



Collective behavior of nonlinear dynamical oscillators

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Abstract This topical issue collects contributions of recent achievements and scientific progress related to the collective behavior of nonlinear dynamical oscillators. The individual papers focus on different questions of present-day interest in this topic.

1 Introduction

Complex networks are formed by the interconnection of many dynamical systems [1]. Based on the type of communication between these systems, they may exhibit different collective behaviors [2]. In real applications, interactions are inevitable, making the research of collective behaviors highly important. An efficient tool for this purpose is connecting nonlinear dynamical systems in complex structures that can be different according to the application.

The interconnection among systems may lead to coherence in their dynamics. The characteristics of this coherence determine the emergence of different collective behaviors. One of the essential collective behaviors is synchronization, generally defined as the time-correlated dynamics of coupled systems [3]. Synchronization has several types, including phase, lag, almost, generalized, explosive, and complete synchronization [4]. Although complete synchronization, i.e., quite similar temporal dynamics, is mainly preferred, it is not achievable in many networks. Therefore, one of the critical issues to study is the possibility of synchrony in a network. Various numerical and analytical methods have been presented to search for this problem [5]. For example, a straightforward and efficacious approach is the master stability function (MSF) [6]. Applying the MSF enables the necessary conditions to be obtained for complete synchronization according to the coupling

scheme. Another important issue for study is investigating various factors in the occurrence of other types of synchronization and their stability area [7–9].

Certain studies have considered the formation of partial synchronization patterns [10, 11]. In these cases, the motivation can be obtaining the emergence conditions, influential factors, stability area, lifetime, and controllability. The chimera state is one of the attractive states among partial synchronization patterns [12]. This state is organized by coexisting coherent and incoherent oscillations in a network. The chimera state can be divided into categories depending on the features of the coherent and incoherent groups, including the nonstationary chimera, traveling chimera, and so on [13, 14]. The relation between the chimera state and the neuronal processes has increased the importance of studying this phenomenon [15]. In addition to the chimera, the solitary state has also attained considerable attention [16]. However, most systems are synchronous in the solitary state; only a few escape from the synchronous state and oscillate differently.

This special issue is devoted to the current state of the art in the research on the collective behavior of nonlinear dynamical oscillators. Many efforts have been made to find the dependence of synchronization on the network structure and increase the possibility of synchronization. In this regard, the authors in [17] report that synchronization can be attained by switching only one link, whereas synchronization is impossible in both alternating structures. According to the master stability function (MSF) method, the requisite for synchronization in a network is that all its normalized coupling parameters are located in the MSF stability region. This paper shows that synchronization can be obtained if the network switches between two structures, each with only one normalized coupling parameter out of the

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stability region. Another significant problem is preserving synchronization during network reduction. Reducing the dimension of large-scale networks helps decrease the computational complexity and required memory. The paper [18] presents an optimization algorithm to preserve synchronization after node reduction. The proposed algorithm tries to keep the eigenvalues of the network unchanged while reducing the network size.

Synchronization and consensus of multi-agent systems have attracted much attention in recent years. In [19], the authors focus on presenting an event-triggered protocol for obtaining global exponential consensus in Lipschitz nonlinear multi-agent systems. Practical constraints such as the unknown time-varying input delay and state quantization constraints are also considered. The result of this study is finding a sufficient condition on the upper bound value of time-varying input delay for the global consensus of multi-agent systems.

Synchronization of coupled chaotic systems and circuits has special applications, for example, in secure communication. The authors in [20] investigate the dynamics of coupled Wang–Zhang–Bao circuits. They show that this system exhibits bistability for a range of parameters. The synchronization of two-coupled and 100-coupled circuits is analyzed using the MSF method and computing the temporally averaged synchronization error. In [21], synchronization of coupled Josephson junction oscillators is considered. Two sliding-mode control strategies, namely integral and terminal, are designed to synchronize the oscillators. The results show that the first method has a smaller error than the second, and its implementation is also more accessible. However, the second method is faster. In [22], the effects of different coupling elements on the collective dynamics of the Chua circuits are examined. It is shown that complete synchronization is only obtained when the circuits are coupled via resistors, while in memristor-coupled circuits, instability emerges for larger couplings. If inductors connect the circuits, increasing the coupling can only synchronize two systems' variables. Furthermore, the nonstationary chimera can be formed in all coupling schemes.

Previous studies have revealed the significance of the synchronized behavior of neurons in brain activity. Hence, several papers in this special issue are dedicated to studying synchronization in neuronal models. Vivekanandhan et al. [23] study the impact of an electric field on the firing pattern and synchronization of the Izhikevich neuron models. Their investigations of the firing patterns show that the electric field intensity and the external electric field parameters greatly influence the firing of neurons. Moreover, synchronization is achieved in stronger couplings in the presence of the electric field. Yang and Ma [24] investigate the dynamics of a star network composed of four photosensitive neurons. The coupling is considered adaptive such that the coupling intensity is increased exponentially to a threshold value before attaining energy balance. Complete synchronization and energy balance can be observed in this network by enhancing the coupling.

