



Novel memristor and memristor-based applications

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Abstract This special issue presents 25 papers regarding memristor systems and their applications. Part of the manuscripts are about memristor-based neural systems, and the systems show rich dynamics. Multistability in memristive systems has aroused much interest, and nonlinear behaviors, such as hidden attractors, extreme multistability, and transient chaotic transition, are observed. There are also two works investigating controllers of fractional-order memristor chaotic systems and several for circuit implementation of memristor chaotic systems. As new research, discrete memristor chaotic maps are reported and show potential application value in the future. Finally, there are two papers about the application of memristive chaotic systems to image encryption fields, indicating that memristive chaotic systems can be used in the information security field.

1 Introduction

In 1971, Leon Chua theoretically demonstrated that there is a two-terminal nonlinear element with variable resistance, and he called it a memristor. In 2008, scientists at Hewlett Packard (HP) Labs realized a physical memristor, which aroused great interest in memristors and memristor-based systems. However, there is a lack of a commercial memristor at present because of the high cost of manufacturing nanoscale electronic components. Therefore, to improve the theoretical framework and applications of memristors, it is necessary to investigate the functional memristor emulator and its intrinsic features. Additionally, the tools for memristor models should be diversified, such as fractional-order derivative, fractional-order difference, analog circuit, and digital circuits. Therefore, memristor-based studies are collected in this volume, including neural systems [1–3], multistability and hidden dynamics [4–10], synchronization controller [11, 12], circuits [13–18], discrete memristor chaotic maps [19–23], and image encryption applications [24, 25]. Some of the contributions discuss several aspects. For instance, Wang et al. [18] reported dynamical behaviors, symmetric coexistence, analog circuit, and hardware implementation of a proposed fractional-order memristor nonlinear chaotic system.

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In this editorial, a detailed overview of the contributions within this special issue are presented, and Sect. 2 is the main body. Section 3 summarizes this paper, and the future prospects for memristors are pointed out.

2 Overview of the contributions

2.1 Memristor-based neural system

Chen et al. [1] proposed a Hopfield neural network by introducing a memristive synaptic weight to the network. This designed memristive neural network model shows interesting dynamics including coexisting bifurcations, coexisting multiple attractors, transient behaviors, and memristor initial boosting behaviors. The authors also validated the facticity of the intricate dynamics by analog circuit simulation.

Feng et al. [2] designed a double time-delayed flux neural system with a magnetron memristor based on the Hindmarsh–Rose neuronal model. Using the Routh–Hurwitz judgment rule and the central manifold theory, the stability, existence of Hopf bifurcation, and direction of bifurcation of the proposed system were investigated. The authors also indicated that a periodic bifurcation solution exists in this time-delayed flux neural system.

Li et al. [3] investigated a time-delayed locally active memristor and analyzed it by plotting its power-off plot,

direct-current (DC) voltage–current (V – I) plot, and pinched hysteresis loops. By introducing the memristor to the Hopfield neural network, a new system was analyzed. In fact, this method has been widely employed, but it shows rich dynamics in the proposed system. The authors also designed an analog circuit and a chaotic image encryption scheme. The results showed that the proposed method has high security and reliable encryption performance.

2.2 Multistability and hidden dynamics

Wu et al. [4] introduced an ideal memristor to a three-dimensional linear system, then a four-dimensional sine-modulation-based memristive chaotic system was designed. They found that this memristor chaotic system has complex dynamics, and initial conditions boosting the plane bifurcation could be observed. Based on the circuit designed for the given memristive nonlinear system, the authors have verified the theoretical and numerical analysis results.

Wan et al. [5] introduced a flux-controlled memristor and a linear feedback term to a three-dimensional (3D) nonlinear chaotic system, and a variable-wing five-dimensional (5D) memristive hyperchaotic system was constructed. The proposed 5D system has a line equilibrium point whose position is determined by the control parameter. Additionally, the 5D system shows many interesting phenomena, including hidden extreme multistability, transient chaotic transition behavior, and variable-wing characteristics.

