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Editorial

Celestial mechanics: new discoveries and challenges for space exploration

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Abstract This special issue entitled "Celestial Mechanics: New Discoveries and Challenges for Space Exploration" presents a series of 27 contributions. Some of them are devoted to the analysis of periodic orbits in the solar system and methods for identifying chaos in non-linear dynamical systems. The topic of satellite attitude determination is also covered, as well as a study of orbital manoeuvres around a large body such as Titania. Researches on near-Earth asteroids are also presented, such as transfers to orbits around the Moon for space mining; prevention of a collision of the asteroid Apophis with the Earth and a dynamical study of a disc of particles around Apophis; the dynamical evolution, fates and lifetimes, besides orbital inclination evolution of current large Near Earth Objects (NEOs). Aerospace engineering is explored through the study of periodic lift and drag on the orbit of rectangular parallelepiped satellites, and also the analysis of orbits of a satellite observing the Earth. Further theoretical work concerned the application of the theory of functional connections via the change of variables and the analysis of the linear stability of a tether system using its Hamiltonian function is analyzed in this paper. Since the number of artificial debris in the near-Earth space has increased in the last decade, this topic has become a current problem, thus discussed in details. Solar sails were also researched in detail in this special issue, for example how solar sails can help with lunar flyby maneuvers. Finally, methods for investigating extra solar planetary systems were also applied.

1 Introduction

Celestial mechanics studies the motion of bodies that are mainly subject to the force of gravity. These can be natural bodies or spacecraft. In the case of artificial satellites, e.g. interplanetary probes, the movement also takes into account forces of a different nature, such as the solar radiation pressure, air resistance and the forces generated by the engines from the propulsion systems. The solar radiation pressure and drag also play a role in the investigation of planetary rings. In the field of applied Celestial Mechanics, scientists and engineers are focused on researching new orbital maneuvering techniques and revolutionary new propulsion systems, such as solar and electric sails, to minimize the cost of current and future interplanetary flights. In this context, Celestial Mechanics is present in the past, present and certainly in the future of space exploration, from the advances in our knowledge of the universe to the thousands of satellites that provide communication, navigation, weather forecasting, Earth observation and entertainment.

However, as a remnant of years of space exploration near Earth, a large amount of space debris orbits around our planet - partly consisting of decommissioned satellites and launched vehicles, partly due to collisions between artificial satellites, lost instruments and tools. Adding to this scenario is the current deployment of super constellations of satellites designed to cover the Earth's low orbits providing internet access. The debate on this topic must therefore be broad and it will be carried out in this special issue.

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This special issue is a carefully reviewed compilation of original research articles in the field of pure and applied Celestial Mechanics related to the scientific expansion of astronomy and space activities in Brazil and the XXI Brazilian Colloquium on Orbital Dynamics. The colloquium took place at the National Institute for Space Research in the city of São José dos Campos, Brazil with researchers from South and North America, Europe and Asia and covered topics related to both pure and applied Celestial Mechanics, such as planetary sciences, dynamical and planetary astronomy, solar system dynamics, orbital mechanics, spacecraft attitude control, dynamical systems and chaos in space exploration.

Below, a brief overview of the contributions within this special issue is given.

2 Methods and applications to celestial mechanics

The best way to explore a celestial body and its environment is *in situ*. Finding the best orbital configuration is the key to the success of a mission. In Searching for Orbits to Observe Iapetus, L. S. Ferreira, A. F. B. A. Prado and R. Sfair [1] analyzed the perturbations caused by Saturn in a spacecraft around the satellite Iapetus. Using a series of lifetime maps that give the lifetime of the spacecraft as a function of its inclination (I) and its initial semi-major axis (a), they found certain values for I and a at which the spacecraft can survive for more than 400 orbital periods of Iapetus. Their results also showed a range of initial values for the longitude of the pericentre and ascending node at which the spacecraft can survive up to 25 years.

