



# Documentation of TauSpinner algorithms: program for simulating spin effects in $\tau$ -lepton production at LHC

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**Abstract**  $\tau$ -leptons produced in  $pp$  collisions allow to measure Standard Model parameters and offer probes for New Physics. The TauSpinner program can be used to modify spin (or production matrix elements) effects in any  $\tau$  sample. It relies on the kinematics of outgoing particles:  $\tau$  lepton(s) (also  $\nu_\tau$  in case of  $W$ -mediated processes, optionally also four-moments of accompanying hard jets) and  $\tau$  decay products. No other information is required from the event record. With calculated spin (or production/decay matrix element) weights, attributed on the event-by-event basis, modifications to the spin/decay/production features, is possible without the need for regenerating events. With TauSpinner algorithms, the experimental techniques developed over years since LEP 1 times are already used and extended for LHC applications. The purpose of the present publication is to systematically document physics basis of the program, and to overview its application domain and systematic errors.

## 1 Introduction

At LHC experiments, interest in  $\tau$  leptons lies in the use of decays for the measurements of the properties of hard production processes, of the properties of resonances like  $W$ ,  $Z$  or Higgs boson or in the searches for New Physics.

That is why, it was of interest to prepare a package called TauSpinner which can be used for changing with weights generated event samples including  $\tau$  lepton decays. In many cases one can argue that such re-weighting of the sample with different physics assumptions may be of little use. Why not simply generate another series of events with different assumptions and compare distributions obtained from them? Indeed one can think re-weighting may bring little benefits and considerable risks, e.g. if the original sample

has some sectors of the phase space biased by a particular technical cut-offs, or if it features zero matrix element on some hyper-surface within the phase space manifold. Arbitrary large weights may then appear. However, in several cases such disadvantages are balanced by the benefits. The weighted events sample is statistically correlated with the original one, thus the statistical errors affect the estimate of the differences only. Even larger CPU gain is expected, if the sample is processed with simulation of the detector effects. A full simulation of the detector response usually requires few orders of magnitude more CPU time than generation of physics events, or the calculation of events weight. Even the inclusion of Next to Leading Order (NLO) QCD effects in the generation can be CPU costly. Re-weighting to a different production process with the same final state (set of final states) can be also of interest. That is why the re-weighting algorithms of TauSpinner were prepared; simultaneously for  $\tau$  lepton pair production and  $\tau$  lepton decays. The weight could depend on ten or even more four momenta taken from kinematical configurations of stored events. The proposed implementation was widely used in experimental analyses at LHC; in the domains of Standard Model precision measurements [1–3], Higgs boson discovery and particle properties measurements [4–8] and searches for New Physics [9, 10].

At the beginning the TauSpinner project was focused on the longitudinal spin effects only [11]. It was used to calculate appropriate weights, to include spin effects into (or removed from) the generated events sample. With time, other options were added. In Ref. [12], an additional weight was introduced to manipulate the production process by adding or replacing the generated production process with an alternative one, including for example exchange of new intermediate particles. Reference [13] brought the possibility of introducing complete spin effects (longitudinal and transverse) in decays of intermediate Higgs bosons, and in Ref. [14] in the Drell–Yan process as well. The production pro-

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cess was modeled with lowest order  $2 \rightarrow 2$  matrix elements. In [15] the implementation of hard processes featuring the parton level matrix elements for production of  $\tau$  lepton pairs and two jets was introduced. This was motivated by the experimental analyses becoming sensitive to Electroweak Higgs and the  $Z$  boson production in the fusion of  $WW$  or  $ZZ$  pairs. The question of systematic uncertainties due to the approximation where  $2 \rightarrow 2$  or  $2 \rightarrow 4$  matrix elements were used for the calculation of spin effects could be addressed since then. With time also more sophisticated options for treatment of spin effects were prepared. In Ref. [13] possibility to assign helicity to  $\tau$  leptons was added. An approximation was used taking only the longitudinal components of density matrix for  $\tau$  lepton pair production into account. In [14] one loop electroweak (EW) corrections in the Drell–Yan process became available for the weight calculation. Finally let us mention that available options allow to configure the `TauSpinner` algorithm to work on samples where only partial spin effects were taken into account and to correct them to full spin effects.

All these different options were introduced into `TauSpinner` because of user's interest and under the pressure of time during converging very fast analyses of LHC Run I data by ATLAS and CMS experiments. That is why there is a need and the opportunity now to document them systematically and archive the code which will be used during the final LHC Run II analyses.

New applications of the `TauSpinner` are currently developed. The most recent example, where re-weighting techniques of `TauSpinner` were used for the precision measurement at LHC is the  $\sin^2 \theta_{eff}^{lep}$  measurement by the ATLAS Collaboration [16]. The `TauSpinner` algorithms served as a tool for introducing genuine EW and other loop corrections to the samples generated with EW leading order (LO) matrix elements.

Contrary to the previous papers we will emphasize here less phenomenological applications, but rather focus on common theoretical/phenomenological basis and relations between different options. Whenever possible we will address the point of the theoretical uncertainties of the proposed algorithms with references to our previous publications on `TauSpinner`.

The focus of the present paper is the documentation of the program. We do not repeat the physics reasons why options or particular approximations were then prepared. The principles of algorithms for the exact treatment of all spin effects for the  $\tau$  leptons production and decay are known since a long time. Also, all approximations used by us and applied in other currently used programs of today were investigated already not later than a decade ago, see e.g. Ref. [17] of the final publication on the LEP precision measurements.

The general purpose Monte Carlo programs for LHC such as Pythia [18], Herwig [19] or Sherpa [20] feature  $\tau$  lepton decays and corresponding spin effects, often without any approximations. This depends on the particular process of  $\tau$  leptons in the final state. Re-weighting algorithms are available in these programs as well, see e.g. [21–23], even though they not necessarily rely on event record formats, presently used for storing events of LHC experiments, or are prepared for  $\tau$  lepton physics. Such re-weighting options are usually not used in the LHC experiments analyses, for events which have been already generated and stored. The `TauSpinner` therefore offers as we will explain later with very little compromising on the physics precision, solution for reweighting.

Discussion, comparisons of different programs and their results for the modelling of spin effects in a multitude of high energy processes is of vital interest but remains out of the scope of the present publication. Fortunately we can refer to conference contributions and workshops, see e.g. [21, 24], with the caveat however that so far details on the  $\tau$  lepton physics at LHC are largely shadowed by the interest and complexity of higher order QCD corrections. The extensive work like the one in Ref. [17] for LEP is to our knowledge at present not available. It should not be a surprise, Ref. [17] was published many years *after* LEP experiments stopped taking data. Likely it will be the case for LHC too and the comprehensive review will have to wait for the time of finalising precision measurements of the full Run II data.

Our paper is organized as follows. In Sect. 2 a theoretical basis for describing  $\tau$  leptons in production and decay matrix elements is introduced. We define weights calculated by the program. Details of kinematics; definition of frames and transformations connecting them, used for calculation of hard parton-level amplitudes or cross section and for calculation of spin effects are given in Sects. 3 and 4. In the sub-sections we explain simplifications and resulting formulae for the differential cross section and spin correlations. Section 5 is devoted to the case when at parton level  $2 \rightarrow 2$  processes are used. Section 6 covers the calculation of density matrices for  $\tau$  lepton decays; the polarimetric vectors. In Sect. 7 the case of  $2 \rightarrow 4$  processes is discussed and extensions to the algorithms which were needed for the case when the final state consists not only of a  $\tau$  lepton pair, but also two additional jets are present are explained. Section 8 is devoted to general discussion how systematic uncertainties related to applications of `TauSpinner` weights can be estimated. The  $\tau$  lepton decay library `Tauola` [25] providing some methods for `TauSpinner` is discussed as well. Section 9, closes the paper. The appendices recall all examples and applications which are included in the most recent release. So far, the documentation for these applications was scattered over several publication appendices and was thus not available in a systematic manner.

## 2 Theoretical basis

We start the discussion from presenting the exact formula on the cross section for  $pp \rightarrow \tau\tau X$  or  $pp \rightarrow \tau \nu_\tau X$  processes, which requires knowledge of the whole process matrix elements, including decays of  $\tau$  leptons. Considering about 20  $\tau$  lepton decay channels we end-up with 400 possible configurations for each  $\tau$  lepton pair production process. The exact formula can therefore not be used in practice. Nonetheless, it can serve as a starting point for more practical ones, also those involving approximations. The formulae presented below for the  $H(Z/\gamma^*) \rightarrow \tau\tau$  can be applied also to the  $H^\pm(W^\pm) \rightarrow \tau \nu_\tau$  case. Then the second  $\tau$  lepton of the formulae should be replaced by  $\nu_\tau$  and the part of formula which describes its decay dropped out.

### 2.1 Exact formula

The basic formula for the cross-section at the process  $pp \rightarrow \tau^+\tau^-X; \tau^+ \rightarrow Y^+\bar{\nu}; \tau^- \rightarrow Y'\bar{\nu}$  reads:

$$d\sigma = |\mathcal{M}|^2 d\Omega = |\mathcal{M}|^2 d\Omega_{prod} d\Omega_{\tau^+} d\Omega_{\tau^-} \tag{1}$$

where  $\mathcal{M}$  denotes complete matrix element. Obviously terms due to average over initial state spin degrees of freedom and the flux factor are omitted. Because of the  $\tau$  lepton width being very small, the  $\tau$  lepton propagator module squared, to a perfect approximation, reduces to the Dirac  $\delta$  function in lepton virtuality and so the infinitesimal element in the kinematical phase space factorizes into  $d\Omega_{prod} d\Omega_{\tau^+} d\Omega_{\tau^-}$ .