Li et al. [6] introduced a tri-stable locally active memristor in which the internal state equation is related to the internal state and voltage as defined by a series of sign functions. A third-order nonlinear chaotic circuit was designed by introducing a capacitor into the second-order oscillator circuit to build a chaotic circuit. Dynamic analysis and circuit implementation were carried out, which showed a simple structure based on the circuit and complex characteristics from the output results.

Ramadoss et al. [7] designed a novel memristive oscillator with infinite equilibria. The dynamics were analyzed with the variation of system parameters. The oscillator displayed attractive features such as bubbles in bifurcation and multistability. Finally, PSpice was used to simulate the proposed circuit and demonstrated its feasibility.

Huang et al. [8] constructed a hyperchaotic jerk circuit based on a cubic memristor using a so-called memristor replacement method. Rich dynamical behaviors such as period circles, chaotic state, and hyperchaotic state were observed. Meanwhile, the attractor displacement phenomenon resulting from the offset constant was observed. Also, the coexistence of asymmetric attractors was found in the proposed system and verified by the circuit simulation.

Xu et al. [9] investigated multistability in a non-autonomous memristive jerk circuit in which the memristor had quadratic memductance and an externally

applied stimulus. Then, multistability near the imperfect bifurcations was investigated by means of a local attraction basin and phase portraits, where four representative sets of system parameters were considered. Moreover, a printed circuit board (PCB)-based analog circuit was obtained, and the experimental results confirmed the effectiveness of the presented analysis.

Wang et al. [10] investigated nonlinear dynamics of a coupled memristive Hindmarsh–Rose neuron system, where the memristor illustrates the effects of electromagnetic induction. The model showed extreme multistability with the initial conditions of the memristor. It also showed that the memristive Hindmarsh–Rose neuron system can generate four clusters in periodic firing, and synchronous variations in different amplitudes were observed.

2.3 Control and synchronization

Liu et al. [11] studied the dynamical behaviors of a fractional-order memristor chaotic system based on Chua's circuit. They concluded that system parameters and fractional-order q could cause complex dynamical behaviors. Through the implementation of a fractional-order memristor chaotic circuit system, the authors also proved that the circuit implementation was consistent with the numerical simulation results.

Huang et al. [12] proposed a finite-time adaptive synchronization method for fractional-order memristor chaotic systems, in which a new type of integral sliding-mode surface with fast convergence speed is considered. Numerical simulations were carried out to verify the effectiveness of the controller, and the synchronization time needed was obtained by adjusting the parameters of the controller.

2.4 Circuit implementation

Sun et al. [13] illustrated the implementation of a 3D crossbar array using purely memristive cells. The authors reported that the proposed 3D crossbar array nonvolatile memory has higher reading/writing speed, higher storage density, and lower power consumption. Moreover, the authors also found that one can increase the storage density to n -bit storage by expanding the memristive clusters or by expanding the memristive memory cells.

Chen et al. [14] designed a generalized charge-controlled memristor with two stable equilibrium states and bistable characteristics. It shows an S-type DC V – I plot which indicates that the proposed memristor has a locally active region of a negative slope. The proposed memristor system is investigated, and complex dynamical behaviors are observed. Finally, an analog circuit was built for this system to verify the design.

Premraj et al. [15] investigated a quasi-periodically driven memristor based on the van der Pol oscillator, and a strange non-chaotic attractor (SNA) was observed. They showed that the torus of the system

can be fractalized and that SNAs emerge by changing one of the external amplitude forces.

Dou et al. [16] designed a fifth-order chaotic circuit with extreme multistability based on a physical memristor and memcapacitor. They showed that the initial condition-dependent extreme multistability emerges based on the plane equilibrium of this chaotic system. By modifying the initial values and parameters of the system, the authors also observed the coexistence of chaotic attractors and the state transition phenomena.

Guo et al. [17] introduced a physical memristor device to a single-T chaotic circuit. The characteristics of the circuit were analyzed by using the dynamic analysis method such as a phase diagram, Lyapunov exponent spectrum, bifurcation diagram, and Poincaré map. In fact, due to the lack of the physical memristor, it is important to investigate application of the physical memristor.