With the aim of observing the smallest primary, e.g. a moon of a planetary system, A. F. S. Ferreira and A. F. B. A. Prado [2] in Periodic Orbits in the Restricted Three-Body Problem for Observations of the Smaller Primary have presented a series of orbits that require only a few corrective maneuvers to allow predicted passages to the main bodies. In this way, these orbits can be useful to support space stations near the smallest primary or, depending on their properties, to form a round-trip connection between two bodies for transfer.

Another study on orbital maneuvers, but now focusing on Titania, the largest satellite of Uranus, is presented in the paper Orbital Maneuvers for a Space Probe Around Titania by J. Xavier, A. B. A. Prado, S. M. Giuliatti Winter and A. Amarante [3]. By expanding the second-order gravitational potential C_{22} of Titania, the zonal coefficient J_2 and the gravitational perturbation of Uranus, many types of maneuvers are investigated. For example, maneuvers to transfer probes between parking orbits, to correct the variation of periapsis, and to avoid collisions with Titania were analyzed to find ways to maximize the lifetime of the probe around this satellite in future missions.

The next three papers deal with the application of different optimization methods to determine the flight attitude. In Attitude Estimation for Remote Sensing Satellite CBERS-4 using Unscented Gaussian Sum Filter by W. R. Silva, R. V. Garcia, P. C. P. M. Pardal, H. K. Kuga, M. C. F. P. S. Zanardi and L. Baroni [4], the so-called Gaussian Sum Filter and Unscented Gaussian Sum Filter (UGSF) are used to estimate the attitude and polarization of the gyroscopes of the Chinese-Brazilian resource satellite CBERS-4, which is currently in operation. In particular, the use of algorithms based on the UGSF has shown that the processing times for attitude estimation are 5 to 20 times shorter than other algorithms based on the Extended Kalman Filter (EKF) and the Descending Kalman Filter.

In The Computational Efficiency of Derivative-free Estimation Methods under the Respective of Attitude Estimation by R. V. Garcia, H. K. Kuga, W. R. Silva, L. Baroni, M. C. F. P. S. Zanardi and P. C. P. M. Pardal [5], compares the performance of algorithms that consider the Central Difference Kalman Filter (CDKF) with other methods such as the EKF-based, the Unscented Kalman Filter (UKF) and the Cubature Kalman Filter. The use of the CDKF outperformed the EKF and is equivalent to the others in terms of processing time for real-world applications.

L. Baroni, H. K. Kuga, R. V. Garcia, W. R. Silva, M. C. Zanardi and P. C. P. M. Pardal [6] in Performance Evaluation of a Central Difference Kalman Filter Applied to Attitude Determination analyzed the performance of the attitude determination of a CDKF and compared it with both the EKF and the UKF. The performance was evaluated using a 3U Cubesat with gyroscopes as attitude sensors with respect to the quaternion and gyroscope errors. One of the results showed that CDKF has a shorter processing time than UKF.

Asteroids are the subject of five subsequent studies. Since the shape of small bodies, such as asteroids and centaurs, has been well determined using various techniques, it has become important to theoretically model the perturbation caused by their irregular shape in their environment. M. L. Mota, S. Aljbaae and A. F. B. A. Prado [7] in The Potential Series Expansion Method: Application to the Asteroid (87) Sylvia calculated the location and stability of the equilibrium points in the gravitational potential of asteroid (87) Sylvia to compare with the new method they have proposed. They derived the gravitational potential analytically by modeling the asteroid in a sample of homogeneous tetrahedral elements. Their method was compared with the classical polyhedral method, which proved to be very efficient. It preserves the accuracy of the position and the stability of the equilibrium points, while reducing the calculation time.

Research on the transfer of Apollo and Athens class Near-Earth Asteroids (NEAs) from their heliocentric orbits to orbit around the Moon for mineral exploration is presented in the paper Trajectories for Mining Space Mission on Asteroids in Near Earth Orbit by E. Vieira Neto, P. Pires and S. Giuliatti Winter [8]. These transfers involve low-cost maneuvers and temporary gravity capture. The results show that about 1000 Apollo class asteroids and 350 Athens class asteroids could be transferred to orbits around the Moon with a variation in the velocity less than 447 m/s. This work also proposes two maneuvers to avoid the collision of the Athens-class asteroid 2021 EJ3 with Earth around March 11, 2103.