The formula (1) is not very convenient, because of the 400 distinct processes and matrix elements. Fortunately, as for the phase space parametrization, because of a narrow  $\tau$  lepton width, matrix elements can be also expressed as the product of amplitudes for production and decays, summed over the  $\tau$  spin projections ( $\lambda = 1, 2$ ) on at this point unspecified axes,

$$\mathcal{M} = \sum_{\lambda_1 \lambda_2=1}^2 \mathcal{M}_{\lambda_1 \lambda_2}^{prod} \mathcal{M}_{\lambda_1}^{\tau^+} \mathcal{M}_{\lambda_2}^{\tau^-}. \tag{2}$$

The cross-section can be expressed then with the rewritten exact formula, which thanks to Fierz identity decouples summations of the decay and production matrix elements. It introduces the correlations back in a form of a spin weight,  $w_{tspin}$

$$d\sigma = \left(\sum_{\lambda_1 \lambda_2} |\mathcal{M}^{prod}|^2\right) \left(\sum_{\lambda_1} |\mathcal{M}^{\tau^+}|^2\right) \left(\sum_{\lambda_2} |\mathcal{M}^{\tau^-}|^2\right) w_{tspin} d\Omega_{prod} d\Omega_{\tau^+} d\Omega_{\tau^-}. \tag{3}$$

This formula is the core of the TauSpinner algorithm, but not only. It is used in the Tauola  $\tau$  decay library interface as foundation of the ‘‘after-burn’’ approach [25] and also in the KKMC Monte Carlo [26]. For details of the geometrical

formulation and naming conventions Ref. [27] may be the most useful.

The spin weight  $w_{tspin}$  is dimensionless, and contains information of all spin effects transmitted from the production to the decay of  $\tau$  leptons,

$$w_{tspin} = \sum_{i,j=t,x,y,z} R_{i,j} h_{\tau^+}^i h_{\tau^-}^j. \tag{4}$$

The indices  $i, j$ , of the adjoint (to 1/2 representations of fermions) rotation group representation run over  $t, x, y, z$ . The polarimetric vector  $h_{\tau^\pm}^i$  is calculated from the formula

$$h_{\tau^\pm}^i = \sum_{\lambda, \bar{\lambda}} \sigma_{\lambda, \bar{\lambda}}^i \mathcal{M}_{\lambda}^{\tau^\pm} \mathcal{M}_{\bar{\lambda}}^{\tau^\pm \dagger}, \tag{5}$$

where  $\sigma_{\lambda, \bar{\lambda}}^i$  stands for Pauli matrices. The  $h_{\tau^\pm}^i$  are normalized further to set their time-like component to 1, taking  $\frac{h_{\tau^\pm}^i}{h_{\tau^\pm}^t} \rightarrow h_{\tau^\pm}^i$ . We will often use shortened form for the notation  $h_{\tau^\pm}^i$  or  $h_{\pm}^i$ . The superscript  $i$  will be dropped if we will have all components in mind.

The spin correlation matrix  $R_{i,j}$  is calculated from the formula

$$R_{i,j} = \sum_{\lambda_1, \bar{\lambda}_1, \lambda_2, \bar{\lambda}_2} \sigma_{\lambda_1, \bar{\lambda}_1}^i \sigma_{\lambda_2, \bar{\lambda}_2}^j \mathcal{M}_{\lambda_1 \lambda_2}^{prod} \mathcal{M}_{\bar{\lambda}_1 \bar{\lambda}_2}^{prod \dagger} \tag{6}$$

and also normalized to set its time-like component to 1, namely  $\frac{R_{i,j}}{R_{i,t}} \rightarrow R_{i,j}$ .

Formula (4) is universal, it does not depend on the production process, and there are no approximations introduced. It features useful properties:  $0 < w_{tspin} < 4$  and weight average  $\langle w_{tspin} \rangle = 1$ . The use of expressions (5) and (6) moves our description from the language of spin amplitudes toward the language of cross-sections and probability distributions; since amplitudes squared contribute only.

All complexities are in the calculation of  $R_{i,j}$  spin correlation matrix for production and  $h_{\tau^+}^i h_{\tau^-}^j$  polarimetric vectors for the decay of  $\tau$  leptons. The  $R_{i,j}$  matrix describes the full spin correlation between the two  $\tau$  leptons as well as the individual spin states of the  $\tau$  leptons. The  $R_{i,j}$  depend on kinematics of the production process only, and  $h_{\tau^+}^i h_{\tau^-}^j$  respectively on the kinematics of  $\tau^+$  and  $\tau^-$  leptons. The explicit definition of the matrix  $R_{i,j}$  and vectors  $h_{\tau^+}^i, h_{\tau^-}^j$  is rather lengthy and also well known, see Refs. [28,29] for detailed definitions. It is decay channel dependent.

So far we discussed the case of a  $\tau$  lepton pair in the final state, being the case of  $H \rightarrow \tau\tau$  or  $Z/\gamma^* \rightarrow \tau\tau$  processes. The case of only one  $\tau$  lepton in the final state, that is the case of  $W^\pm \rightarrow \tau \nu_\tau$  or  $H^\pm \rightarrow \tau \nu_\tau$  is much simpler. The  $R_{i,j}$  matrix is replaced by the vector of components  $R_t, R_x, R_y, R_z = 1, 0, 0, \mp 1$  respectively for  $W$  and charged Higgs decays. The sum over two indices is thus reduced to the sum over one index only, the axis  $z$  is the direction of  $\nu_\tau$

from  $W^\pm/H^\pm$  decay as seen from the  $\tau$  lepton rest-frame, and the spin weight read thus  $wt_{spin} = 1 \pm h_\tau^z$ . The frames choices (presented in the next Section) for that case could have been simpler than for  $Z/H$  decays, no details of transverse directions are then needed.

### 2.2 Formula using parton level amplitudes

We do not have at our disposal matrix elements for the complete  $pp \rightarrow \tau^+\tau^-X$  process, but instead, following factorization theorems, we can represent the cross-section with the help of the parton level amplitudes, convoluted with the parton density functions (PDFs) and summed over all *flavours* (flavour configurations of incoming partons).<sup>1</sup> Thus, from formula (3) we obtain:

$$d\sigma = \sum_{flav.} \int dx_1 dx_2 f(x_1, \dots) f(x_2, \dots) d\Omega_{prod}^{part. lev.} d\Omega_{\tau^+} d\Omega_{\tau^-} \left( \sum_{\lambda_1, \lambda_2} |\mathcal{M}_{part. lev.}^{prod}|^2 \right) \left( \sum_{\lambda_1} |\mathcal{M}^{\tau^+}|^2 \right) \left( \sum_{\lambda_2} |\mathcal{M}^{\tau^-}|^2 \right) wt_{spin}. \tag{7}$$

At this point we would like to decouple the calculation of  $wt_{spin}$  as much as possible from the PDFs. Note, that this is needed for the  $Z/\gamma^*$  mediated processes due to  $V$  and  $A$  for  $Z$  boson and  $\gamma^*$  couplings to fermions. For the  $H$  or  $W$  mediated processes the spin polarization state does not depend on flavours of incoming partons. For the  $Z/\gamma^*$  the  $R_{i,j}$  used in calculation of  $wt_{spin}$ , is taken as the weighted average (with PDFs and production matrix elements squared) over all flavour configurations, as in the following eq.

$$R_{i,j} \rightarrow \frac{\sum_{flav.} f(x_1, \dots) f(x_2, \dots) \left( \sum_{\lambda_1, \lambda_2} |\mathcal{M}_{parton level}^{prod}|^2 \right) R_{i,j}}{\sum_{flav.} f(x_1, \dots) f(x_2, \dots) \left( \sum_{\lambda_1, \lambda_2} |\mathcal{M}_{parton level}^{prod}|^2 \right)}. \tag{8}$$

No approximation is introduced in this way. The denominator of Eq. (8) cancels explicitly the corresponding factor of Eq. (7) (that is also the integrand of the following Eq. (9), first line).

To introduce the changes due to different spin effects in modified production process or the decay models, one can define in the generated sample (i.e. without re-generation of events) the weight  $WT$ , representing the ratio of the new to old cross-sections at each point in the phase space.

Equation (7) for modified cross-section, takes then the form

$$d\sigma = \sum_{flavours} \int dx_1 dx_2 f(x_1, \dots) f(x_2, \dots) = \left( \sum_{\lambda_1, \lambda_2} |\mathcal{M}_{parton level}^{prod}|^2 \right) d\Omega_{prod}^{parton level}$$

<sup>1</sup> The double sum runs in principle over all possible flavours for the two incoming partons:  $g, u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}, b, \bar{b}$ .

$$\left( \sum_{\lambda_1} |\mathcal{M}^{\tau^+}|^2 \right) d\Omega_{\tau^+} \left( \sum_{\lambda_2} |\mathcal{M}^{\tau^-}|^2 \right) d\Omega_{\tau^-} \times wt_{spin} \times WT. \tag{9}$$

A variant of Eq. (9) is used in Tauola.<sup>2</sup>

In general the weight  $WT$  factorizes into production ( $wt_{prod}$ ), decay ( $wt_{decay}^{\pm}$ ) and the ratio of spin correlation/polarization weights, new to the old one ( $wt_{spin new}/wt_{spin old}$ ):

$$WT = wt_{prod} wt_{decay}^{\tau^+} wt_{decay}^{\tau^-} wt_{spin new}/wt_{spin old}. \tag{10}$$

The first three terms of the weight, represent modification of the matrix elements for production and decays

$$wt_{prod} = \frac{\sum_{flav.} f(x_1, \dots) f(x_2, \dots) \left( \sum_{spin} |\mathcal{M}_{part. lev.}^{prod}|^2 \right) \Big|_{new}}{\sum_{flav.} f(x_1, \dots) f(x_2, \dots) \left( \sum_{spin} |\mathcal{M}_{part. lev.}^{prod}|^2 \right) \Big|_{old}}, \tag{11}$$

$$wt_{decay}^{\pm} = \frac{\sum_{spin} |\mathcal{M}_{new}^{\pm}|^2}{\sum_{spin} |\mathcal{M}_{old}^{\pm}|^2}. \tag{12}$$

The combined weight as in Eq. (10), is nothing else than ratios of spin averaged amplitudes squared for the whole process; new to old. The eventual changes in the PDFs parametrizations should be taken into account in calculation of  $wt_{prod}$  and also in the ratio  $wt_{spin new}/wt_{spin old}$  in formula (10). In case the production process or decay models are not modified, respectively  $wt_{prod}$  and  $wt_{decay}^{\pm}$  are equal to 1. The ratio of spin weights  $wt_{spin new}/wt_{spin old}$  allows for introduction of new spin effects. In case of originally unpolarized sample, the  $wt_{spin old} = 1$  and  $wt_{spin new}$  alone allows to introduce the desired spin effects.

