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Editorial

Noncommutativity and physics

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Noncommutative geometry has opened exciting new avenues in mathematics and physics. It has brought together mathematicians and physicists in fruitful interdisciplinary interaction. This special volume on Noncommutativity and Physics in European Physical Journal Special Topics brings together some of the world's leading experts in the field to address various aspects of noncommutative geometry with relevance in physics, focusing on applications to the Standard Model and beyond, quantum gravity, superstring theory, condensed matter physics, and other fields of physics. The range of methods used include noncommutative differential geometry, deformation quantization, star products, fuzzy spaces, matrix models, and more recent developments like generalized geometry. The articles included are concise but comprehensive reviews that introduce the reader to these subjects while also presenting the current state of the art. The basic idea of this special volume is to collect the knowledge needed for current research in the field in one place, while avoiding ballast or dead ends. The focus is on physics applications rather than pure mathematics, illustrative examples are given. While a few textbooks on aspects of noncommutative geometry have already been written, a collection of dedicated review articles with the breadth of topics, focus on physics application and aim to enable research as outlined, does not yet exist and here we hope to fill a gap. Given the popularity of recent international workshops and conferences in the field (predominantly in Corfu and Bayrischzell and occasionally in other places such as Belgrade, Bratislava, Dubna, Dubrovnik, Karpacz, Munich, Porto, Sofia, Varna, Vienna, Zagreb, and Zakopane), we believe that such a volume will be well received.

Historically, the idea of noncommutativity has its roots in quantum mechanics. In the early twentieth century, quantum theory challenged classical notions of commutativity. The Heisenberg uncertainty principle fundamentally altered our understanding of observables in quantum systems. It introduced the notion that certain pairs of physical properties, such as position and momentum, may not be compatible and must be described by non-commuting operators. Noncommutative geometry extends this idea to the description of spaces. This mathematical shift has profound implications, in particular when applied to physics. Noncommutative geometry provides a powerful framework for addressing some of the most pressing questions in modern physics. Together with more recent developments like fuzzy, twisted, generalized, higher, and graded geometry, it has found applications in a variety of areas, including the Standard Model of particle physics, quantum field theory, quantum gravity, superstring theory, cosmology, and condensed matter physics.

One of the early inspirations for noncommutative geometry came from the work of the renowned mathematician Alain Connes in the 1980s. Connes introduced the concept of a spectral triple, a mathematical structure that elegantly encapsulates the geometric and topological information of a space. This groundbreaking work laid the foundation for applying noncommutative geometry to physics and in particular to the Standard Model of particle physics as outlined in Connes' contribution to this volume together with Chamseddine and van Suijlekom. Inspired by research on quantum integrable models, the 1980s also saw the advent of Quantum Groups (a term coined by Drinfel'd and Jimbo for quasi-triangular Hopf algebras) and the application of noncommutative differential geometric methods in this context by Woronowicz and others. This work in turn led to a surge of interest in the physics community in the 1990s to explore these novel quantum symmetries and corresponding quantum spaces like, for example, the Wess–Zumino quantum hyperplane.

One of the motivations for this line of research was the quest to find a natural regularization of ultraviolet divergences in quantum field theory and gravity in a quantum geometric setting, which could provide either a fundamental or an effective description of quantum gravitational effects. (Although in many models the regularization

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is limited by UV–IR mixing effects.) A similar motivation led to the introduction of truncated matrix geometries with Madore's pioneering work on the fuzzy sphere, as outlined by Madore's long-time scientific collaborator Buric in this volume. Matrix models are still among the prime candidates for a theory of everything and can be motivated from string theory and certain limits of supersymmetric gauge theory.

In parallel to these developments, there was tremendous progress in the understanding of the deformation quantization of classical Poisson structures to noncommutative star products in the work of Fedosov, Kontsevich and others and its link to noncommutative geometry and string theory popularized in the influential paper by Seiberg and Witten, building on and inspiring the work of many other physicists and mathematicians in the 2000s. These methods allowed the construction of particle physics models on noncommutative spaces with experimentally testable predictions, setting first lower bounds on the scale of noncommutativity from particle accelerator experiments.

Noncommutative geometry is not the only possible generalization of geometry motivated by quantum gravitational physics. Generalized geometry, a unified approach to symplectic, complex and Riemannian geometry introduced by Hitchin, Gualtieri and others turned out to be the perfect setting for the construction of effective gravity actions that are compatible with stringy symmetries. There has been a lot of activity in this direction since the 2010s including even work on nonassociative structures. Generalized geometries can in turn be derived from graded geometry, which is related to supersymmetry and it has also been understood how noncommutative gauge theory arises in this context.

Recently, a renewed interest in noncommutative gauge theory can be seen—including the rediscovery of some previously known facts from different perspectives. Among the current motivations is the construction of novel quantum integrable models. These models were one of the topics where it all started, so in a sense the field has started to come full circle.

With new data from cosmic microwave background and gravitational wave observations, as well as quantum interferometry experiments, it has become feasible to start looking for measurable signatures of quantum spacetime and explore its physical and cosmological consequences.