Wang et al. [18] constructed a fractional-order memristive chaotic system and investigated the dynamical behaviors of this system. The system showed excellent and various dynamic behaviors with a quadratic nonlinear memristor. It was observed that the symmetric coexistence attractors could be found with derivative order and system parameters. Meanwhile, the analog circuit and hardware implementation were carried out to verify the numerical simulation results.

2.5 Discrete memristor chaotic maps

Peng et al. [19] introduced three discrete memristors into a classical sine map after summarizing their characteristics, and then three different kinds of two-dimensional discrete memristive sine maps were designed. Dynamical analysis results demonstrated that the proposed method can expand the scope of chaos and improve the value of the maximum Lyapunov exponent. Compared with classical chaotic maps, the proposed discrete memristive chaotic maps have better performance. Employing the parameter identification method showed that the proposed chaotic memristor maps have lower identification rate and higher security.

Li et al. [20] proposed a memristor with window function by considering the boundary effect, and the discrete fractional mathematical model of the memristor was established based on the fractional difference theory. It was then applied to a logistic map for constructing a discrete fracmemristor logistic map.

Li et al. [21] reported two generalized models of a discrete memristor based on sampling discretization of a continuous memristor model. Then the discrete memristive logistic map (DM-L map) and 2D discrete memristive Hindmarsh–Rose neuron model (DM-HR model) were constructed based on the proposed DM models. The results indicate that the designed DM-L map has linear fixed points. As a result, the stability is related to system parameters. Since the DM-HR system does not have fixed points, the proposed system can have hidden periodic and chaotic attractors.

Wang et al. [22] proposed a discrete memristor-based chaotic map (3D-MCM) and three S-boxes to enhance the security of an encryption scheme. Applying the 3D-MCM and S-boxes to design an encryption algorithm, it is interesting that each half-pixel is substituted with a half-pixel substitution box (HPS-box) before S-box substitution with random time. The simulation results in this work also verify the effectiveness of the proposed image encryption scheme.

Dong [23] proposed a two-dimensional non-degenerate memristive nonlinear chaotic map. And it was applied to design a keyed chaos-based hash function with a dynamic S-box and parallel mode, showing the potential values for real applications in block chain and mass data.

2.6 Applications to image encryption

Sha et al. [24] designed a cross-plane color image encryption method based on the binary sort tree model and memristor-based chaotic system. According to the experimental results and performance evaluations, the proposed image cryptosystem has superior security and high efficiency.

Xu et al. [25] proposed a new four-dimensional memristive chaotic system that can exhibit complex dynamical behaviors according to the analysis results by means of bifurcation diagrams, phase diagrams, Lyapunov exponents, correlation dimension, and Kolmogorov entropy. Then the system was applied to design an irreversible key expansion algorithm, which can effectively avoid the problems of linear round-constant and reversible transformation.

3 Conclusion and discussions

At present, memristor-based nonlinear systems design is a hot topic. Although there are many different aspects about memristor-based nonlinear chaotic systems reported, such as a memristor-based neural system [1], multistability [8], synchronization [11], and discrete memristor [19], some of the memristor-based applications are not discussed, such as logical circuit, storage circuit, and artificial neuron system. Meanwhile, physical memristor devices [17] should receive more attention in the future since they will ultimately enable the realization of commercial applications. We hope that this special issue will contribute to constructive discussions on memristor-based nonlinear system modeling and stimulate further results in related fields.

Future directions Increasing interest among scientists to create high-performance computing devices with less energy consumption has been made possible by the introduction of memristor-based elements. Artificial intelligence has witnessed rapid growth in several

domains, and higher efficiency to store data and perform higher-level computations are the basic requirements. These requirements can be fulfilled by memristive elements that are developed using lithium-based oxides. Proficiency, responses, and adaptability to new environments makes memristor elements ideal components for future. Thus, it is clear that the understanding and implementation of the memristive devices to enhance neuromorphic computing which is vital in development of artificial intelligence are to be initiated. The dynamical state of memristive elements and the circuit implementation for its application in neuromorphic computing need to be further explored. Choosing the memristive elements to be incorporated for better performance of systems and storage of data could be a possible direction for future research.

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