Formula (9) can be rewritten to the equivalent form, which underlines the separation useful for the organization of the program, with segments for production of  $\tau$  leptons, their decays and the spin weights. Let us recall that the spin weight  $wt_{spin}$  is the only part depending simultaneously on the production and decay kinematics. For the production of  $\tau$  lepton pair it has always the form of Eq. (4). Ratio of the weight calculated from the new and old matrix elements has to be taken. Reorganizing some terms we can finally write:

$$d\sigma = \left[ d\sigma_{prod} d\Gamma_{decay}^{\tau^+} d\Gamma_{decay}^{\tau^-} \right] wt_{spin old} \left( \frac{wt_{spin new}}{wt_{spin old}} \right), \tag{13}$$

<sup>2</sup> It can seem natural to choose the initial flavours, on the basis of probabilities obtained from production density  $1/N_{normalization} f(x_1, \dots) f(x_2, \dots) \left( \sum_{\lambda_1, \lambda_2} |\mathcal{M}_{parton level}^{prod}|^2 \right)$  and then calculate weights for such flavour configuration. It would not be correct. The dependence on the decay configuration, already fixed and stored for the event, would be then ignored. In case of the Tauola interface incoming flavours are obtained from the event record and decays are generated accordingly.

where

$$\begin{aligned}
 d\sigma_{prod} &= \sum_{flavours} \int dx_1 dx_2 f(x_1, \dots) f(x_2, \dots) \\
 &\quad \times d\Omega_{parton\ level}^{parton\ level} \left( \sum_{spin} |\mathcal{M}_{parton\ level}^{prod}|^2 \right) w_{t_{prod}} \\
 d\Gamma_{decay}^{\tau^+} &= d\Omega_{\tau^+} \left( \sum_{spin} |\mathcal{M}^{\tau^+}|^2 \right) w_{t_{decay}}^{\tau^+}, \\
 d\Gamma_{decay}^{\tau^-} &= d\Omega_{\tau^-} \left( \sum_{spin} |\mathcal{M}^{\tau^-}|^2 \right) w_{t_{decay}}^{\tau^-}. \tag{14}
 \end{aligned}$$

All four components of the overall weight (10) are given separately to underline, wherever possible, their dependence on all or only sub-set of phase space dimensions. Note that for  $W^\pm/H^\pm \rightarrow \tau \nu_\tau$  we have not prepared so far any option which allows to modify production processes.

### 3 WT calculations in TauSpinner

The formulae listed above are used in TauSpinner for the calculation of distinct components of  $WT$ , see Eq. (10). Let us start with the main weight of the spin effects,  $w_{t_{spin}}$  defined by Eq. (4). With this weight, for the samples where spin effects of  $\tau$  lepton production are absent, they can be inserted into decay distributions. Alternatively, its inverse can be used to remove spin effects from a sample where they are taken into account. As a building block of this weight introducing the dependence on the production process, the matrix  $R_{i,j}$ , is given by the formula (8). The other weights,  $w_{prod}$  and  $w_{decay}^{\tau^\pm}$  are defined by formulae (11, 12).

It is useful to introduce the notation  $P_\tau^z = R_{t,z} = R_{z,t}$ , which represents the longitudinal polarization of the single  $\tau$  lepton (if one integrates out possible configurations of the other  $\tau$  lepton). In usually sufficient approximation of helicity states, that is when transverse momenta of  $\tau$  decay products are neglected, as is the case of ultra-relativistic  $\tau$  leptons, the  $P_\tau^z$  is the only non-trivial (dynamic dependent) element of matrix  $R$ . All others are equal to  $\pm 1$ , or can be set to 0. We will return to the details later.

In the calculation of  $w_{t_{spin}}$  and  $w_{prod}$  the physics uncertainty depends on the accurateness of the factorization assumptions for separation of parton level matrix elements and PDFs and also on the choice of a particular PDFs parametrization. Nothing in principle would change if instead of  $2 \rightarrow 2$  body production matrix element one would use the ones for  $2 \rightarrow 2 + n$ , where  $n$  denotes additional partons/jets or other particles (of summed spin-states). In addition to the choice of PDFs (eventually also model of underlying event interactions) for parton level matrix elements one has to make a careful choice of how hard scattering kinematics is reconstructed from information available in the event.

Calculation of  $w_{t_{decay}}^{\tau^\pm}$  involves modeling of  $\tau$  lepton decays only. This weight is useful in studies of the resulting systematic uncertainties. This part of the code is using function of the Tauola library.

We will return to these points later, but one can keep it in mind already now, while reading the following.

#### 3.1 Kinematical frames

The components necessary for the  $WT$  weight are calculated in different frames. This is a correct approach as long as details of boosts and rotations connecting frames are meticulously followed. Also spin states of the  $\tau$  leptons may be defined in different frames than four momenta of the hard process. Bremsstrahlung photons and properties of the matrix element in their presence, require a dedicated treatment.

The basic formula (1) can provide exact results in principle. In practice, we have to introduce approximations to adapt to the fact that matrix elements are calculated from the parton level amplitudes, and to the way how these amplitudes are reconstructed by the TauSpinner algorithm where only four momenta of outgoing  $\tau$  leptons and partons can be used. The definition of quantization frames is an essential element of the arrangements, let us provide now the necessary details.

For calculating all components of the event weights (to prepare kinematical configurations of events necessary for that purpose) we define in total four frames (group of frames):

- A : The basic frame (starting point) for kinematic transformations and all other frame definitions. The rest frame of the  $\tau$  lepton pair is used (and not of  $\tau$  lepton pair with the final state bremsstrahlung photons combined). Such choice is possible thanks to the properties of bremsstrahlung amplitudes. Emitted photons do not carry out the spin. This non-trivial observation is exploited also in Photos Monte Carlo phase space parametrization, see Refs. [30,31]. The definition of the rest frame of the  $\tau$  lepton pairs is completed with incoming partons set along the  $z$ -axis.
- A' : The rest frame of the  $\tau$  lepton pairs and the final state bremsstrahlung photons combined. Incoming partons are again set along the  $z$ -axis. This frame is used for calculating the production weight  $w_{t_{prod}}$  and the spin correlation matrix  $R_{i,j}$ ; if no bremsstrahlung photons are present it is equivalent<sup>3</sup> to frame A. In every case we reconstruct  $x_1, x_2$ , the arguments of the PDF function, from the virtuality  $M$  and the longitudinal to the beam direction component ( $p_L$ ) of the intermediate state (sum of momenta of  $\tau^+ \tau^- n\gamma$ )

<sup>3</sup> In case when photons are present and for  $2 \rightarrow 4$  matrix elements modifications are needed to assure that on-mass shell kinematic configuration is passed to the routines calculating spin amplitudes.

**Table 1** Frames used for the calculation of components used in weights calculation, Eq. (10). Specified are also frames to which these components need to be transformed

Object	Frame of object calculation	Frame of object use	Comment
$R_{i,j}$ for $wt_{spin}$	$A$ (or $B$ )	$C^\pm$	$B$ if transverse spin included
$h_{\tau^+}^i, h_{\tau^-}^j$ for $wt_{spin}$	$D^\pm$	$C^\pm$	
$wt_{prod}$	$A$ ( $A'$ )		$A'$ is used if bremsstrahlung photons are present
$wt_{decay}$	$D^\pm$	$D^\pm$	

momentum in lab frame. The center of mass energy of  $pp$  scattering is used to calculate  $x_1 x_2$  through relations  $x_1 x_2 CMS_{ENE}^2 = M^2(\tau^+ \tau^- n\gamma)$ ,  $(x_1 - x_2) CMS_{ENE} = 2p_L(\tau^+ \tau^- n\gamma)$ . The  $n\gamma$  is assumed to correspond to the final state bremsstrahlung associated with the  $\tau$  lepton pair production and its momentum has to be taken into account, but only in the calculation of the virtuality for the intermediate  $Z/\gamma^*$  state in case of  $2 \rightarrow 2$  amplitudes.<sup>4</sup> Note that  $A'$  is used for  $x_1, x_2$  calculation only. The  $2 \rightarrow 2$  hard process scattering angle is calculated in frame  $A$ .

- $B$  : The rest frame of the  $\tau$  lepton pairs, with  $\tau$  leptons along the  $z$ -axis. It is used for common orientation of production and decay coordinate systems. In this frame the polarimetric vectors  $h_\pm$  are defined. Transformation  $A \rightarrow B$  requires rotation.
- $C_{\tau^\pm}$  : The rest frames of individual  $\tau$  leptons with the direction of the boost to  $\tau$  lepton pair rest-frame along the  $z$ -axis. Transformation  $B \rightarrow C_{\tau^+}$  or  $C_{\tau^-}$  requires boost only.
- $D_{\tau^\pm}$  : The rest frames of individual  $\tau$  leptons, with  $v_\tau$  along the  $z$ -axis. It is used for the calculating decay weights. In fact in the frames  $D_{\tau^\pm}$  also the polarimetric vectors  $h_{\tau^\pm}$  are calculated and then rotated back to frame  $C_{\tau^\pm}$ .

In Table 1, we summarize which frames are used for calculations of different variables and then to which frames they are boosted (rotated) before being used in formula (7). Let us stress that it is not only important to define all the frames of type  $A - D$  but also the Lorentz transformations between them.

The frame of type  $A$  is used for calculation of the hard process amplitudes. If only longitudinal components of spin density matrix are taken into account, we do not need to define in detail the transverse directions (quantization frame vectors) with respect of  $\tau$  lepton momenta, and control on the corresponding details of boost methods to the rest frames of  $\tau$  leptons is not needed. However, the code of `TauSpinner`

controls such details of the boosts, because it is prepared to handle also cases when the complete spin density matrix is used.

The frames of type  $B$  and of type  $C$  have common direction of the  $z$ -axis, also  $x$ - and  $y$ - axes coincide. Frame  $B$  is used for  $R_{i,j}$  calculation if transverse degrees of freedom are taken into account. Calculation of the spin weight, that is contraction of indices in  $wt_{spin} = \sum_{ij} R_{i,j} h_{\tau^+}^i h_{\tau^-}^j$ , is performed in frames  $C_{\tau^+}, C_{\tau^-}$ . It enables control on transverse spin effects. Frames of type  $D$  are used for calculation of decay matrix elements and  $h_{\tau^+}^i, h_{\tau^-}^i$  polarimetric vectors with methods from the `Tauola` library, see Ref. [29] for details.