Studying the Prevention of Collision of Asteroid Apophis with Earth by Kinetic Impact, by B. S. Chagas, A. F. B. A. Prado and O. C. Winter [9], shows a technique of deflection by using a swing by with the Earth in a previous close encounter. The central idea is to intensify the velocity of the orbit of the asteroid by taking advantage of its first maximum approach with the Earth. They found that the smallest impulse Δ_v needed to deflect Apophis is − 5 mms^{-1}. According to the authors, this shows the efficiency of the proposed technique compared to the size and speed of the impactor required for such a deflection.

The Dynamical Structure of a Hypothetical Disc of Particles Around the Asteroid 99942 Apophis is presented by R. M. Oliveira, G. Valvano, O. C. Winter, R. Sfair and G. Borderes-Motta [10]. They analyse the fate of a hypothetical disc of massless particles around this NEA under the effects of its highly irregular shape. They found stable and unstable regions and showed how spin-orbit resonances associated with an irregularly elongated asteroid affect the disc. The results show a pattern where particles orbiting Apophis in the equatorial plane are stable, while the unstable particles oscillate strongly out of plane, with some exceptions.

In the paper Analysis of Symmetric Periodic Orbits in a Tripolar System with a Segment, by G. A. S. Boaventura, A. F. S. Ferreira, S. M. Giuliatti Winter and O. C. Winter [11], the modeling of the gravitational field of elongated asteroids with three protuberances, similar to the Near-Earth Asteroid 2003 SD220, is performed. Once the gravitational potential of this peculiar type of asteroid has been determined, the existence of families of periodic and heteroclinic orbits, collision orbits with the asteroid, zero velocity curves and equilibrium points are investigated, with the main aim of analyzing missions to asteroids with this type of mass distribution.

At this time, two studies whose focus are the large NEOs are presented. The first one is The Known Large Near-Earth Objects' Highways: Dynamical Evolution, Fates and Lifetimes, by L. Liberato, R. A. N. Araújo and O. C. Winter [12]. They examined the transfers of NEOs between different regions (near-Earth space, the main asteroid belt, the Jupiter family - comets, the Centaurs and trans-Neptunian) to determine their most common routes and fates. Their numerical results showed that 70% of the studied objects are removed by solar thermal perturbation, while 13% of the objects are lost in space. Only about 14% of the bodies remain in the NEO region, although they could collide with one of the terrestrial planets.

The second one is On the Orbital Inclination Evolution of the Current Large NEOs, by R. A. N. Araújo, L. Liberato, and O. C. Winter [13]. Here, the current distribution of the NEOs' orbital inclination are investigated. Through numerical simulations of the N-body gravity problem, they found that the orbital inclination of NEOs changes during their time evolution under the gravity of the terrestrial planets. As their results have shown, the NEO environment favors the increase in orbital inclination, especially for initially low-inclination orbits, suggesting that NEOs with an inclination less than 5° have only recently arrived in the NEO region. NEOs with highly inclined orbits are more likely to decrease their inclination and leave the region. An average inclination around $\sim$20° to $\sim$60° is favorable for the long-lived NEOs.

Selecting orbits for a satellite to explore an object in space is a very important task in aerospace engineering. The next paper entitled Selecting Orbits for Earth Observations, by L. B. T. Santos, A. F. B. A. Prado, T. F. A. Santos and N. B. Lima [14], covers this topic. Orbits that can be used to collect data from platforms located around the Earth and send them to specific ground stations are analyzed. The geometric configuration of a satellite orbit is studied to optimize certain important factors for space missions. One of the factors studied was to understand the key parameters that optimize the time the satellite is exposed to sunlight. It was found that the higher the satellite altitude and the greater the orbital inclination are, the longer the satellite is exposed to sunlight per orbital period. As the satellites' batteries are charged with the help of sunlight, this information is crucial for extending the useful time of a space mission. Thus, it is possible to choose orbital configurations that optimize the time the satellite is exposed to sunlight and significantly extend the time a given terrestrial latitude remains within a satellite's access range.

