g_X}X, \tag{8}$$

where $e^{-2} = g_L^{-2} + g_R^{-2} + g_X^{-2}$. Let

$$Z = (g_L^2+g_Y^2)^{-1/2}\left(g_L W_{3L} - \frac{g_Y^2}{g_R}W_{3R} - \frac{g_Y^2}{g_X}X\right), \tag{9}$$

$$Z' = (g_R^2+g_X^2)^{-1/2}(g_R W_{3R} - g_X X), \tag{10}$$

where $g_Y^{-2} = g_R^{-2} + g_X^{-2}$, then the 3×3 mass-squared matrix spanning (Z, Z', S) has the entries:

$$M_{ZZ}^2 = \frac{1}{2}\left(g_L^2+g_Y^2\right)\left(v_1^2+v_2^2\right), \tag{11}$$

$$M_{Z'Z'}^2 = \frac{1}{2}\left(g_R^2+g_X^2\right)v_R^2 + \frac{g_X^4 v_1^2 + g_R^4 v_2^2}{2\left(g_R^2+g_X^2\right)}, \tag{12}$$

$$M_{SS}^2 = 18 g_S^2 v_S^2 + \frac{1}{2}g_S^2\left(v_R^2+v_2^2\right), \tag{13}$$

$$M_{ZZ'}^2 = \frac{\sqrt{g_L^2+g_Y^2}}{2\sqrt{g_R^2+g_X^2}}\left(g_X^2 v_1^2 - g_R^2 v_2^2\right), \tag{14}$$

$$M_{ZS}^2 = \frac{1}{2}g_S\sqrt{g_L^2+g_Y^2}\,v_2^2, \tag{15}$$

$$M_{Z'S}^2 = -\frac{1}{2}g_S\sqrt{g_R^2-g_X^2}\,v_R^2 - \frac{g_S g_R v_2^2}{2\sqrt{g_R^2+g_X^2}}. \tag{16}$$

Their neutral-current interactions are given by

$$\mathcal{L}_{NC} = eA_\mu j_Q^\mu + g_Z Z_\mu\left(j_{3L}^\mu - \sin^2\theta_W j_Q^\mu\right) + \left(g_R^2+g_X^2\right)^{-1/2} Z'_\mu\left(g_R^2 j_{3R}^\mu - g_X^2 j_X^\mu\right) + g_S S_\mu j_S^\mu, \tag{17}$$

where $g_Z^2 = g_L^2 + g_Y^2$ and $\sin^2\theta_W = g_Y^2/g_Z^2$.

In the limit $v_{1,2}^2 << v_R^2, v_S^2$, the mass-squared matrix spanning (Z', S) may be simplified if we assume

$$\frac{v_S^2}{v_R^2} = \frac{\left(g_R^2+g_X^2+g_S^2\right)^2}{36 g_S^2\left(g_R^2+g_X^2-g_S^2\right)}, \tag{18}$$

and let

$$\tan\theta_D = \frac{\sqrt{g_R^2+g_X^2}-g_S}{\sqrt{g_R^2+g_X^2}+g_S}; \tag{19}$$

then

$$\begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_D & \sin\theta_D \\ -\sin\theta_D & \cos\theta_D \end{pmatrix}\begin{pmatrix} Z' \\ S \end{pmatrix}, \tag{20}$$

with mass eigenvalues given by

$$M_{D_1}^2 = \sqrt{g_R^2+g_X^2}\sqrt{g_R^2+g_X^2+g_S^2}\,\frac{v_R^2}{2\sqrt{2}\cos\theta_D}, \tag{21}$$

$$M_{D_2}^2 = \sqrt{g_R^2+g_X^2}\sqrt{g_R^2+g_X^2+g_S^2}\,\frac{v_R^2}{2\sqrt{2}\sin\theta_D}. \tag{22}$$