Let us note that we have used such frames in many applications already at the time of LEP analyzes. They were useful for comparisons and tests. These choices once established can be easily modified whenever necessary, e.g. to fulfill the constraints of particular conventions of spin amplitude calculations, see e.g. in Ref. [27].

Let us comment on the possible future improvement of the definition of the frame  $A$ , to allow useful freedom of choice. At present in the `TauSpinner` the simplest possible ansatz is used in definition of lepton pair rest-frame  $A$ . For the frame  $A$  beam direction (necessary to define Born-level scattering angle), it is assumed that no jets of substantial  $p_T$  are present and  $p_T$  can be ignored. This can be improved without redoing the design of the above tree of frame definitions. The studies for matrix elements featuring one or two high  $p_T$  jets are already documented in Refs. [32,33]. The corresponding modifications of the code were not yet introduced into `TauSpinner`, as there are no clear indications of their numerical importance, but it is a path for the possible forthcoming improvement.

### 3.2 Production weight $wt_{prod}$ and spin correlation matrix

$$R_{i,j}$$

The  $R_{i,j}$  matrix is not Lorentz invariant. In fact, its indices run over coordinates in *two* frames,  $C_{\tau^+}$  and  $C_{\tau^-}$ . As input for matrix element calculation the four momenta of  $A$  frame are used. For details of relations between frames and elements used in weight calculations, see also Table 1.

<sup>4</sup> In case when configurations with two-jets are evaluated ( $2 \rightarrow 4$  processes), not only more attention for the bremsstrahlung photons will be needed, but also the momenta of the jets have to be taken into account in evaluation of  $p_L$  and  $M^2$ . We will return to this point in the future.

For calculation of the production weight  $w_{t_{prod}}$  and the spin correlation matrix  $R_{i,j}$ , the centre-of-mass frame of the  $\tau$  lepton pair with incoming partons being along the  $z$ -axis (frame of type  $A$ ) is not the optimal one. It is nonetheless used if there is no interest in the transverse spin degrees of freedom. Frame of type  $B$ , with  $\tau$  leptons along the  $z$ -axis, is more convenient and with the  $x$  and the  $y$  axes parallel to the ones of frames  $C_{\tau^\pm}$ , represents the most convenient setup. In fact all these three frames  $B$  and  $C_{\tau^\pm}$  are used *simultaneously* for calculation of the spin weight of Eq. (4).

The kinematical frames for  $\tau$  lepton pair production and  $\tau$  decays need to be explicitly related by the set of Lorentz transformations with the frames used in matrix element calculations. Let us recall details in a form of algorithmic steps, which need to be performed on all decay products of  $\tau$  leptons before calculation of  $R_{i,j}$  and  $h_{\tau^\pm}^{i/j}$  (for  $h_{\tau^\pm}^{i/j}$  further steps will be still needed):

- Boost both  $\tau$  leptons and their decay products from the laboratory frame to the rest-frame of  $\tau$  lepton pair.
- Rotate all to the frame, where the direction of incoming partons is along the  $z$ -axis (frame  $A$ ).
- Rotate so that  $\tau$  leptons are set along the  $z$ -axis and incoming partons remain in the  $z - y$  plane (frames  $C_{\tau^\pm}$  have to be correlated with  $B$  by a boost along the  $z$  axis). These frames are used for  $R_{i,j}$  spin states definition too.
- Presence of the final state bremsstrahlung from  $\tau$  lepton pair brings complication, because the rest frame of the  $\tau$  lepton pair is not the rest frame of the resonance which decayed into  $\tau$  lepton pair. For the sake of kinematical transformations, frames  $A$  and  $B$  defined as above are used. However, for the calculation of matrix elements we use frame  $A'$ , where effectively bremsstrahlung photons are absorbed into  $\tau$  lepton momenta. For calculation of  $x_1, x_2$  we use the invariant mass, which can be easily calculated in frame  $A'$ .

### 3.3 Decay weight $w_{t_{decay}}^{\tau^\pm}$ and polarimetric vectors $h_{\tau^\pm}$

The decay weight  $w_{t_{decay}}^{\tau^\pm}$  and the polarimetric vectors  $h_{\tau^\pm}$  are calculated for each outgoing  $\tau$  lepton in its rest frame, denoted as frame  $D_{\tau^\pm}$  (in this frame neutrino of  $\tau$  lepton decay is along the  $z$ -axis). Lorentz transformations of the previous sub-section have to be supplemented with the one relating frames  $C_{\tau^\pm}$  with  $D_{\tau^\pm}$ .

- Boost all decay products along the  $z$ -axis to the rest frame of the  $\tau^\pm$  lepton (frames  $C_{\tau^\pm}$ ).
- Rotate again  $\tau$  lepton daughters so that  $\tau$  neutrino is along the  $z$ -axis (frames  $D_{\tau^\pm}$ ).
- Calculate in frames  $D_{\tau^\pm}$  polarimetric vectors  $h_{\tau^\pm}^i$ , rotate them back from  $D_{\tau^\pm}$  to  $C_{\tau^\pm}$ . Only then they can be contracted with  $R_{i,j}$  for  $w_{t_{spin}}$  calculation.

Decay weights and polarimetric vectors for  $\tau^+$  and  $\tau^-$  are calculated in the frame  $D_{\tau^\pm}$ , using code from `TAUOLA`. That is why  $v_\tau$  along the  $z$ -axis is required.

## 4 Exact and approximate spin weight $w_{t_{spin}}$

Basic formula (3) of Sect. 2.1 is exact. Because of approximated calculations (introduction of parton level amplitudes) or as explained later in the section, to obtain a physical picture easier to interpret (valid in ultra-relativistic case only) we can introduce simplifications. No such simplification was introduced yet. Let us recall first that according to  $R_{i,j}$  and  $h_{\tau^\pm}^i$  definitions,  $R_{t,t} = 1$  and  $h_{\tau^\pm}^t = 1$  (we can use shorter notation  $h_{\tau^\pm}^i = h_{\tau^\pm}^i$ ).

### 4.1 Neglecting $m_\tau^2$ terms

With the choices of the kinematical frames as defined in Sect. 3.1 and neglecting  $m_\tau^2$  terms following the ultra-relativistic limit, one gets that  $R_{z,z} = sign = \pm 1$  (the *sign* depends on whether the decaying object is a scalar or a vector boson). Also non-diagonal terms  $R_{t,x} = 0, R_{t,y} = 0, R_{x,t} = 0, R_{y,t} = 0, R_{z,x} = 0, R_{z,y} = 0, R_{x,z} = 0, R_{y,z} = 0$ , independently whether the decay of spin zero (scalar) or spin one (vector) state is considered. Formula (4) reduces to:

$$w_{t_{spin}} = 1 + sign \ h_{\tau^+}^z h_{\tau^-}^z + R_{z,t} h_{\tau^+}^z + R_{t,z} h_{\tau^-}^z + \sum_{i,j=x,y} R_{i,j} h_{\tau^+}^i h_{\tau^-}^j, \tag{15}$$

or if notation of individual  $\tau$  lepton longitudinal polarization  $P_\tau^z$  is used (for Higgs decays  $P_\tau^z = 0$ ),

$$w_{t_{spin}} = 1 + sign \ h_{\tau^+}^z h_{\tau^-}^z + P_{\tau^+}^z h_{\tau^+}^z + P_{\tau^-}^z h_{\tau^-}^z + \sum_{i,j=x,y} R_{i,j} h_{\tau^+}^i h_{\tau^-}^j. \tag{16}$$

Let us stress that the formulae (15, 16) are quantization frame dependent. One has to be careful especially with signs if the kinematical transformations, as explained in Sect. 3.1 are not followed. Distinct conventions for defining those frames are used in the literature.

### 4.2 Neglecting transverse spin correlations and spin state probabilities

A further approximation (not always used) to include longitudinal spin effects only, means that terms  $R_{i,j}$  are set to zero for  $i, j = x, y$ . In general those terms can be large, but as they result in dependencies of transverse with respect to  $\tau$  lepton direction components of  $\tau$  lepton decay products momenta, they often do not lead to measurable effects and can be dropped.

With this approximation, formula (16) becomes

$$wt_{spin} = 1 + \text{sign } h_+^z h_-^z + P_\tau^z h_{\tau+}^z + P_\tau^z h_{\tau-}^z, \tag{17}$$

Using formula (17), one will be able to introduce probabilities for spin states  $p_\tau^z$  which will be defined later.

For the decay of a scalar resonance (e.g. Higgs) we define (equal) probabilities for  $\tau$  pair configurations being left-right (+) or right-left (-). We rewrite formula (17) and decompose  $wt_{spin}$  into a sum of  $wt_{spin}^+$  and  $wt_{spin}^-$ :

$$wt_{spin} = 0.5(1 + h_{\tau+}^z)(1 - h_{\tau-}^z) + 0.5(1 - h_{\tau+}^z)(1 + h_{\tau-}^z) = wt_{spin}^+ + wt_{spin}^- \tag{18}$$

For the decay of a scalar into  $\tau$  lepton pair, the spin state probability is  $p_\tau^z = \frac{1}{2}$  (the subscript  $z$  denotes spin projection on the  $z$  axes of frames type  $C$ ).

For the decay of a vector resonance, the probability  $p_\tau^z$  depends on the intermediate boson virtuality and  $\tau$  lepton direction, thus we can rewrite formula (17) as

$$wt_{spin} = p_\tau^z(1 + h_{\tau+}^z)(1 + h_{\tau-}^z) + (1 - p_\tau^z)(1 - h_{\tau+}^z)(1 - h_{\tau-}^z) = wt_{spin}^+ + wt_{spin}^- \tag{19}$$

Note that  $p_\tau^z$  can be interpreted as the probability of creating first  $\tau$  lepton of the pair in a spin state (+) along the  $z$  direction, whereas  $P_\tau^z$  as its longitudinal polarization. Obviously, from comparison of above Eqs. (17) and (19), following relation occurs:

$$p_\tau^z = 0.5 - 0.5P_\tau^z \tag{20}$$

which can be rewritten as

$$P_\tau^z = 2(1 - p_\tau^z) - 1. \tag{21}$$

### 4.3 Longitudinal versus transverse spin correlations

In the formalism discussed above, there is a seemingly rather minor difference between the situations when the complete spin effects or only longitudinal ones are taken into account. The case of longitudinal spin effects only means that all  $R_{i,j}$  components are set to zero except  $R_{t,z}$ ,  $R_{z,t}$  and  $R_{t,t} = 1$ ,  $R_{z,z} = \pm 1$ ; the sign depends on whether the decaying object is a scalar or a vector and  $R_{t,z}$  is equal to longitudinal  $\tau$  lepton polarization  $P_\tau^z$ . No difference in the re-weighting algorithm is needed with this definition of  $R_{i,j}$ . The transverse spin correlations are introduced with non-zero  $R_{x,x}$ ,  $R_{y,y}$  and off-diagonal  $R_{x,y}$ ,  $R_{y,x}$ , without any modifications of the  $t$  and  $z$  components.