Talking specifically about forces acting on satellites with a certain shape, the Influence of Periodic Lift and Drag on the Orbit of Rectangular Parallelepiped Satellites, by J. O. Murcia Piñerosa, R. V. de Moraes and A. F. B. A. Prado [15], is used to analyse the influence of periodic variations of lift and drag on the orbital elements of the satellite, caused by the rotation of the satellite at constant angular velocity. To calculate the lift and drag coefficients, the panel method for free molecular flow is used assuming three selected rectangular parallelepiped satellites similar to the CubeSats standards. The results show that the aerodynamic perturbation can be simplified to a sinusoidal function at constant values for the roll angle and angular velocity. The effects of periodic variations in drag lead to secular perturbations of the semi-major axis and the eccentricity of the orbit, with large differences at the lowest angular velocity. As for lift, it affects the inclination at lower angular velocities because it helps to maintain the values and direction of a given coefficient for a long time. The angular velocity of the satellite changes the faces of the satellite relative to the flow, which leads to oscillations in the values of the drag and lift coefficients. For angular velocities larger than 1.0×10^{-4} $^\circ/s$, the effects of the periodic lift and drag can be neglected and considered as an average value due to the high frequency of the effect. Compared to drag, the lift does not significantly influence the trajectory, since the coefficient is about two orders of magnitude lower than the drag. In this case, the models had a low ratio of surface area to mass. This approach is important to reduce the uncertainty during the decay before re-entry into the Earth's atmosphere.

The amount of artificial space debris, regardless its size, has increased over the last decade and poses a problem for the low Earth orbits.

The paper Ground-based Laser Effect on Space Debris Maneuvering, by J. K. S. Formiga, D. P. Souza Santos, A. F. B. A. Prado, R. Vilhena de Moraes and J. A. A. Diaz [16], investigates the efficiency of an orbital maneuver, taking into account the gravitational effect and a ground-based laser, when space debris approaches the Earth in a heliocentric orbit in the range of 1 cm to 10 cm at an altitude of 100 to 1000 km. The analysis was performed by varying the velocity and energy after the approach considering a single pulse laser. This is important when evaluating the orbital properties of space debris for a better re-entry into Earth's atmosphere or to avoid collisions considering the impulse magnitude performed by a ground-based laser. The analytical model is applied to various factors, such as the fluency of the laser, the inclination of the debris, and the relative motion between the laser and the debris. Some results show that the laser can perform a small change in delta V and has a maximum efficiency in energy variation of 12%, which can be accumulated and provide energy for re-entry. This study can be used to facilitate the fast calculations to obtain better conditions for maneuvering the debris. Therefore, it is possible to describe the limits and real advantages of the maneuver considering the laser and to analyse which are the ideal initial conditions for the objectives of the mission. This area of space maneuvering could be used for ground-based applications as a method to support space debris mitigation.

Among several possibilities to mitigate the debris problem, in Study of Orbital Relocation of Objects in GEO Via Orbital Perturbations and Solar Sail, L. F. Brejão, A. F. B. A. Prado, J. P. S. Carvalho and D. C. Mourão [17] studied a sun-synchronous solar sail on board of a satellite as a propulsion system to bring the satellite into a safe orbit (graveyard orbit) or to guide the inoperative satellite to re-entry in a short time. The dynamical model consists of a satellite under the gravitational influences of the Earth (the zonal and sectoral coefficients are considered up to the sixth degree and up to the second degree and second order, respectively), the Moon and the Sun, as well as the effects of solar radiation pressure. The variation of the eccentricity and inclination of the satellite are analyzed using the planetary Lagrange equations. The most important result is that a solar sail with an area-to-mass ratio of 10 m^2/kg can remove the satellite in about 15 years.

Solar sails have been investigated and explored since costs in space missions may be significantly reduced with the exploration of a renewable energy source. The next papers show three applications using solar sails.