Following are brief summaries of all contributed articles to the volume, sorted by topic. These articles collectively contribute to a comprehensive exploration of noncommutative geometry's applications in physics, from string theory to gauge theories, particle physics and gravity, offering an in-depth understanding of the field's diversity and potential impact. We refer to these individual contributions for a comprehensive and up to date list of references to the research literature. A short list of references to some of the work mentioned in the editorial follows at the end.

Noncommutative geometry and spectral action

"Noncommutativity and Physics: A non-technical review" by Chamseddine, Connes and van Suijlekom offers a conceptual exploration of noncommutative geometry's applications in physics. This review delves into the theoretical foundations, demonstrating the relationship between noncommutativity, canonical time evolution, discrete and continuous variables, spacetime geometry, and the emergence of spin geometry and non-Abelian gauge theory.

"Spectral interactions between strings in the Higgs background" by Bochniak and Sitarz derives the exact spectral interaction of strings mediated by a constant scalar field. This is achieved through methods derived from noncommutative geometry in a modified Connes–Lott model offering a fresh perspective on the scalar field's interpretation.

Fuzzy physics and matrix models

"A Road to Fuzzy Physics" by Buric aims to develop an intuitive understanding of noncommutative spaces and their applications. The paper discusses fundamental concepts like continuity, differential calculus, and geometry applied to noncommutative algebras, introducing the noncommutative frame formalism.

"Intertwining noncommutativity with gravity and particle physics" by Manolakos, Manousselis, Roumelioti, Stefas and Zoupanos provides an overview of how noncommutativity impacts both gravity and particle physics and examines the potential unification of these two fields from a matrix-realized perspective with noncommutative gauge theories playing the central role.

"Fuzzy scalar field theories" by Tekel reviews scalar field theories on fuzzy spaces, addressing the UV/IR mixing and non-uniform order phases. It discusses the matrix model approach and recent results in this field.

"Kronecker coefficients from algebras of bipartite ribbon graphs" by Ramgoolam explores an algebra of bipartite ribbon graphs with applications in matrix and tensor models, particularly in computing Kronecker coefficients.

"Selective Multiple Power Iteration" by Ouerfelli, Rivasseau and Tamaazousti presents a new algorithm for Tensor PCA problem solving, significantly improving recovery performance in noisy tensors, particularly at low signal-to-noise ratios.

String theory, gauge theory and matrix models

"Noncommutative Instantons in Diverse Dimensions" by Szabo provides a mini-review focusing on generalized instantons of noncommutative gauge theories in various dimensions. It places emphasis on their realization in type II string theory and their applications in enumerative geometry.

"Progress in the numerical studies of the type IIB matrix model" by Azuma, Anagnostopoulos, Hatakeyama, Hirasawa, Ito, Nishimura, Papadoudis, and Tsuchiya reviews the IKKT model, a candidate for a non-perturbative formulation of superstring theory. It specifically highlights recent results obtained through the complex Langevin method, addressing challenges like the sign problem.

"Eight-dimensional non-geometric heterotic strings and enhanced gauge groups" by Kimura discusses eight-dimensional non-geometric heterotic strings and their relation to the unbroken $\mathfrak{e}_8 \oplus \mathfrak{e}_7$ gauge algebra. It explores the moduli space and non-Abelian gauge group enhancements within these theories.

Generalized and higher geometry

"Instances of Higher Geometry in Field Theory" by Chatzistavrakidis explores the role of higher geometry in the context of field theory and quantization. This mini-review highlights algebroid structures on bundles and Q-manifolds, and their relation to Batalin–Vilkovisky quantization, higher gauge theories, and tensor gauge theories in terms of graded geometry.

"Generalized symmetries as homotopy Lie algebras" by Jonke explores the use of homotopy Lie algebras to describe generalized gauge symmetries. It highlights specific examples related to non-commutative gauge symmetry and double field theory.

Noncommutative gauge theory and gravity

"Revisiting NCQED and scattering amplitudes" by Trampetic and You discusses the progress made in non-commutative gauge theories on the Moyal space, with an emphasis on Seiberg–Witten maps and their role in understanding the formal equivalence of on-shell DeWitt background field effective action and tree-level scattering amplitudes in noncommutative quantum electrodynamics (NCQED).

"Noncommutative gauge and gravity theories and geometric Seiberg–Witten map" by Aschieri and Castellani offers a pedagogical account of noncommutative gauge and gravity theories. It discusses the Seiberg–Witten map and its role in noncommutative Einstein gravity.

"Noncommutative $SO(2,3)_\star$ Gauge Theory of Gravity" by Dimitrijevic Ciric delves into a noncommutative star-product deformation of the AdS gauge theory of gravity. It examines perturbative noncommutative corrections and their impact on field equations.

"Review of Twisted Poincaré Symmetry" by Balachandran, Kürkcüoglu and Vaidya takes a closer look at twisted Poincare covariant quantum fields on the Moyal plane. It explores the Drinfel'd twist's role, applications to discrete groups, spacetime noncommutativity, and the difference between Moyal and Voros quantum fields.

References to some of the early work on noncommutativity and physics mentioned in the editorial follow. This is just a very small historical selection including a few books and review articles and by no means complete. There are many important works with great impact on the field and we refer to the individual contributions in this volume on noncommutativity and physics for comprehensive up to date lists of references to the research literature.

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