## 5 The 2 → 2 parton level processes

The complete event weight  $WT$  is factorized into multiplicative components: production ( $wt_{prod}$ ), decay ( $wt_{decay}^{\tau^\pm}$ ) and spin correlation/polarization ( $wt_{spin}$ ):

$$WT = wt_{prod} wt_{decay}^{\tau^+} wt_{decay}^{\tau^-} wt_{spin}. \tag{22}$$

If the sample is polarized the last factor should read  $wt_{spin}^{new}/wt_{spin}^{old}$ .

The implemented methods calculate internally a complete set of weights but as a default one, the spin correlation/polarization weight, as most often used in the applications is returned.<sup>5</sup> Other weights are returned by supplementary methods.<sup>6</sup>

### 5.1 Production weight

The production weight  $wt_{prod}$  which is a sum over all configurations of  $\tau$  leptons spin and flavours of incoming partons of matrix element squared multiplied by PDFs is calculated in frame  $A'$ . The parton level angular kinematic configuration is usually taken in the  $A$  frame. The  $R_{i,j}$  are calculated in  $A$  or  $A'$  frame, except the case when transverse spin effects are taken into account and frame  $B$  has to be used.

### 5.2 Decay weights

Calculation of the decay weights  $wt_{decay}^{\tau^+}$  and  $wt_{decay}^{\tau^-}$  is a by-product of polarimetric vectors of  $h_{\tau+}^i$  and  $h_{\tau-}^j$  calculation. This calculation is performed in frames  $D_{\tau^\pm}$ . The methods of Tauola used for  $h_{\tau-}^j$  calculation, provide at the same time decay matrix elements squared summed over the  $\tau$  lepton spin which are used in  $wt_{decay}^{\tau^\pm}$ .

### 5.3 Spin correlation matrix $R_{i,j}$

In most cases the spin correlation matrix  $R_{i,j}$  is calculated in frame  $A$  or  $A'$ . Frame  $B$  is used if transverse spin effects are taken into account.

Calculations of the production weight and of spin correlation matrix  $R_{i,j}$  are related with each other. It is important to realize that in the collinear approximation

<sup>5</sup> Calculation is invoked with the call to `calculateWeightFromParticlesH` method. The code organization is a consequence of the history of implemented functionalities and benchmarks. Not all features were expected to be included at the beginning:

At first, only longitudinal spin effects were expected to be taken into account and for  $2 \rightarrow 2$  at parton level processes only. That is why, methods from KORALZ [34] could have been used. Later processes  $2 \rightarrow 4$  with two jets in final state made original methods focused around calculation of  $\tau$  lepton polarization i.e.  $R_{t,z}$  less intuitive/convenient. Introduction of options featuring complete spin effects made such organization even less suitable. Still we preserve backward compatibility, of the functioning code and stability of results for tests and examples, accumulated over years.

<sup>6</sup> There is technical detail specific for the  $2 \rightarrow 2$  processes. In the sum of formula (11) over configurations of the incoming partons with flipped flavours it is enough to change the sign of the  $\cos \theta$ , as used e.g. in formula (39) of Appendix B.9.

$$R_{t,z} = \frac{\sigma_{prod}^+ - \sigma_{prod}^-}{\sigma_{prod}^+ + \sigma_{prod}^-} \tag{23}$$

where  $\sigma_{prod}^\pm$  denotes the sum of matrix element squared multiplied by PDFs, and the first  $\tau$  lepton spin is  $\pm$ . In this approximation and for Eq. (23) the difference between frames  $A$  and  $A'$  can be ignored.

Note, that the weight  $wt_{prod}$  is obtained as a by-product of  $R_{i,j}$  calculation, in fact of its  $R_{t,z}, R_{z,t}$  components.<sup>7</sup> In case of using directly the formula for Born cross-section with vector/axial couplings and  $Z/\gamma^*$  propagators (implemented in TauSpinner) this connection is less transparent.

### 5.3.1 Case of a spin zero resonance

If the  $\tau$  lepton mass is small in comparison to the  $\tau$  lepton pair virtuality then  $R_{z,z} = -1$ . For the scalar Higgs boson holds  $R_{x,x} = R_{y,y} = 1$  and  $R_{x,x} = R_{y,y} = -1$  for the pseudo-scalar, while the mixed scalar-pseudoscalar configuration the formula includes scalar-pseudoscalar mixing angle and non-zero off-diagonal  $R_{x,y}, R_{y,x}$  terms. This extension is important for the simulation of Higgs CP parity signatures.

### 5.3.2 Case of a spin one resonance (Drell–Yan)

For a neutral vector boson  $R_{z,z} = 1$ , if the  $\tau$  lepton mass is small in comparison of  $\tau$  lepton pair virtuality. The non-zero off-diagonal terms are

$$R_{t,z} = R_{z,t} = 2 \cdot p_\tau^z - 1. \tag{24}$$

In the present implementation, if the EW corrections are neglected the transverse spin correlations for neutral vector boson/photon exchange are neglected as well and  $R_{x,x} = R_{y,y} = R_{x,y} = R_{y,x} = 0$ . We are simply missing the code for the Born level expressions taking into account the transverse spin effects. Fortunately, if EW corrections calculation is activated, thanks SANC library [35] tables ported from the Tauola, not only the first order EW corrections become available, but the transverse spin effects as well.

### 5.3.3 Case of a spin two resonance X (non SM Higgs)

In this case  $R_{z,z} = 1$  as in the Drell–Yan case. The non-zero off-diagonal terms are

$$R_{t,z} = R_{z,t} = 2 \cdot p_\tau^z - 1. \tag{25}$$

<sup>7</sup> Relation between  $R_{t,z}, R_{z,t}$  and modules of amplitudes is transparent in case of calculation as of Sect. 7.

the difference to Drell–Yan case is in  $p_\tau^z$  angular dependence (defined in  $\tau$  lepton pair rest frame  $B$ ). It is described by spherical polynomial of the fourth order and not of the second order. This case was introduced in the paper [12] and corresponding example of the TauSpinner application.

### 5.4 Calculating the $wt_{spin}$ weight

For the  $wt_{spin}$  calculation formulae (15, 16) are available. However, unless required differently by the user, the default version is to omit transverse spin correlations, i.e.  $R_{x,x} = R_{y,y} = R_{x,y} = R_{y,x} = 0$ , corresponding to formula (17). In the following we will exploit components of  $R_{i,j}$  for the individual cases prepared in the previous sub-section.

#### 5.4.1 Case a of spin zero resonance (scalar Higgs)

$$wt_{spin} = 1.0 + sign \cdot h_+^z h_-^z + R_{x,x} h_+^x h_-^x + R_{yy} h_+^y h_-^y + R_{x,y} h_+^x h_-^y + R_{y,x} h_+^y h_-^x \tag{26}$$

where  $sign = -1$ .

#### 5.4.2 Case a of spin two resonance X (non SM Higgs)

$$wt_{spin} = 1.0 + sign \cdot h_+^z h_-^z + h_+^z + h_-^z + P_\tau^z h_+^z + P_\tau^z h_-^z + R_{x,x} h_+^x h_-^x + R_{y,y} h_+^y h_-^y + R_{x,y} h_+^x h_-^y + R_{y,x} h_+^y h_-^x \tag{27}$$

where  $sign = +1$ . Note, that this form depends on Born level cross section provided (by a pointer) at the execution time with the user function. It is used in calculating  $R_{i,j}$ . At present only  $P_\tau^z$  is non-zero in implemented examples.

#### 5.4.3 Case a of spin one resonance (Drell–Yan)

$$wt_{spin} = 1.0 + sign \cdot h_+^z h_-^z + h_+^z + h_-^z + P_\tau^z h_+^z + P_\tau^z h_-^z + R_{1,1}^{EW} R_{x,x} h_+^x h_-^x + R_{2,2}^{EW} R_{y,y} h_+^y h_-^y + R_{1,2}^{EW} R_{x,y} h_+^x h_-^y + R_{2,1}^{EW} R_{y,x} h_+^y h_-^x \tag{28}$$

where  $sign = +1$  and  $R_{1,1}^{EW}, R_{1,2}^{EW}, R_{2,1}^{EW}$  and  $R_{2,2}^{EW}$  denote the averaged (with the production cross-section) pre-tabulated transverse spin correlations terms, calculated using  $\mathcal{O}(\alpha)$  EW corrections from the SANC program.<sup>8</sup>  $R_{x,x} = R_{y,y} = R_{x,y} = R_{y,x} = 1$  should be set, in case we want to use  $\mathcal{O}(\alpha)$  EW corrections from SANC. Or one can also set some of them to 1 and the remaining ones to 0 in case one is

<sup>8</sup> Nothing in principle prevents us to perform similar arrangements for  $R_{t,z}$ , that is for  $P_\tau^z$  which can be taken from pre-tabulated EW results as well. Discussion of the systematic uncertainty is then nonetheless needed.

interested in studying only some components of transverse-spin effects.

The  $R_{x,x}, R_{y,y}, R_{x,y}, R_{y,x}$  are used directly as transverse components for  $R_{i,j}$  in case of non-standard model calculation. If one is not interested in the actual size of transverse effects, as predicted by  $\mathcal{O}(\alpha)$  EW corrections, one can also modify hard coded pre initialized to zero values, or use non SM Higgs options discussed previously.

