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Editorial

Focus point on uncertainty quantification of modeling and simulation in physics and related areas: from theoretical to computational techniques

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The main goal of this Focus Point volume is to explore new advances on theoretical and computational techniques for uncertainty quantification (UQ) of modeling and simulation in relevant problems appearing in physics sciences. The papers submitted to this volume were received by invitation. As a result we have collected a number of high-quality contributions dealing with a rich and update picture of UQ analysis that includes its interplay with laws in physics problems, mainly, formulated via differential equations.

The following text highlights some of the primary aspects of the topics covered as well as the contributions of the published papers.

The consideration of randomness in the analysis of mathematical models for analyzing relevant problems related to health is currently a main research topic. The paper "**The role of noise in the tumor dynamics under chemotherapy treatment**" proposes a two-dimensional stochastic nonlinear differential system, taking into account the impact of chemotherapy on both the tumor and the healthy cells. The randomness is driven by two uncorrelated Gaussian noises with different levels of intensity. Authors analyze this interesting problem combining a number of techniques that include bifurcation diagrams, phase portraits, time series and probability density plots. The contribution is aimed at offering a trustworthy explanation of tumor growth under medical treatment.

The Focus Point volume continues with the study of a second model also belonging to Medicine, specifically, to Epidemiology. In "**Random resampling numerical simulations applied to a SEIR compartmental model**," authors revisit the SEIR model addressed to study the dynamics of COVID-19 in Spain during its first pandemic-wave in 2020. Authors concentrate on estimating model parameters taking into account data uncertainties by means of bootstrap sampling. The validation of the stochastic analysis is performed by different goodness-of-fit measures. They show that the point estimates obtained by the bootstrap samples improve the ones of the original data.

The use of fractional and stochastic calculus has emerged, mainly, during the last decades with important applications in Physics. In "**Hölder continuity of mild solutions of space–time fractional stochastic heat equation driven by colored noise**," authors combine the Caputo fractional derivative with the fractional Laplace operator to analyze the heat equation with noise driven by a colored noise. The paper establishes Hölder continuity of mild random field solutions of the aforementioned PDE by, first, taking advantage of key properties associated to the space–time fractional Green function.

In "**Asymptotic controllability of nonlinear Fokker–Planck equations**," the author studies a problem related with stabilization of the control for a Fokker–Plank PDE. This equation often appears in kinetic theory of statistical mechanics and, contains, as a particular case, the classical Einstein–Smoluchowski equation, which describes the time evolution of the probability density of Brownian particles in a field of forms with a given potential. In the paper, one identifies a class of final states (reachable states) for which the problem has solution. The results can have interesting applications in control theory and physics.

The stochastic of multiphysics systems is often a challenging problem because different physical scales and physical effects must be properly combined to accurately conduct the corresponding analysis. When the study is done in the practical setting, it usually involves the rigorous treatment of large sets of data. In the paper "**Random stick–slip oscillations in a multiphysics system**" (Fig. 12), one combines stick–slip oscillations, electromechanical coupling and uncertainties to characterize and quantify the stochastic response of a multiphysics system in terms of two physical quantities, duration and position, which are dependent random variables that must be jointly studied.

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The development of innovative methods for estimating relevant quantities when dealing with stochastic physical system has a great interest in the scientific UQ community. In the paper "**State and parameter estimation of stochastic physical systems from uncertain and indirect measurements**," authors present a deep analysis on various issues related to inference methods that play an essential role for estimating parameters in stochastic models formulated via stochastic differential equations. Particularly, they analyze the Fisher information matrix and the estimation of confidence intervals in connection with the evaluation of the fitting-innovation processes as Gaussian white noise. The results are nicely illustrated by several examples including a stochastic oscillator.

Designing strategies to protect the state of quantum systems from environmental noise is currently a crucial problem under study. In the paper: "**Quantum state and entanglement protection in finite temperature environment by quantum feed-forward control**" (Fig. 2), authors present the quantum feed-forward and its reversal scheme for protection of one-qubit state from finite temperature thermal noise. They describe how to use the aforementioned scheme to protect an entangled state of two qubits discussing the theoretical results with some experiments. As authors point out in this contribution, the results established in the paper will help treating the more complex case of multiple-qubit system.