In addition to the assumption of Eq. (18), let us take for example

$$2g_S = \sqrt{g_R^2+g_X^2}, \tag{23}$$

Table 2 Particle content of proposed model under $(T_{3R}+S)\times Z_2$

Particles	Gauge $T_{3R}+S$	Global S'	Z_3	Z_2
u, d, ν, l	0	0	1	+
(ϕ_L^+, ϕ_L^0), (η_2^+, η_2^0), ϕ_R^0	0	0	1	+
n, ϕ_R^+, ζ	1	1	ω	+
$h, (\eta_1^0, \eta_1^-)$	-1	-1	ω^2	+
ψ_{2R}^+, χ_R^+	$3/2, -3/2$	0	1	$-$
ψ_{1R}^-, χ_R^-	$3/2, -3/2$	0	1	$-$
ψ_{1R}^0, ψ_{2R}^0	$5/2, 1/2$	$1, -1$	ω, ω^2	$-$
χ_{1R}^0, χ_{2R}^0	$-1/2, -5/2$	$1, -1$	ω, ω^2	$-$
σ	3	0	1	+

then $\sin\theta_D = 1/\sqrt{10}$ and $\cos\theta_D = 3/\sqrt{10}$. Assuming also that $g_R = g_L$, we obtain

$$\frac{g_X^2}{g_Z^2} = \frac{\sin^2\theta_W\cos^2\theta_W}{\cos 2\theta_W}, \quad \frac{g_S}{g_Z} = \frac{\cos^2\theta_W}{2\sqrt{\cos 2\theta_W}}, \tag{24}$$

$$\frac{v_S^2}{v_R^2} = \frac{25}{108}, \quad M_{D_2}^2 = 3M_{D_1}^2 = \frac{5\cos^4\theta_W}{4\cos 2\theta_W} g_Z^2 v_R^2. \tag{25}$$

The resulting gauge interactions of $D_{1,2}$ are given by

$$\begin{aligned}\mathcal{L}_D = \frac{g_Z}{\sqrt{10}\sqrt{\cos 2\theta_W}} \Big\{ &\Big[3\cos 2\theta_W j_{3R}^\mu - 3\sin^2\theta_W j_X^\mu \\ &+ (1/2)\cos^2\theta_W j_S^\mu\Big] D_{1\mu} \\ &+ \Big[-\cos 2\theta_W j_{3R}^\mu + \sin^2\theta_W j_X^\mu \\ &+ (3/2)\cos^2\theta_W j_S^\mu\Big] D_{2\mu}\Big\}. \end{aligned} \tag{26}$$

Since D_2 is $\sqrt{3}$ times heavier than D_1 in this example, the latter would be produced first in pp collisions at the Large Hadron Collider (LHC).

5 Fermion sector

All fermions obtain masses through the four vacuum expectation values of Eq. (6) except ν_R, which is allowed to have an invariant Majorana mass. This means that neutrino masses may be small from the usual canonical seesaw mechanism. The various Yukawa terms for the quark and lepton masses are

$$\begin{aligned} -\mathcal{L}_Y = &\frac{m_u}{v_2}\left[\bar u_R(u_L\eta_2^0 - d_L\eta_2^+) + \bar h_R(-u_L\eta_2^- + d_L\eta_1^0)\right] \\ &+ \frac{m_d}{v_1}\left(\bar u_L\phi_L^+ + \bar d_L\phi_L^0\right) d_R + \frac{m_h}{v_R}\left(\bar u_R\phi_R^+ + \bar h_R\phi_R^0\right) h_L \\ &+ \frac{m_l}{v_2}\left[\left(\bar\nu_L\eta_1^0 + \bar l_L\eta_1^-\right) n_R + \left(\bar\nu_L\eta_2^+ + \bar l_L\eta_2^0\right) l_R\right] \\ &+ \frac{m_D}{v_1}\bar\nu_R\left(\nu_L\phi_L^0 - l_L\phi_L^+\right) \\ &+ \frac{m_n}{v_R}\bar n_L\left(n_R\phi_R^0 - l_R\phi_R^-\right) + H.c. \end{aligned} \tag{27}$$

These terms show explicitly that the assignments of Tables 1 and 2 are satisfied.

As for the exotic ψ and χ fermions, they have masses from the Yukawa terms of Eqs. (4) and (5), as well as from

$$(\phi_{1R}^0\psi_{2R}^0 - \psi_{1R}^-\psi_{2R}^+)\sigma^*, \quad \chi_R^-\chi_R^+\sigma, \quad \chi_{1R}^0\chi_{2R}^0\sigma. \tag{28}$$

As a result, two neutral Dirac fermions are formed from the matrix linking χ_{1R}^0 and ψ_{1R}^0 to χ_{2R}^0 and ψ_{2R}^0. Let us call the lighter of these two Dirac fermions χ_0, then it is one component of dark matter of our model. The other will be the scalar ζ, to be discussed later. Note that χ_0 communicates with ζ through the allowed $\chi_{1R}^0\chi_{1R}^0\zeta$ interaction. Note also that the allowed Yukawa terms

$$\bar d_R h_L\zeta, \quad \bar n_L\nu_R\zeta \tag{29}$$

enable the dark fermions h and n to decay into ζ.