Generated Drell–Yan samples may feature only partial spin effects, e.g. just the  $(1.0 + sign \cdot h_+^z h_-^z)$  part of the spin effects. Then  $w_{tspin}$  has to take this into account, the denominator of such form has to be introduced, as to remove this partial effect. The numerator given in Eq. (28) will introduce the required (usually full) spin effect. Let us list the formulae for the typical cases:

- If in the generated sample spin correlations between two  $\tau$  leptons are included but no other effects, the weight takes the form:

$$w_{tspin}^{no\ pol} = \frac{w_{tspin}}{1.0 + sign \cdot h_+^z h_-^z}. \tag{29}$$

Such choice for the sample introduces dominant spin effect (the correlation), which at the same time is free of systematic errors. It is a consequence of the spin 1 state decaying to  $\tau$  lepton pair.

- If in the generated sample spin correlations and partial (no angular dependence and no incoming quark flavour dependence) polarization effects are included, the weight should take the form:

$$w_{tspin}^{no\ angular} = \frac{w_{tspin}}{1 + sign \cdot h_+^z h_-^z + P_\tau^{z,part} h_+^z + P_\tau^{z,part} h_-^z} \tag{30}$$

where the denominator again represents spin correlations present in the sample. One can expect the  $P_\tau^{z,part} = P_\tau^z(s, \cos(\theta = 0))$  calculated for incoming electrons being the easiest way to obtain an average correction. This approach reproduces correctly the average individual  $\tau$  lepton polarization for the events of  $\tau$  lepton pair virtuality close to the  $Z$  boson peak. Such choice for the generated sample could be motivated as follows. It features all spin effects except those which depend on the incoming partons PDFs, thus it is free of related systematic uncertainties, making it convenient for studying systematic uncertainty from PDFs. The corrections introduced with  $w_{tspin}^{no\ angular}$  are even smaller than those of  $w_{tspin}^{no\ pol}$ .

### 5.5 Use of spin correlation/polarization weight in special cases

If the generated sample feature spin effects then the  $w_{tspin}$  of formula (3) can be used as weight  $WT = 1/w_{tspin}$  to remove spin effects.

The cases when only a part of spin effects is taken into account, like in Ref. [36], more specifically the spin correlation but no effects due to vector and axial couplings to the intermediate  $Z/\gamma^*$  state, can be corrected with the help of the appropriate weights as in examples for Eqs. (29, 30). On the other hand, such partial spin effects can be also removed completely, e.g. for consistency checks. The case, when spin correlations and average  $\tau$  lepton polarization are present, but angular dependence is absent, can be treated as usual with Eq. (10).

### 5.6 Born cross-section

Until now, we were expressing formulae in the language of spin amplitudes and spin weights (spin correlation matrix  $R_{i,j}$ ). However, often cross-sections calculated at the parton level are available for the explicit helicity states of the outgoing  $\tau$  leptons. Several such lowest order formulae for calculating cross-sections of an analytical form are implemented in the code of the TauSpinner itself, or of its examples. Some of them represent the *legacy* code, ported from different projects, others were coded for the TauSpinner application as benchmarks on the EW parameters setting and are used in the published tests. For completeness of the documentation we describe them in Appendix B.9.

## 6 Polarimetric vectors, polarization and helicity states

Calculation of the polarimetric vectors  $h_\tau$  is performed using the code from the Tauola library. The convention is that  $h_\tau = (h_\tau^x, h_\tau^y, h_\tau^z, h_\tau^t)$  (i.e. the last component is time-like) following the convention used in FORTRAN. The same FORTRAN code calculates decay matrix element squared, which are returned as well, as auxiliary information.

Depending on which decay channel is present the calculation is performed using the respective functions from the Tauola library. Not for all  $\tau$  lepton decay channels, see Table 2,  $h_\tau$  is nonetheless calculated. About 97.5% of the decay width is covered, i.e. the polarimetric vector  $h$  is calculated, otherwise its spatial components are set<sup>9</sup> to 0.

<sup>9</sup> This is not the case for the Tauola library if used for generating  $\tau$  lepton decays. Then only for channels featuring more than 5 pions polarimetric vector is calculated with approximation. Still then, it is not set to zero.

**Table 2** Summary of  $\tau$  lepton decay modes implemented for calculation of polarimetric vectors. Branching fractions for each decay mode according to [37] illustrate the completeness of the algorithm. The ‘‘Other’’ decay modes are treated as unpolarized ones

$\tau$ lepton decay mode	Branching fractions %
$e^- \bar{\nu}_e \nu_\tau$	17.85
$\mu^- \bar{\nu}_\mu \nu_\tau$	17.36
$\pi^- \nu$	10.91
$\pi^- \pi^0 \nu$	25.51
$\pi^- \pi^0 \pi^0 \nu, \pi^- \pi^+ \pi^- \nu$	9.29, 9.03 (incl. $\omega$ )
$K^- \nu$	0.70
$K^- \pi^0 \nu, K^0 \pi^- \nu$	0.43, 0.84
$\pi^- \pi^+ \pi^- \pi^0 \nu$	4.48 (incl. $\omega$ )
$\pi^- \pi^0 \pi^0 \pi^0 \nu$	1.04
Other	2.5

To calculate decay matrix elements,  $\tau$  lepton decay products have to be boosted into the  $\tau$  lepton rest-frame: we first boost all products to the  $\tau$  lepton pair rest frame, then rotate from  $A$  to  $B$  so that a  $\tau$  lepton of  $Z/H$  decay is along the  $z$ -axis. An intermediate step is to boost from the  $\tau$  lepton pair rest-frame to  $C_\tau$ , followed by a rotation to have the neutrino from  $\tau$  lepton decay along the  $z$ -axis ( $D_\tau$  frame). The rotation angles  $\phi_2, \theta_2$  are stored for rotating back of the polarimetric vector to the  $\tau$  lepton rest-frame  $B$  or  $C_\tau$ .

As a by-product of the  $h_\tau$  calculations, the  $w_{decay}^{\tau^\pm}$  of Eq. (12) is obtained.

### 6.1 Longitudinal polarization

The calculation of components of the  $R_{i,j}$  spin density matrix in the ultra-relativistic limit is then equivalent to the calculation of modules for matrix elements of  $\tau$  lepton pair production in given helicity configurations. Depending on the approach, either  $p_\tau^z$  or  $P_\tau^z$  is calculated from the other one, using respectively formula (21) or (20). The case when EW effects are included, technically differs only slightly,<sup>10</sup> despite the fact that then the transverse spin effects can be taken into account. In all cases the relation  $R_{i,z} = P_\tau^z$  holds, no pre-tabulated EW results are used for this component of  $R_{i,j}$ . This can be easily activated though, e.g. if results for large  $s$  are needed. Around the  $Z$  peak presently implemented EW library may be less suitable. Formula (19) is used for attribution of helicity states.

<sup>10</sup> The same formulae using  $R_{i,j}$  matrix are still used, individual parton flavour contributions to  $R_{i,j}$  are obtained numerically from the pre-tabulated values for some invariant masses and scattering angles. These pre-tabulated values are read from the stored files. Then results are interpolated to obtain results for required arguments. The  $x$  and  $y$  components of  $R_{i,j}$  are pre-tabulated and available for use.

#### 6.1.1 Case of a spin = 0 resonance (scalar (Higgs))

In  $H \rightarrow \tau\tau$  lepton decays the probability of the helicity state denoted  $p_\tau^z$  is equal to 0.5 for  $P_\tau^z = 1$  and for  $P_\tau^z = -1$  as well, see Table 3.

#### 6.1.2 Case of spin = 1 resonance (Drell–Yan)

In  $Z/\gamma^* \rightarrow \tau\tau$  lepton decays the probability of the helicity state denoted  $p_\tau^z$  is a function of the  $\tau$  lepton scattering angle,  $\theta$ , and the center of mass energy squared of the hard process,  $s$ . It is true at Born level, and in the ultra-relativistic limit [36].

Note that the rest frame of the production process and the rest frame of the  $\tau$  lepton pair might not be the same due to photon bremsstrahlung in the  $\tau$  lepton pair production vertex. We have discussed this already in Sect. 3.1, but let us complete some more technical details now.

The center-of-mass system of  $\tau$  lepton pairs is used for calculating  $\cos \theta$  and the direction of incoming partons is set along  $z$ -direction in the laboratory frame. The following definitions are used:

$$\cos \theta_1 = \frac{\boldsymbol{\tau}^+ \cdot \boldsymbol{b}^+}{|\boldsymbol{\tau}^+| |\boldsymbol{b}^+|}, \quad \cos \theta_2 = \frac{\boldsymbol{\tau}^- \cdot \boldsymbol{b}^-}{|\boldsymbol{\tau}^-| |\boldsymbol{b}^-|}, \quad (31)$$

where  $\boldsymbol{b}^+, \boldsymbol{b}^-$  denote 3-vectors of incoming beams, in the laboratory frame defined along the  $z$ -axis. The definition of an effective  $\cos \theta$  is constructed following Ref. [38]

$$\cos \theta = \frac{\cos \theta_1 \cdot \sin \theta_2 + \cos \theta_2 \cdot \sin \theta_1}{\sin \theta_1 + \sin \theta_2}. \quad (32)$$

At Born level, for a given flavour of incoming partons and in the ultra-relativistic limit, the  $p_\tau^z$  is defined, Ref. [36], as

$$p_\tau^z = \frac{\frac{d\sigma}{d \cos \theta}(s, \cos \theta, P_\tau^z = 1)}{\frac{d\sigma}{d \cos \theta}(s, \cos \theta, P_\tau^z = 1) + \frac{d\sigma}{d \cos \theta}(s, \cos \theta, P_\tau^z = -1)}, \quad (33)$$

where

$$\frac{d\sigma}{d \cos \theta}(s, \cos \theta, P_\tau^z) = (1 + \cos^2 \theta) F_0(s) + 2 \cos \theta F_1(s) - P_\tau^z [(1 + \cos^2 \theta) F_2(s) + 2 \cos \theta F_3(s)] \quad (34)$$

and  $F_i(s)$  are four form factors which depend on the initial and the final state fermion couplings to the  $Z$  boson and its propagator.