In the contribution "**Dynamics of stochastic nonlocal partial differential equations**," one studies the asymptotic behavior of solutions to nonlocal stochastic partial differential equations with multiplicative and additive noise driven by a standard Brownian motion, respectively. First, the stochastic nonlocal differential equations are transformed into their associated conjugated random differential equations. Then, one constructs the dynamical systems generated by the original problems via the properties of conjugation. Next, in the multiplicative case, one establishes the existence of the random attractor when it absorbs every bounded deterministic set. Particularly, it is shown the pullback random attractor, which is also forward attracting, becomes a singleton when the external forcing term vanishes at zero. Eventually, in the case of additive noise, two approaches are applied to prove the existence of pullback random attractors with the help of energy estimations. Actually, these two attractors turn out to be the same one.

Recurrent neural networks are artificial neural networks that use temporal sequence of data as input. They have been applied to model different problems as language translation and processing, statistical machine translation, speech recognition and image captioning, etc. In "**Dynamics of continuous-time recurrent neural networks with random connection weights and unbounded distributed delays**," authors extend some recent results for recurrent neural networks in the scenario that they have random connection weights and unbounded distributed time delays. They formulated the abovementioned neural network by means of a random lattice system that generates a random nonautonomous dynamical system that possesses a pullback attractor. They then establish a comparison theorem that is applied to prove the existence of extremal complete quasi-trajectories for the cocycle generated by the solutions to the lattice system. The results, though theoretical, have applicability to represent different problems in physics as, for instance, the study of the resistively connected network of electrical amplifiers.

The analysis of damped mechanical systems subject to uncertainties is a key topic in Physics since it has a wide range of real-world applications. In the paper "**Steady-state density preserving method for stochastic mechanical systems**," one proposes an explicit stochastic numerical method for the long-term integration of a class of nonlinear oscillators formulated via an Itô-type differential equation. Examples shown throughout the numerical experiments illustrate the main advantages of the proposed numerical method, namely, one reproduces asymptotic probabilistic characteristics of the exact solution with high accuracy. The contribution opens new avenues to devise new numerical integrators for analyzing other classes of damped mechanical systems.

In the paper "**Propagation of dust acoustic waves in multi-components plasma with random parameters**," authors study a randomized dusty plasma model formulated by a coupled nonlinear system of partial differential equations. Authors justify such randomization because of inhomogeneity of temperature fluctuations through the plasma medium, hence they assume that ion-to-electron temperature ratio and the electron-to-ion number density ratio are continuous random variables. Under very general assumptions, authors can determine the density of relevant characteristics of the dusty plasma wave. The results are numerically illustrated in different simulations that show acceptable consistency with physical situations.

It has been already mentioned the importance of studying nonlinear oscillators in Physics. In "**Probabilistic analysis of random nonlinear oscillators subject to small perturbations via probability density functions: theory and computing**," one combines the stochastic perturbation and entropy methods to study a class of one-degree-of-freedom oscillators whose restoring function is affected by small nonlinearities and excited by stationary Gaussian stochastic processes. In the paper, one obtains different statistical properties of the steady-state response, that includes the first moments, the correlation and probability density function. Both advantages and limitations of the method are carefully discussed at the end of the paper.

In the paper "**Robustness of a dynamical systems model with a plastic self-organising vector field to noisy input signals**," authors propose a model for studying the neural activity in the brain subject to signals. The model is established through two coupled systems of differential equations. Although the results focus on the deterministic formulation of the model, authors also have included their interpretation in the noisy setting as well as a discussion on their practical implication. Some surprising results then emerge for the reader.

It is well-known that one of the most important applications of fractional derivatives is the analysis of physical systems where memory-effects play a major role. Many physical models can be formulated via a class of linear second-order differential equations where the first derivative is missing, and the coefficient of the unknown follows a power law. It includes, for example, the free undamped motion of a mass. In the paper "**Probabilistic analysis of a foundational class of generalized second-order linear differential equations in classic mechanics**," authors analyze the abovementioned class of fractional differential equations assuming that all model parameters are random variables. They construct a generalized random power series solution, and prove, under very general assumptions, that is mean square convergent. In a second step, they obtain convergent approximations to the probability density function of the solution by means of the so-called probabilistic transformation method. The numerical experiments show the rapid convergence of the proposed method.

The Focus Point volume finishes with the paper "**The probability density function of interspike intervals in an FHN model with α-stable noise**," where one extends the analysis of the Fitzhugh–Nagumo model including α-stable noise and studying the firing rate and coherence resonance via probability density function of the interspike intervals.

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Data availability There are no data availability for this paper since it is an editorial on a Focus Point volume.

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