6 Scalar sector

Consider the most general scalar potential consisting of $\Phi_{L,R}$, η, and σ. Let

$$\eta = \begin{pmatrix}\eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0\end{pmatrix}, \quad \tilde\eta = \sigma_2\eta^*\sigma_2 = \begin{pmatrix}\bar\eta_2^0 & -\eta_1^+ \\ -\eta_2^- & \bar\eta_1^0\end{pmatrix}; \tag{30}$$

then

$$\begin{aligned} V = &-\mu_L^2\Phi_L^\dagger\Phi_L - \mu_R^2\Phi_R^\dagger\Phi_R - \mu_\sigma^2\sigma^*\sigma - \mu_\eta^2 Tr(\eta^\dagger\eta) \\ &+ [\mu_3\Phi_L^\dagger\eta\Phi_R + H.c.] \\ &+ \frac{1}{2}\lambda_L(\Phi_L^\dagger\Phi_L)^2 + \frac{1}{2}\lambda_R(\Phi_R^\dagger\Phi_R)^2 + \frac{1}{2}\lambda_\sigma(\sigma^*\sigma)^2 \\ &+ \frac{1}{2}\lambda_\eta[Tr(\eta^\dagger\eta)]^2 + \frac{1}{2}\lambda'_\eta Tr(\eta^\dagger\eta\eta^\dagger\eta) \\ &+ \lambda_{LR}(\Phi_L^\dagger\Phi_L)(\Phi_R^\dagger\Phi_R) + \lambda_{L\sigma}(\Phi_L^\dagger\Phi_L)(\sigma^*\sigma) \\ &+ \lambda_{R\sigma}(\Phi_R^\dagger\Phi_R)(\sigma^*\sigma) \\ &+ \lambda_{\sigma\eta}(\sigma^*\sigma)Tr(\eta^\dagger\eta) \end{aligned}$$

$$+\lambda_{L\eta}\Phi_L^\dagger\eta\eta^\dagger\Phi_L + \lambda'_{L\eta}\Phi_L^\dagger\tilde{\eta}\tilde{\eta}^\dagger\Phi_L + \lambda_{R\eta}\Phi_R^\dagger\eta^\dagger\eta\Phi_R + \lambda'_{R\eta}\Phi_R^\dagger\tilde{\eta}^\dagger\tilde{\eta}\Phi_R. \quad (31)$$

Note that

$$2|det(\eta)|^2 = [Tr(\eta^\dagger\eta)]^2 - Tr(\eta^\dagger\eta\eta^\dagger\eta), \quad (32)$$
$$(\Phi_L^\dagger\Phi_L)Tr(\eta^\dagger\eta) = \Phi_L^\dagger\eta\eta^\dagger\Phi_L + \Phi_L^\dagger\tilde{\eta}\tilde{\eta}^\dagger\Phi_L, \quad (33)$$
$$(\Phi_R^\dagger\Phi_R)Tr(\eta^\dagger\eta) = \Phi_R^\dagger\eta^\dagger\eta\Phi_R + \Phi_R^\dagger\tilde{\eta}^\dagger\tilde{\eta}\Phi_R. \quad (34)$$

The minimum of V satisfies the conditions

$$\mu_L^2 = \lambda_L v_1^2 + \lambda_{L\eta}v_2^2 + \lambda_{LR}v_R^2 + \lambda_{L\sigma}v_S^2 + \mu_3 v_2 v_R/v_1, \quad (35)$$
$$\mu_\eta^2 = (\lambda_\eta + \lambda'_\eta)v_2^2 + \lambda_{L\eta}v_1^2 + \lambda_{R\eta}v_R^2 + \lambda_{\sigma\eta}v_S^2 + \mu_3 v_1 v_R/v_2, \quad (36)$$
$$\mu_R^2 = \lambda_R v_R^2 + \lambda_{LR}v_1^2 + \lambda_{R\eta}v_2^2 + \lambda_{R\sigma}v_S^2 + \mu_3 v_1 v_2/v_R, \quad (37)$$
$$\mu_\sigma^2 = \lambda_\sigma v_S^2 + \lambda_{L\sigma}v_1^2 + \lambda_{\sigma\eta}v_2^2 + \lambda_{R\sigma}v_R^2. \quad (38)$$