In [25], a neural network model proposed for the up-to-down state oscillations of the cortex is used. The model consists of excitatory and inhibitory neurons. The authors investigate synchronization by constructing a network of this neural model. Their results show that the synchronization relies significantly more on the excitatory connections than the inhibitory ones.

Map-based models can decrease the computational cost even though they can reproduce many neuron behaviors. Therefore, the use of map-based models is beneficial for investigating neural synchrony. Sayari et al. [26] consider a random network of Rulkov maps with burst-timing-dependent plasticity. They also investigate the effects of periodic and random external perturbations on the network synchronization. They report that the perturbations can induce synchronization and synchronization states which are dependent on the initial synaptic weights. In [27], Wang et al. study synchronization in coupled map-based neurons by considering the memristive synapse. They demonstrate that the synchronization is dependent on the memristor coupling coefficient and the initial condition of the flux variable. When the memristor coupling coefficient increases, the network alternates between synchrony and asynchrony and, finally, becomes synchronous. The initial condition of the flux variable also determines the range of the instability zone. In [28], Shang et al. use a discrete memristor for the coupling of Chialvo neurons. This study considers two coupled neurons and a ring of coupled neurons, and the occurrence of synchronization and emergence of a chimera state are investigated.

Wave propagation and pattern formation are other important subjects in neuronal networks. In [29], a two-layer network of neurons with gradient field coupling is considered. Two types of gradient coupling are assumed as step-like and cone-like. For the small central intensity of the gradient field, the cone-like coupling causes less damage to the target waves in the first layer. As the central intensity of the gradient field increases, the target waves are destroyed, and spiral waves are formed. Furthermore, larger external stimulations induce spiral waves in the second layer. The authors in [30] study turbulent exotic waves in a network of piecewise linear learning neuron models containing magnetic flux coupling and periodic excitation. It is shown that as the excitation amplitude increases, the exotic waves become very turbulent.

The chaotic activity can influence the electrical activity and firing rhythms of the single and coupled neurons. In [31], the spiking inhibition behavior of Hodgkin–Huxley (HH) neurons induced by the chaotic activity is studied. The mean firing rate is observed to reach a minimum according to the chaotic activity, which is called the inverse chaotic resonance. It is found that the types of synaptic currents, the number of spiking neurons, and each neuron's firing rate affect the network's collective firing rate. The rhythmic behaviors also have essential roles in the respiratory network. To study the firing patterns or rhythms and the transitions between different patterns, bifurcation analysis

can be used. The paper [32] studies the rhythm transitions in coupled sigh and eupnea compartments of the pre-Bötzinger complex, which is the respiratory network in mouse brainstem slices. The authors investigate the effects of different ionic currents and analyze the underlying mechanism of bifurcations.

Another part of the special issue is devoted to the contributions concerning partial synchronization patterns. In [33], Ramamoorthy et al. report transitions between different collective behavior induced by the repulsive coupling in Stuart–Landau oscillators by varying the control parameter. For example, by decreasing the control parameter, the state of the network transitions from a traveling wave to an imperfect traveling chimera, then to the synchronized state. Other collective behaviors such as cluster chimera death, cluster oscillation death, imperfect amplitude chimera, and multi-chimera death state are also observed in this study. In [34], the authors concentrate on the mechanism of the formation of the solitary state in complex networks. This study considers three different network topologies: a symmetric nonlocally coupled ring, a random, and a scale-free network. The occurrence of the solitary state is studied for various parameters. It is found that the solitary state is more likely to form in asymmetric networks with scale-free properties. Fan et al. [35] consider a two-layer multiplex network of discrete Hindmarsh–Rose neurons with a flux-controlled memristor. The interlayer connections are through chemical synapses, while different synapses are examined for intralayer links. The simulation results for this network show that the synchronization of the neurons mostly depends on the intralayer connections. Moreover, the chemical interlayer links have an essential role in the emergence of the chimera state in layers.

Finally, the special issue ends with a study on the attenuation rate of an electromagnetic wave in plasma. Although the interaction between the electromagnetic wave and the plasma has been studied extensively, its attenuation in an unmagnetized collisionless cold plasma has not been considered. In [36], the authors explore this issue through two-dimensional (2D) particle-in-cell simulation. The evolution of electromagnetic waves through plasma with different sizes is considered at different frequencies. It is found that electromagnetic energy reduction is more remarkable in plasma with larger sizes.

Thus, this special issue provides a broad spectrum of current research on the collective behavior of complex networks, and we hope that the related researchers in this field will find it useful. We wish to express our appreciation to the authors of all the papers in this special issue for their excellent contributions, as well as to the many reviewers for their high-quality work in reviewing the manuscripts.

Data availability No data has been used in this study.