Solar sail Dynamics in the Sun-Earth System: Effects of SRP in the Earth Hill's Region, by G. A. Braz, M. O. Terra and A. F. B. A. Prado [18], investigates the effects on the phase space dynamics of a solar sail in the presence of the gravitational fields of the Sun and the Earth. For this purpose, the Circular Restricted Three-Body Problem, including the pressure acceleration of the solar radiation, prescribes the temporal evolution of the initial conditions located in the Hill region of the Earth. To obtain essential data for the study, the behavior of the equilibrium points and the value of the Jacobi constant associated with them, as well as the zero velocity curves for different values of β (defined as the ratio between the solar radiation pressure acceleration and the gravitational acceleration of the Sun on the sail) in the Hamiltonian system were evaluated. Poincaré sections were obtained for the Hamiltonian case for different values of β. Some remarkable dynamic properties are reported. Possible applications and practical implications for trajectory design are discussed. From this study, it is concluded that by knowing the behavioral changes of the analyzed dynamic properties, better planning of space missions becomes possible. By choosing suitable initial conditions, the natural dynamics of the sail can benefit space missions with different goals.

Low-cost Mars Transfer from the Earth Using Solar Sails and Lunar Swing-bys, by L. G. Meireles, R. S. Ribeiro, C. F. de Melo and A. F. B. A. Prado [19], presents a strategy that combines the use of solar sails and lunar swing-by to reduce the velocity increments (ΔV) required to escape the Earth-Moon system and destinations as far as Mars in the restricted four-body Sun-Earth-Moon-spacecraft problem. Starting from a circular low-Earth orbit, the speed of the spacecraft is increased and the solar sail is controlled to maximize the gain of its orbital energy. This continues until the spacecraft performs a swing-by with the Moon. With the goal of performing a transfer to Mars, this technique resulted in a 16.7% reduction in required speed increases compared to the Patched Conics approach, considering a mission of approximately three years. If the time frame is extended up to ten years, the reduction due to the sail is more than 98%. This proves the exceptional potential that solar sails offer in reducing costs and increasing accessibility of space exploration.

In the paper Optimal Geometry for Lunar Swing-by Maneuvers Aided by Solar Sails in the Restricted Four-Body Problem, by R. S. Ribeiro, L. G. Meireles, C. F. de Melo and A. F. B. A. Prado [20], presents a study of the relative positioning between Earth and Moon in order to define the conditions for the best transfers to the inner terrestrial planets of the solar system using the Restricted Four-Body Sun-Earth-Moon-spacecraft Problem. At the beginning of the numerical simulation, a solar sail with a load of 164.7 g/m^2 is already used to perform transfers to Venus and Mars. The combination of these swing-by maneuvers and the use of the solar sail enabled transfer trajectories that could reach the orbits of Venus and Mars. However, the results are related to specific swing-by conditions, and further studies on the variation of these parameters could be carried out.

The following three studies take the advantage of advanced computing tools to develop their methodologies. In the first, a code in a high-level programming language - Python - for the simulation of stellar occultation light curves by Planet 9 (P9) is presented. Stellar occultation is an observational technique that is often used to study trans neptunian objects. Through this technique it is possible to determine the shape, size and albedo with high precision and also to investigate the existence of atmosphere, rings and satellites around the body. Stellar Occultation Simulator: Application to Planet 9, by W. G. Ferrante and F. Braga-Ribas [21], presents a code written in Python that generates the light curves of the stellar occultation by P9, although the noise from the cameras and the Earth's atmosphere is not yet implemented.

In the second, GPU Accelerated Stability Maps for Extrasolar Planetary Systems: The Kepler-46 System, by A. C. de Souza, F. Roig and X. Saad-Olivera [22], presents a code to compute different chaos indicators for the planetary N-body problem. The code is able to map the stability of the phase space in detail. The Kepler-46 system of exoplanets was analyzed through this code. Their results revealed the main resonant structure of the system, and also constrained the possible radial migration of the Jovian planets.