The 4×4 mass-squared matrix spanning $\sqrt{2}Im(\phi_L^0, \eta_2^0, \phi_R^0, \sigma)$ is then given by

$$\mathcal{M}_I^2 = \mu_3\begin{pmatrix} -v_2v_R/v_1 & v_R & v_2 & 0 \\ v_R & -v_1v_R/v_2 & -v_1 & 0 \\ v_2 & -v_1 & -v_1v_2/v_R & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (39)$$

and that spanning $\sqrt{2}Re(\phi_L^0, \eta_2^0, \phi_R^0, \sigma)$ is

$$\mathcal{M}_R^2 = \mu_3\begin{pmatrix} -v_2v_R/v_1 & v_R & v_2 & 0 \\ v_R & -v_1v_R/v_2 & v_1 & 0 \\ v_2 & v_1 & -v_1v_2/v_R & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + 2\begin{pmatrix} \lambda_L v_1^2 & \lambda_{L\eta}v_1v_2 & \lambda_{LR}v_1v_R & \lambda_{L\sigma}v_1v_S \\ \lambda_{L\eta}v_1v_2 & (\lambda_\eta+\lambda'_\eta)v_2^2 & \lambda_{R\eta}v_2v_R & \lambda_{\sigma\eta}v_2v_S \\ \lambda_{LR}v_1v_R & \lambda_{R\eta}v_2v_R & \lambda_R v_R^2 & \lambda_{R\sigma}v_Rv_S \\ \lambda_{L\sigma}v_1v_S & \lambda_{\sigma\eta}v_2v_S & \lambda_{R\sigma}v_Rv_S & \lambda_\sigma v_S^2 \end{pmatrix}. \quad (40)$$

Hence there are three zero eigenvalues in $\mathcal{M}_I^2$ with one nonzero eigenvalue $-\mu_3[v_1v_2/v_R + v_R(v_1^2+v_2^2)/v_1v_2]$ corresponding to the eigenstate $(-v_1^{-1}, v_2^{-1}, v_R^{-1}, 0)/\sqrt{v_1^{-2}+v_2^{-2}+v_R^{-2}}$. In $\mathcal{M}_R^2$, the linear combination $H = (v_1, v_2, 0, 0)/\sqrt{v_1^2+v_2^2}$, is the standard-model Higgs boson, with

$$m_H^2 = 2[\lambda_L v_1^4 + (\lambda_\eta+\lambda'_\eta)v_2^4 + 2\lambda_{L\eta}v_1^2v_2^2]/(v_1^2+v_2^2). \quad (41)$$

The other three scalar bosons are much heavier, with suppressed mixing to H, which may all be assumed to be small enough to avoid the constraints from dark-matter direct-search experiments. The addition of the scalar ζ introduces two important new terms:

$$\zeta^3\sigma^*, \quad (\eta_1^0\eta_2^0 - \eta_1^-\eta_2^+)\zeta. \quad (42)$$

The first term breaks global S' to Z_3, and the second term mixes ζ with η_1^0 through v_2. We assume the latter to be negligible, so that the physical dark scalar is mostly ζ.

7 Present phenomenological constraints

Many of the new particles of this model interact with those of the standard model. The most important ones are the neutral $D_{1,2}$ gauge bosons, which may be produced at the LHC through their couplings to u and d quarks, and decay to charged leptons (e^-e^+ and $\mu^-\mu^+$). As noted previously, in our chosen example, D_1 is the lighter of the two. Hence current search limits for a Z' boson are applicable [7,8]. The $c_{u,d}$ coefficients used in the data analysis are

$$c_u = \left(g_{uL}^2 + g_{uR}^2\right)B = 0.0273\ B,$$
$$c_d = \left(g_{dL}^2 + g_{dR}^2\right)B = 0.0068\ B, \quad (43)$$

where B is the branching fraction of Z' to e^-e^+ and $\mu^-\mu^+$. Assuming that D_1 decays to all the particles listed in Table 2, except for the scalars which become the longitudinal components of the various gauge bosons, we find $B = 1.2\times 10^{-2}$. Based on the 2016 LHC 13 TeV data set from ATLAS [9], this translates to a bound of about 4 TeV on the D_1 mass.