References

1. S.H. Strogatz, Exploring complex networks. *Nature* **410**(6825), 268–276 (2001)
2. S. Boccaletti et al., Complex networks: structure and dynamics. *Phys. Rep.* **424**(4–5), 175–308 (2006)
3. A. Arenas et al., Synchronization in complex networks. *Phys. Rep.* **469**(3), 93–153 (2008)
4. S. Boccaletti et al., The synchronization of chaotic systems. *Phys. Rep.* **366**(1–2), 1–101 (2002)
5. L.M. Pecora and T.L.J.P.r.l. Carroll, Synchronization in chaotic systems. **64**(8), 821 (1990)
6. L.M. Pecora, T.L. Carroll, Master stability functions for synchronized coupled systems. *Phys. Rev. Lett.* **80**(10), 2109 (1998)
7. G.A. Leonov, Phase synchronization: theory and applications. *Automation remote control* **67**(10), 1573–1609 (2006)
8. M.G. Rosenblum, A.S. Pikovsky, J. Kurths, From phase to lag synchronization in coupled chaotic oscillators. *Phys. Rev. Lett.* **78**(22), 4193 (1997)
9. L. Kocarev, U. Parlitz, Generalized synchronization, predictability, and equivalence of unidirectionally coupled dynamical systems. *Phys. Rev. Lett.* **76**(11), 1816 (1996)
10. L. Ramlow, et al., Partial synchronization in empirical brain networks as a model for unihemispheric sleep. *EPL (Europhys. Lett.)*, **126**(5), 50007 (2019)
11. B. Ao and Z. Zheng, Partial synchronization on complex networks. *EPL (Europhys. Lett.)*, **74**(2), 229 (2006)
12. D.M. Abrams, S.H. Strogatz, Chimera states for coupled oscillators. *Phys. Rev. Lett.* **93**(17), 174102 (2004)
13. A.V. Slepnev, A.V. Bukh, T. Vadivasova, Stationary and non-stationary chimeras in an ensemble of chaotic self-sustained oscillators with inertial nonlinearity. *Nonlinear Dyn.* **88**(4), 2983–2992 (2017)
14. O. Omel'chenko, Traveling chimera states. *J Phys. A*, **52**(10), 104001 (2019)
15. S. Majhi et al., Chimera states in neuronal networks: a review. *Phys. Life Rev.* **28**, 100–121 (2019)
16. N. Semenova, T. Vadivasova, V. Anishchenko, Mechanism of solitary state appearance in an ensemble of nonlocally coupled Lozi maps. *Eur. Phys. J. Spec. Top.* **227**(10), 1173–1183 (2018)
17. T. Moalemi, F. Parastesh, T. Kapitaniak, When switching makes impossible synchronization possible. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00692-6>
18. N. Naseri et al., An optimization method to keep synchronization features when decreasing network nodes. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00626-2>
19. F. Golestani, M.S. Tavazoei, Event-based consensus control of Lipschitz nonlinear multi-agent systems with unknown input delay and quantization constraints. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00634-2>
20. R. Lu et al., Synchronization and different patterns in a network of diffusively coupled elegant Wang–Zhang–Bao circuits. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00690-8>

21. F. Serrano, D. Ghosh, Sliding mode synchronization of complex resonant Josephson junction network. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00695-3>
22. R. Lu et al., Network dynamics of coupled Chua circuits: comparison of different coupling elements. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00632-4>
23. G. Vivekanandhan et al., Firing patterns of Izhikevich neuron model under electric field and its synchronization patterns. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00636-0>
24. F. Yang, J. Ma, Synchronization and energy balance of star network composed of photosensitive neurons. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00698-0>
25. J. Kang et al., Complete synchronization analysis of neocortical network model. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00630-6>
26. E. Sayari et al., Dynamics of a perturbed random neuronal network with burst-timing-dependent plasticity. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00694-4>
27. Z. Wang et al., Synchronization in a network of map-based neurons with memristive synapse. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00691-7>
28. C. Shang et al., Dynamics and chimera state in a neural network with discrete memristor coupling. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00699-z>
29. Y. Wu et al., Pattern formation induced by gradient field coupling in bi-layer neuronal networks. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00628-0>
30. A. Karthikeyan et al., Complex network dynamics of a memristor neuron model with piecewise linear activation function. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00700-9>
31. D. Yu et al., Inverse chaotic resonance in Hodgkin-Huxley neuronal system. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00629-z>
32. H. Hua, H. Gu, Bifurcations underlying sigh and eupnea rhythmic transition in a pre-Bötzinger complex model. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00631-5>
33. R. Ramamoorthy et al., Impact of repulsive coupling in exhibiting distinct collective dynamical states. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00627-1>
34. L. Schülen et al., Solitary states in complex networks: impact of topology. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00713-4>
35. W. Fan et al., Synchronization and chimera in a multiplex network of Hindmarsh-Rose neuron map with flux-controlled memristor. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00720-5>
36. D.-N. Gao, S.-M. Lin, W.-S. Duan, Attenuation of electromagnetic waves in an unmagnetized collisionless plasma by particle-in-cell method. *Eur. Phys. J. Spec. Top.* (2022). <https://doi.org/10.1140/epjs/s11734-022-00633-3>