In the last one, X. Saad-Olivera, C. F. Martínez, M. J. García and F. Roig [23] used the MiultiNest Bayesian inference tool and a N-body integrador in order to reanalyse the transit timing variations data in the Kepler-27 system. One of the missions that revolutionised our knowledge on planetary systems is the Kepler Mission. Among all the planetary systems discovered by this mission is Kepler-27 system. Their paper entitled The planetary System Kepler-27 Revisited explored the dynamics of the planets and the candidates within this system. Their results confirmed that Kepler-27b and c are low density planets with masses smaller than 8 $M_{\oplus}$. Both planets are close to the 2:1 mean motion resonance (MMR). They also constrained the mass of Kepler-27d to be 4^{+3}_{-2} $M_{\oplus}$, which is also close to a fourth order MMR (7:3) with Kepler-27b. Their results ruled out the existence of the candidate KOI−841.04.

Finally, theoretical papers that deal with chaos identification; integral manifolds of the N-body problem; theory of Functional Connections via change of variables, and with Linear Stability analysis in tether system, respectively, are presented.

Several methods have been used to identify chaos in nonlinear dynamical systems, such as Poincaré Surfaces of Section and the Fast Lyapunov Indicator. The study Chaotic Diffusion in the Action and Frequency Domains: Estimate of Instability Times, by G. T. Guimarães and T. A. Michtchenko [24] analyzed the performance of different methods for estimating chaotic diffusion coefficients and diffusion time scales applied to the Circular Restricted Three Body Problem. One of their results showed that the study of chaotic diffusion in frequency space proved to be better in terms of accuracy and computational time, while the Shannon entropy method (SEM) proved to be time consuming. The estimation of the diffusion time scales was analyzed with the frequency-based methods and compared with the numerical integrations. The agreement regarding the instability times was better with the frequency-based methods than with the SEM. As stated by the authors, this work provided a concise and simple study of the main tools used to determine instability times.

The integral manifolds are level sets in the N-body problem of the conservation variables parameterized by the angular momentum c and the energy h. Alain Albouy has identified two categories of singular values of h: finite bifurcations (relative equilibria: solutions of the N-body problem revolving around the centre of mass) and infinite bifurcations. In Patterns in Bifurcation Levels of the Integral Manifolds of the N Body Problem, H. G. Havel and C. K. McCord [25] applied the analysis proposed by Albouy to study the patterns of these two types of bifurcations, finite and infinite. One of their results showed that the conjecture that all infinite bifurcations occur at levels smaller than the finite ones is false. They also concluded that the separation between the two types of bifurcations does not hold as the value of N grows.

In the paper Orbit Transfer Using Theory of Functional Connections Via Change of Variables, by A. K. de Almeida Jr, A. F. B. A. Prado and D. Mortari [26], the authors show that a class of astrodynamics problems subject to mission constraints can be efficiently solved using the theory of functional connections (TFC) by a specific change of coordinates and how to apply the coordinate change method to the perturbed Hohmann-type and the one-tangent burn transfer problems. In these transfer problems, the boundary conditions are typically nonlinear and coupled when expressed in the traditional rectangular coordinates. By changing variables, these nonlinear constraints are transformed into linear ones. This constraint is required by the TFC framework to solve the associated boundary value problems. In addition to the linear constraints, this change of variables also allows obtaining decoupled equations. These results were obtained by exploiting the symmetries of the problem. These properties are also very useful for other approaches to solve the transfer problems. The change from rectangular to polar coordinates enabled the use of the TFC frame to solve the one-tangent burn transfer method. In this work, the transfer is performed from a LEO to a geosynchronous orbit, but it could also be performed to another orbit. Another application is the evaluation of transfers for near-Earth satellites, where perturbations such as drag and other terms of the Earth's gravitational potential can also be taken into account.