The would-be dark-matter candidate n is a Dirac fermion which couples to $D_{1,2}$, which also couples to quarks. Hence severe limits exist on the masses of $D_{1,2}$ from underground direct-search experiments as well. The annihilation cross section of n through $D_{1,2}$ would then be too small, so that its relic abundance would be too big for it to be a dark-matter candidate. Its annihilation at rest through s-channel scalar exchange is p-wave suppressed and does not help, barring of course any accidental resonance enhancement. As for the t-channel diagrams, they also turn out to be too small. Suggestions of previous studies [2,3] where n is chosen as dark matter are now ruled out.

8 Dark sector

Dark matter is envisioned to have two components. One is a Dirac fermion χ_0, which is a mixture of the four neutral fermions of odd Z_2, and the other is a complex scalar boson which is mostly ζ, with the added assumption that m_{χ_0} is significantly greater than m_ζ. The annihilation $\chi_0\bar{\chi}_0 \to \zeta\zeta^*$ determines the relic abundance of χ_0, and the annihilation $\zeta\zeta^* \to HH$, where H is the standard-model Higgs boson, determines that of ζ. The direct $\zeta\zeta^* H$ coupling is assumed small to avoid the severe constraint in direct-search experiments.

Let the interaction of ζ with χ_0 be $f_0 \zeta \chi_{0R} \chi_{0R} + H.c.$, then the annihilation cross section of $\chi_0 \bar{\chi}_0$ to $\zeta\zeta^*$ times relative velocity is given by

$$\langle \sigma \times v_{rel} \rangle_\chi = \frac{f_0^4}{4\pi m_{\chi_0}} \frac{\left(m_{\chi_0}^2 - m_\zeta^2\right)^{3/2}}{\left(2m_{\chi_o}^2 - m_\zeta^2\right)^2}. \tag{44}$$

This determines the relic abundance of χ_0.

As the Universe cools below m_{χ_0}, χ_0 decouples from the thermal bath. We assume that m_ζ is much below m_{χ_0} so that χ_0 is essentialy frozen out at m_ζ. The relic abundance of ζ is then mostly determined by $\zeta\zeta^* \to HH$. Let the effective interaction strength of $\zeta\zeta^*$ with HH be λ_0, then the annihilation cross section of $\zeta\zeta^*$ to HH times relative velocity is given by

$$\langle \sigma_\zeta \times v_{rel} \rangle_\zeta = \frac{\lambda_0^2}{16\pi} \frac{\left(m_\zeta^2 - m_H^2\right)^{1/2}}{m_\zeta^3}. \tag{45}$$

Note that λ_0 is the sum over several interactions. The quartic coupling $\lambda_{\zeta H}$ is assumed negligible, to suppress the trilinear $\zeta\zeta^* H$ coupling which contributes to the elastic ζ scattering cross section off nuclei. However, the trilinear couplings $\zeta\zeta^* Re(\phi_R^0)$ and $Re(\phi_R^0)HH$ are proportional to v_R, and the trilinear couplings $\zeta\zeta^* Re(\sigma)$ and $Re(\sigma)HH$ are proportional to v_S. Hence their effective contributions to λ_0 are proportional to $v_R^2/m^2[\sqrt{2}Re(\phi_R^0)]$ and $v_S^2/m^2[\sqrt{2}Re(\sigma)]$, which are not suppressed. Whereas there are other possible contributions to Eqs. (44) and (45), we assume that the f_0 and λ_0 interactions are in fact dominant.

As a rough estimate, we will assume that

$$\langle \sigma \times v_{rel} \rangle_\chi^{-1} + \langle \sigma_\zeta \times v_{rel} \rangle_\zeta^{-1} = (4.4 \times 10^{-26}\ \mathrm{cm}^3/s)^{-1} \tag{46}$$

to satisfy the condition of dark-matter relic abundance [10] of the Universe. For given values of m_ζ and m_{χ_0}, the parameters λ_0 and f_0 are thus constrained. We show in Fig. 1 the plots of λ_0 versus f_0 for $m_\zeta = 150$ GeV and various values of m_{χ_0}. Since m_ζ is fixed at 150 GeV, λ_0 is also fixed for a given fraction of Ω_ζ/Ω_{DM}. To adjust for the rest of dark matter, f_0 must then vary as a function of m_{χ_0} according to Eq. (44).