In the TauSpinner algorithm the flavour of the initial partons is assumed to be unknown, the probability  $p_\tau^z$  is calculated as a weighted average over all possible initial state quark configurations. This requires averaging (summing) over all production matrix elements square convoluted with the structure functions

**Table 3** Probability for the helicities of the of  $\tau$  leptons from different origins [36]

Origin	Hel. $\tau_1$	Hel. $\tau_2$	Prob.
Neutral Higgs boson: $H$	+	−	0.5
	−	+	0.5
Neutral vector boson: $Z/\gamma^*$	+	+	$p_\tau^z$
	−	−	$1 - p_\tau^z$
Charged Higgs boson: $H^\pm$	+	−	1.0
Charged vector boson: $W^\pm$	−	−	1.0

$$p_\tau^z = \frac{\sum_{(flav.:i,j)} f_i(x_1, Q^2) f_j(x_2, Q^2) \cdot \sum_{spin} |\mathcal{M}_{i,j}^{prod}|^2 \cdot p_\tau^z(i, j)}{\sum_{(flav.:i,j)} f_i(x_1, Q^2) f_j(x_2, Q^2) \cdot \sum_{spin} |\mathcal{M}_{i,j}^{prod}|^2}, \tag{35}$$

where the indices  $i, j$  denote flavours of incoming partons.  
 For calculating the spin weight, the polarization  $P_\tau^z$  of the single  $\tau$  lepton in a mixed quantum state is calculated neglecting transverse spin degrees of freedom (ultra-relativistic limit). The  $P_\tau^z$  is then a linear function of the probability for the helicity state,<sup>11</sup> Eq. (21).

6.1.3 Case of a spin two resonance  $X$  (not a Higgs)

In this case a user provided function required to replace  $\frac{d\sigma}{d\cos\theta}$ , the non-Standard physics effective Born parametrizations, is not included in the TauSpinner library, and a dummy function is provided only. It can be replaced with a user one, see Ref. [12] for details and in particular the `TAUOLA/TauSpinner/examples/tau-reweight-test.cxx` directory. This  $2 \rightarrow 2$  parton level process function `nonSM_adoptH` (or `nonSM_adopt`) has the following arguments (int ID, double S, double cost, int H1, int H2, int key). It can be activated in the user code at the execution time, see Appendix B.2.

6.2 Attributing  $\tau$  (or  $\tau$  pair) helicity states

For the individual event, the  $\tau$  helicity (+/−) is attributed stochastically. The probability for given configurations of the longitudinal polarization of  $\tau$  leptons of different origins [36] is shown in Table 3. These probabilities can be used, when decay kinematical configurations of the decays are not yet constructed.

However, once the configurations of decay products are generated the event-by-event calculation of the probability for helicity states has to take into account polarimetric vectors

<sup>11</sup> Note that in paper [11] formula  $P_\tau^z = (2p_\tau^z - 1)$  is used instead of Eq. (21). There, different (rotated by  $\pi$  angle with respect to axis perpendicular to reaction) frame is used. Similar convention matching may be necessary for other programs as well, see e.g. `nonSM_adopt()`; `nonSM_adoptH()` methods of the `tau-reweight-test.cxx`.

$h_+$  and  $h_-$ . The (+) is chosen if probabilities, calculated with the formulae below, are smaller than a generated random number RRR sampled uniformly in [0,1] range, otherwise (−) is taken:

– **Case of a spin zero resonance (scalar Higgs)**

$$p(-) = \frac{1 + sign \cdot h_+^z \cdot h_-^z + h_+^z + h_-^z}{2 + 2 \cdot sign \cdot h_+^z \cdot h_-^z} \tag{36}$$

where  $sign = -1$ .

– **Case of a spin two resonance  $X$  (not a Higgs)**

$$p(-) = (1 + P_\tau^z) \frac{1 + sign \cdot h_+^z \cdot h_-^z + h_+^z + h_-^z}{2 + 2 \cdot sign \cdot h_+^z \cdot h_-^z + 2.0 \cdot P_\tau^z \cdot (h_+^z + h_-^z)} \tag{37}$$

where  $sign = +1$ .

– **Case of a spin one resonance (Drell–Yan)**

$$p(-) = (1 + P_\tau^z) \frac{1 + sign \cdot h_+^z \cdot h_-^z + h_+^z + h_-^z}{2 + 2 \cdot sign \cdot h_+^z \cdot h_-^z + 2.0 \cdot P_\tau^z \cdot (h_+^z + h_-^z)} \tag{38}$$

where  $sign = +1$ .

7 The  $2 \rightarrow 4$  process

At this point we have completed the discussion for the case when the TauSpinner is used assuming a parton level processes of  $2 \rightarrow 2$  type. Let us turn now to the case when  $2 \rightarrow 4$  parton level processes are used. This extension has been introduced recently, see Ref. [15] for a detailed description.

The baseline algorithm remains essentially the same. The differences can be grouped into few parts

– **Kinematics**

For calculation of factorization scale, instead of virtuality of  $\tau$  lepton pair, virtuality of lepton pair combined with

jets is used. In fact a few options to define the  $Q^2$  factorization scale have been implemented. The definitions of frames type  $A$ ,  $B$ ,  $C_{\tau^\pm}$  and  $D_{\tau^\pm}$  as explained for case of  $2 \rightarrow 2$  processes are used. In principle the additional rotation by the angle  $\pi$  around the axis  $x$  would be again needed for the  $\tau^-$  in transformations from  $B$ ,  $C_{\tau^-}$  frames. This is due to convention used for spin amplitudes.<sup>12</sup> The definition of the previously discussed frame  $A'$  needs to be modified. Not only bremsstrahlung photons need to be added to  $\tau$  leptons to obtain kinematic configurations for matrix element calculation, but outgoing partons (jets) four-momenta have to be taken into account as well.

#### – Matrix Element Calculation

Matrix element squared are calculated for given helicity states of  $\tau$  leptons and flavours of incoming/outgoing partons in frame  $A'$ . In particular formula (23) is used for calculation of  $R_{z,t}$  and  $R_{t,z}$ , which are then used as of frame  $B$ , without any modifications. The EW scheme used and  $\alpha_s(Q^2)$  can be configured using several options defined in Ref. [15].

#### – Extension due to flavours of accompanying partons/jets

All sums  $\sum_{flavour}$  as used in Sect. 2.2 should read now as sums over flavours of incoming *and* outgoing partons. Of course PDFs convolution with distributions is used for incoming partons only. This modification is extending re-weighting algorithm dependence on the final state components consisting of partons (hadronic jets). This technically simple change may be thus of a concern because of additional types of the systematic uncertainties involved.

Except these changes, the algorithms are not modified. However only longitudinal spin effects can be simulated because we use helicity states, in the ultra-relativistic approximation in matrix element calculation. In future, if necessary, we can extend the case to the complete spin effects treatment. Minor extension of the program will be needed. Approximation is in the matrix element calculation only. It may even be possible to use for that purpose presently available MadGraph amplitudes discussed in Ref. [15]. Better control of the complex phases will be however needed.

## 8 On the systematic errors

In the following the limitations of the program and its systematic uncertainties will be discussed.

One of the questions is whether TauSpinner can be used to modify spin/production/decay features of events;

<sup>12</sup> In practice, as no transverse spin effects are taken into account, the convention change is performed while calculating `double W[2][2]` (amplitudes squared for the helicity states) in `wbfdistr.cxx`.

when  $\tau$  lepton decay library other than Tauola was used, when its physics initialization was different, or when events were generated with QCD matrix elements of higher order than implemented in TauSpinner? If yes, what is the systematic uncertainty or bias which should be attributed to the predictions obtained using a Monte Carlo sample with weights calculated by TauSpinner?

The evaluation of them is not universal, it needs to be repeated for every new observable. We therefore cannot focus the discussion on the tool itself, but rather present general observations concerning re-weighting techniques for the  $\tau$  lepton physics. The topic obviously can not be all-embracingly covered in the present paper.

Nowadays Monte Carlo programs such as Herwig [39], Sherpa [20], Pythia [40] have their own  $\tau$  lepton decay algorithms and physics initializations. One can argue, if in some aspects, they are not better than the ones used in publicly available version of the Tauola library. One can also argue that the re-weighting with algorithms like TauSpinner is nowadays an obsolete technique, and that it is better to generate new event samples with modified required physics input, or to use generator own re-weighting methods.

In some cases this might be true but it is not always easy to apply in practice. For example, experiments store events in a format which must be universal for all generators. Inevitably some intermediate information is lost. Also, re-generation of such experiment-specific samples with modified assumptions is time-consuming, because of the detector response simulation and often also because of the CPU needed for each event of the desired phase space region. Event generation became CPU expensive in modern Monte Carlo generators, because of the complexity of the phase space and multiplicity of the outgoing legs in the matrix-elements.

The evaluation of systematic uncertainties of the re-weighting technique implemented in TauSpinner is nonetheless mandatory. It can be divided into three main domains:

- A. Physics driven uncertainty, related to approximations used for particular matrix element calculations of the production and the  $\tau$  lepton decays, finally of the parton distribution functions.
- B. Approximations related to the program design.
- C. Technical uncertainties related to the mismatch in the way how information is being passed to the program or of approximations used in the generation of samples to be re-weighted.

#### A.

Technically, any  $\tau$  lepton decay generator, to a great degree, can be understood as consisting of three elements: (i) a phase space generator, (ii) a calculator of matrix elements and finally (iii) a model for hadronic current for semilep-

tonic decays. The last one (iii) is prone to large systematic uncertainties. `TauSpinner` shares the code for (ii) and (iii) with `Tauola`. That is also why part of the systematic uncertainties is common. The precision of models used in construction of hadronic currents is optimistically at 5 % precision level and realistically one can expect a precision rather at the level of 10 or 20 % only. Smaller ambiguities are expected for inclusive distributions like branching ratios or one-dimensional spectra, larger ones for multidimensional distributions. It is crucial, that the work on related comparisons with the data, fits and evaluation of systematic uncertainties is performed within the experiments. Let us focus on the systematic uncertainty evaluation and practical consequences for `TauSpinner` when applied to events where  $\tau$  lepton decays are simulated with a different library than `Tauola`. The particular choice of hadronic currents used in samples generated with `Herwig`, `Sherpa` or `Pythia` needs to be known and the same choice has to be used for `TauSpinner` too. The techniques how to check if proper choice was taken are documented in Ref. [13] (Section 4.5 “Consistency checks”). Consequences of mismatches are of numerical importance.