The aim of the paper Linear Stability Analysis in Tether System Using its Hamiltonian Function, by D. P. Souza Santos, J. L. Menezes Neto, V. T. Azevedo and J. K. S. Formiga [27], is to investigate the behavior of a tethered system consisting of two masses connected by a cable, which are in Keplerian orbit around a Newtonian centre of attraction, with no external forces acting on the system. Starting from the problem of tethers in a Keplerian orbit around a Newtonian centre of attraction, after some reductions, the two equations of motion are given in terms of the angle of rotation of the tethers about their centre of mass in the orbital plane, the angle of elevation relative to the plane of the orbit, the eccentricity of the orbit, and the true anomaly. A Hamiltonian function is obtained which is linked to these equations. Four stationary solutions are found, two of which are stable. The parametric linear stability is studied with respect to the parameter of the eccentricity of the orbit and another parameter related to the angle between the projection of the tether and the plane of the orbit. Using the Deprit-Hori method and numerical calculations, curves are plotted that separate regions of stability and instability in the plane of these two parameters. An analytical analysis of the linear stability is presented for small values of eccentricity.

3 Future challenges

Celestial mechanics is a well-established field. However, as has been shown, there are still some open problems and more to come. Researches in this field are based on a combination of theoretical models, observations by telescopes or by cameras on board of spacecraft and numerical simulations. Progress in the accuracy of observational capabilities, in parallel with advances on theoretical methods and computational tools, keep the field moving forward.

As an example, asteroids can have irregular shapes. Such distribution of mass associated with spin-orbit resonances can model the ring distribution of particles around them. Modeling this asymmetry is a problem under analysis nowadays, for instance the dynamical environment around (99942) Apophis and (87) Sylvia, taking into account their peculiar shape, are currently under investigation. The asteroid Apophis will pass very close to Earth in 2029, although it does not pose as a threat for our planet, it will pass around 32000 km from Earth, where geosynchronous satellites orbit. Many efforts have been made to collect as much as possible information before and after the close approach.

The large number of inoperative satellites, along with others that will soon reach this condition, countless debris generated by the remains of launch vehicles, collisions between satellites, accidental and intentional explosions, tools lost in missions over 66 years form a dangerous cloud of debris around the Earth and could, in a short term, make the use of space unfeasible, jeopardising the operation of active satellites, manned missions and even astronomical observation activities. The control and mitigation of space debris is therefore an important area of current research. As the amount of debris near Earth is increasing every year, the improvement of methods to mitigate the problem is a great challenge, for example the use of solar sail is been studied in many ways, as a renewable energy source for propulsion system, besides the effect of the use of ground-based laser.

According to data from [28], in 2018 there were around 2000 active satellites in orbit, and we will end 2023 with around 7700 operational satellites. Of these, 5500 are in LEO and are from the Starlink super constellation for fast internet owned by a private company, SpaceX, and this number could soon reach 42,000. The other 2200 satellites carry out Earth observation, communications, weather forecasting, navigation and scientific missions, among other purposes. There are still around 3600 satellites in orbit that are currently out of operation. Thus, researches on control theory, computer technology, networks, the Internet of Things, robotics and machine learning/artificial intelligence are essential areas to support the exploration of space around the Earth, as well as providing detailed and mass exploration of deep space: planets, moons and asteroids, mainly considering the long-distance between the controlling station and the intended site of exploration.

Another possible future challenge in aerospace engineering is the extraction of materials (minerals, metals and water) from asteroids. Many aspects are linked to the future of space mining, such as regulatory agreements and the impact on the space environment, but also studies in the orbital dynamics area. Choosing the best trajectory for the spacecraft to and from the target is crucial for the success of the mission. The project should include low-cost maneuvers and temporary gravity capture, most likely taking advantage of gravity assist.

As has been said, modern computer tools play an important role in expanding the knowledge about our and extrasolar systems, as they make it possible to process large amounts of data, perform a large number of simulations, as well as reduce data from telescopes or satellites using specialized softwares. Through statistical analysis and data manipulation, it is possible to study extrasolar planets. Among the positive results from NASA's Kepler mission was the discovery of Kepler-27 system. The investigation of such systems expands the potential for finding life in the universe. The diversity of the characteristics and configurations of planets outside our solar system are enormous, as has been shown since 1992. The extrasolar systems probably have the most unlimited problems waiting to be revealed.

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