As for direct detection, both χ_0 and ζ have possible interactions with quarks through the gauge bosons $D_{1,2}$ and the standard-model Higgs boson H. They are suppressed by making the $D_{1,2}$ masses heavy, and the H couplings to χ_0 and ζ small. In our example with $m_\zeta = 150$ GeV, let us choose $m_{\chi_0} = 500$ Gev and the relic abundances of both to

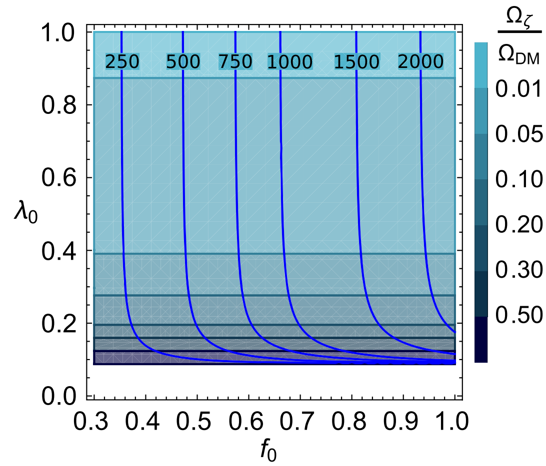

Fig. 1 Relic-abundance constraints on λ_0 and f_0 for $m_\zeta = 150$ GeV and various values of m_{χ_0}

be equal. From Fig. 1, these choices translate to $\lambda_0 = 0.12$ and $f_0 = 0.56$.

Consider first the $D_{1,2}$ interactions. Using Eq. (26), we obtain

$$g_u^V(D_1) = 0.0621, \quad g_d^V(D_1) = 0.0184, \quad g_\zeta(D_1) = 0.1234, \tag{47}$$

$$g_u^V(D_2) = -0.1235, \quad g_d^V(D_2) = -0.0062, \quad g_\zeta(D_2) = 0.3701. \tag{48}$$

The effective ζ elastic scattering cross section through $D_{1,2}$ is then completely determined as a function of the D_1 mass (because $M_{D_2} = \sqrt{3} M_{D_1}$ in our example), i.e.

$$\mathcal{L}_{\zeta q}^V = \frac{(\zeta^* \partial_\mu - \zeta \partial_\mu \zeta^*)}{M_{D_1}^2} \Big[(-7.57 \times 10^{-3}) \bar{u} \gamma^\mu u + (1.51 \times 10^{-3}) \bar{d} \gamma^\mu d \Big]. \tag{49}$$

Using the most recent XENON result [11] at $m_\zeta = 150$ GeV for which $\sigma < 2 \times 10^{-46}$ cm^2 and Eq. (25), we obtain $v_R > 35$ TeV which translates to $M_{D_1} > 18$ TeV, and $M_{W_R} > 16$ TeV. These are a few percent more restrictive than the most recent LUX result [12].

The $\bar{\chi}_0 \gamma_\mu \chi_0$ couplings to $D_{1,2}$ depend on the 2×2 mass matrix linking (χ_1, ψ_1) to (χ_2, ψ_2), which has two mixing angles and two mass eigenvalues, the smaller one being m_{χ_0}. By adjusting these parameters, it is possible to make the effective χ_0 interaction to any particular nucleus through $D_{1,2}$ negligibly small. Hence there is no useful limit on the D_1 mass in this case. Note that the amplitude cancellation here is through $D_{1,2}$ and not necessarily through u and d quarks (which are not adjustable in this model), as would be necessary in models with only one vector mediator.

Direct search also constrains the coupling of the Higgs boson to ζ (through a possible trilinear $\lambda_{\zeta H} \sqrt{2} v_H \zeta^* \zeta$ interaction) or χ_0 (through an effective Yukawa coupling ϵ from

H mixing with σ_R and ϕ_R^0). Let their effective interactions with quarks through H exchange be given by

$$\mathcal{L}^S_{\zeta q} = \frac{\lambda_{\zeta H} m_q}{m_H^2} \zeta^* \zeta \bar{q} q + \frac{\epsilon f_q}{m_H^2} \bar{\chi}_0 \chi_0 \bar{q} q, \tag{50}$$

where $f_q = m_q/\sqrt{2} v_H = m_q/(246 \text{ GeV})$. The spin-independent direct-detection cross section per nucleon in the former is given by

$$\sigma^{SI} = \frac{\mu_\zeta^2}{\pi A^2} [\lambda_p Z + (A - Z)\lambda_n]^2, \tag{51}$$

where $\mu_\zeta = m_\zeta M_A/(m_\zeta + M_A)$ is the reduced mass of the dark matter, and [13]

$$\lambda_N = \left[\sum_{u,d,s} f_q^N + \frac{2}{27} \left(1 - \sum_{u,d,s} f_q^N \right) \right] \frac{\lambda_{\zeta H} m_N}{2 m_\zeta m_H^2}, \tag{52}$$

with [14]

$$f_u^p = 0.0139, \quad f_d^p = 0.0253, \quad f_s^p = 0.113, \tag{53}$$
$$f_u^n = 0.0116, \quad f_d^n = 0.0302, \quad f_s^n = 0.113. \tag{54}$$

For $m_\zeta = 150$ GeV, we have

$$\lambda_p = 4.30 \times 10^{-8} \lambda_{\zeta H} \text{ GeV}^{-2},$$
$$\lambda_n = 4.35 \times 10^{-8} \lambda_{\zeta H} \text{ GeV}^{-2}. \tag{55}$$

Using $A = 131$, $Z = 54$, and $M_A = 130.9$ atomic mass units for the XENON experiment [11], and twice the most recent bound of 2×10^{-46} cm^2 (at $m_\zeta = 150$ GeV) because ζ is assumed to account for only half of the dark matter) at this mass, we find

$$\lambda_{\zeta H} < 6.2 \times 10^{-4}. \tag{56}$$

As noted earlier, this is negligible for considering the annihilation cross section of ζ to H.

For the H contribution to the χ_0 elastic cross section off nuclei, we replace m_ζ with $m_{\chi_0} = 500$ GeV in Eq. (51) and $\lambda_{\zeta H}/2m_\zeta$ with $\epsilon/\sqrt{2} v_H$ in Eq. (52). Using the experimental data at 500 GeV, we obtain the bound.

$$\epsilon < 6.4 \times 10^{-4}. \tag{57}$$

From the above discussion, it is clear that it is possible for future improvements in direct-search experiments to yield positive results within the framework of our model.

9 Conclusion and outlook

In the context of the alternative left–right model, a new gauge $U(1)_S$ symmetry has been proposed to stabilize dark matter. This is accomplished by the addition of a few new fermions to cancel all the gauge anomalies, as shown in Table 1. As a result of this particle content, an automatic unbroken Z_2 symmetry exists on top of $U(1)_S$, which is broken to a conserved residual Z_3 symmetry. Thus dark matter has two components. One is the Dirac fermion $\chi_0 \sim (\omega, -)$ and the other the complex scalar $\zeta \sim (\omega, +)$ under $Z_3 \times Z_2$. We have shown how they may account for the relic abundance of dark matter in the Universe, and satisfy present experimental search bounds.

Whereas we have no specific prediction for discovery in direct-search experiments, our model will be able to accommodate any positive result in the future, just like many other existing proposals. To single out our model, many additional details must also be confirmed. Foremost are the new gauge bosons $D_{1,2}$. Whereas the LHC bound is about 4 TeV, the direct-search bound is much higher, provided that ζ is a significant fraction of dark matter. If χ_0 dominates instead, the adjustment of free parameters of our model can lower this bound to below 4 TeV. In that case, future $D_{1,2}$ observations are still possible at the LHC as more data become available.

Another is the exotic h quark which is easily produced if kinematically allowed. It would decay to d and ζ through the direct $\bar{d}_R h_L \zeta$ coupling of Eq. (29). Assuming that this branching fraction is 100%, the search at the LHC for 2 jets plus missing energy puts a limit on m_h of about 1.0 TeV, as reported by the CMS Collaboration [15] based on the $\sqrt{s} =$ 13 TeV data at the LHC with an integrated luminosity of 35.9 fb^{-1} for a single scalar quark.

If the $\bar{d}_R h_L \zeta$ coupling is very small, then h may also decay significantly to u and a virtual W_R^-, with W_R^- becoming $\bar{n} l^-$, and $\bar{n}$ becoming $\bar{\nu} \zeta^*$. This has no analog in the usual searches for supersymmetry or the fourth family because W_R is heavy ($>$ 16 TeV). To be specific, the final states of 2 jets plus $l_1^- l_2^+$ plus missing energy should be searched for. As more data are accumulated at the LHC, such events may become observable.

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