If the matrix elements used in the  $\tau$  lepton decay sample generation do not match those assumed to be used for the calculation of `TauSpinner` weights, results can be largely inaccurate. Particularly numerical instabilities may occur if the matrix element used for the weight denominators would approach zero in non-depleted regions of the phase space. The problem may appear for multi-scalar final states, but for  $\tau^\pm \rightarrow l^\pm \nu \bar{\nu}$ ,  $\tau^\pm \rightarrow \pi^\pm \nu$  or even  $\tau^\pm \rightarrow \pi^\pm \pi^0 \nu$  decays, which amount for 2/3 of all  $\tau$  lepton decays, this problem is generally absent. Let us concentrate on the case of  $\tau^\pm \rightarrow (3\pi)^\pm \nu$  because it contributes more than 20 % to the  $\tau$  lepton decay rate. We have observed that in some regions of the phase space the modeling of  $\tau$  lepton decays can lead to distributions which differ by a factor of 2, or even more, depending on the data used as an input. At the same time, for more inclusive results like one-dimensional distributions differences can be of the order of 10 % or less. For numerical evaluation see Ref. [41] (Section 10). Eight variants of hadronic currents were presented and used for the discussion in Section 2 of Ref. [41], most of them as available in `Tauola`, but one of them was taken from `Pythia 8`, in the version of the year 2015. Tests how to check if indeed sufficient level of precision was achieved are discussed in the summary of that paper. It is stressed that at least 3-dimensional distributions are necessary to appropriately identify differences. Tests of modelling for  $\tau$  lepton decays are particularly important for applications based on Machine Learning techniques. Such tests were performed in Ref. [42], for Higgs CP parity sensitive observables in  $H \rightarrow \tau^+ \tau^-$  with  $\tau$  lepton decays to three scalars.

## B.

Let us note that not only the matrix elements, used for  $\tau$  lepton decays in the re-weighted sample, have to match the one used in the weight calculation. Also the level of approximation used for calculation of production matrix element and in particular spin effects have to match the one used in spin weight denominators. See Ref. [13], Section 4, for details on the consistency checks.<sup>13</sup>

The matrix element for  $pp \rightarrow \tau^+ \tau^- X$  production are generally not available. They are obtained from the factorization theorems separating parton distribution (parton showers) from the hard matrix elements. For the case of  $pp$  collisions, references [32,33] can be used to provide necessary input from the theoretical side for the algorithmic solutions. The tests of consistency are presented<sup>14</sup> in Section 5.4 of Ref. [13]. Following the results of references [32,33] tests consisting of comparing spin effects implemented in different  $\tau$  lepton pair rest frames can be very useful on top of the ones for the dependence of results on EW schemes studied in these papers.<sup>15</sup>

The comparisons of results obtained from `TauSpinner` working in  $2 \rightarrow 2$  and  $2 \rightarrow 4$  matrix element modes offer the way to check if factorization used in `TauSpinner` algorithm is sufficiently precise for the configurations featuring high  $p_T$  jets.

## C.

One should bear in mind, that information on intermediate and also final state particles stored by many of today's generators use distinct variants of encoding conventions, even if the same format like `HEP-MC` is used in every case. That can be a problem for many applications used by experiments, e.g. in the estimation if reconstruction algorithms work properly, and if the initial input of the generation can be deconvoluted from the event response of the detector. Inescapably this may cause problems for `TauSpinner`, too. Implicitly, the event record design relies on the assumption that the event is represented by a tree-like structure. This generally can not be assured, in particular because of quantum interference and quantum entanglement. Some approximations need to

<sup>13</sup> One of such test is to verify whether with the help of `TauSpinner` weight, spin polarization and spin correlations can be completely removed from the event sample. It is relatively easy to observe if it is indeed the case. The decay product spectra should then not depend on anything else but properties of decay matrix elements and  $\tau$  lepton momenta.

<sup>14</sup> They are also distributed together with `Tauola` and `TauSpinner` source code.

<sup>15</sup> Once completed, they open the way for interchanging the EW schemes with the help of the weights. With the improving precision, this may turn out to be important for  $\tau$  leptons final states including decays as for light leptons production studied in Ref. [16].

be used, or information of intermediate states, such as EW bosons abandoned.<sup>16</sup>

Several options of `TauSpinner` initialization are prepared. They can help to understand if the samples were e.g. generated particular, partial spin implementation only. Then they can be used to re-weight to the required physics assumptions, see Ref. [13].

## 9 Summary

In this paper we have presented the theoretical basis and description of algorithms used in the `TauSpinner` program for simulating spin effects in the production and the decay of  $\tau$  leptons in proton–proton collisions at the LHC. The primary source of  $\tau$  leptons are Drell–Yan processes of single  $W$  and  $Z$  boson production, often accompanied by additional jets. Since the Higgs boson discovery by ATLAS and CMS Collaboration in 2012, the  $H \rightarrow \tau\tau$  lepton decay became the most promising channel for studying CP properties of its couplings to fermions. With more data available from Run II of the LHC also  $\tau$  lepton decays in case of multi-boson production will become part of interesting signatures.

The algorithms discussed here allow for the separation of the  $\tau$  lepton production and decay in phenomenological studies. We have discussed options used for simplifying descriptions and approximations used.

Although the framework of `TauSpinner` is prepared for implementation of the non Standard Model couplings, we have not given too much attention to this possible development path. We have recalled in the Appendices most of examples and applications. They are available with the distributed code. References, covering details wherever possible are given. The technical information is brief, details are delegated to the README files. Techniques of how to estimate systematic uncertainties for the results obtained with `TauSpinner` were briefly reviewed.

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**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors' comment: The code of `TauSpinner` is distributed only as a part of the `Tauolapp` [25]

<sup>16</sup> That is not convenient, if properties of such intermediate states are to be measured. For example, reconstruction algorithms are more difficult to debug. From that perspective `TauSpinner` can be understood as supplementary debugging tool. If its results are far from the expected ones, this often mean, that the content of the (`HepMC`) event records used in experimental environment was not up to the expectations. Results obtained from `TauSpinner` can point to the source of difficulties. See Ref. [13], Section 5.4 for examples of such applications.

package. It is available from webpage: <http://tauolapp.web.cern.ch/tauolapp/>. At the time of present publication, the head release for `Tauola` was 1.1.6c (and for `TauSpinner` 2.0.3.)]

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## A Project software organization

The code of `TauSpinner` is distributed as a part of the `Tauolapp` [25] package. The main modules of the library source code reside in `/src` and `/src/VBF` sub directories of `TAUOLA/TauSpinner`. The compiled libraries are placed in the `TAUOLA/TauSpinner/lib` and copied to the `TAUOLA/lib` directory during installation of the `TAUOLA` library with the `TauSpinner`. Technical details on the installation and execution of main examples are provided in README files of sub-directories:

`TAUOLA/TauSpinner`,  
`TAUOLA/TauSpinner/examples`,  
`TAUOLA/TauSpinner/examples/example-VBF`,  
`TAUOLA/TauSpinner/examples/example-VBF/SPIN2`. The `changelog.txt` file is also provided, however, since Nov. 2012 logs are set into the `TAUOLA/changelog.txt`. Note that the file `html/index.html` is of the old `TauSpinner` versions prior to its merge with `Tauolapp` distribution too.

The distribution includes extended set of examples and applications, collected in the `TAUOLA/TauSpinner/examples` sub-directories:

- `CLEO-to-RCHL`, see App. B.7,
- `CP-tests`, see Apps. B.3 B.3.1 B.3.2, and also introduction to EW effects implementation, App. B.4.
- `example-VBF`, App. B.5 and B.6,
- `testsERW`, App. B.8,
- `applications`, general methods, useful to deal with event record contents and README file is included only.
- `applications/applications-fits`, App. B.1,
- `applications/applications-plots-and-paper`, App. B.1,
- `applications/applications-rootfiles`, App. B.1,
- `applications/test-bornAFB`, App. B.1,
- `applications/test-ipol`, App. B.1.

Some of published results obtained with the program are stored in further sub-directories of the `TAUOLA/TauSpinner`, that is in

- `paper`, see App. B.1, and Ref. [11],
- `paper-spin2`, see App. B.2, and Ref. [12],
- `paper-spin2-higgs`, see App. B.1, and Ref. [12],
- `paper-application-studies`, see App. B.1, and Ref. [13] (redirects to `TAUOLA/TauSpinner/examples/applications/applications-plots-and-paper`),
- `paper-CP` see App. B.3 and Ref. [14].

There is no need for the user to read details of all this multitude of files, except the one of the interest.

The code of the basic examples is stored in the following two files: `TAUOLA/TauSpinner/examples/tau-reweight-test.cxx` for  $2 \rightarrow 2$  processes, `TAUOLA/TauSpinner/examples/example-VBF/example-VBF.cxx` for  $2 \rightarrow 4$  processes.

The methods needed to read events from data-files are stored separately as they depend on the particular data-file format: `TAUOLA/TauSpinner/examples/read_particles_from_TAUOLA.cxx` for  $2 \rightarrow 2$  processes, and `TAUOLA/TauSpinner/examples/read_particles_for_VBF.cxx` for the  $2 \rightarrow 4$  processes.<sup>17</sup>

The FORTRAN examples of the user functions for  $2 \rightarrow 2$  processes `ggHXtautau.f`, `distr.f` (accessible through the C++ wrappers) are stored in `TAUOLA/TauSpinner/examples/` and example library for implementing user  $2 \rightarrow 4$  processes in `TAUOLA/TauSpinner/examples/example-VBF/SPIN2`.

## B General information on the user interface methods

Over the years several methods of initializing and operating the `TauSpinner` were developed [11–15]. Technical descriptions and discussions on systematic uncertainties were provided, but remained scattered over several publications. Collecting them or providing references from one place is now of convenience.

Correct reading and interpretation of information in the event record, namely finding outgoing  $\tau$  leptons and their decay products<sup>18</sup> is a rather nontrivial component of the analysis code. The standard of the `HepMC` format [44] of the event

<sup>17</sup> The `lhef-to-hepmc.cxx`, include program to convert necessary for the `TauSpinner` event content from `LHEF` format [43] to `HepMC` format [44].

<sup>18</sup> Also bremsstrahlung photons from  $\tau$  pair production process. In case of multitude entries for the single particle, identification of the one matching the kinematical constraints is necessary. It often requires navigation back and forth through the event record, finding links to mothers, etc.

record is an usual input format for the `TauSpinner`, but some helping classes are prepared in the examples to allow also for reading `LHEF` format [43]. These interface classes are usually kept separately from the examples code and are described in the appropriate `README` files of the particular example directory. In several cases also input files with the small number of generated events are provided together with the examples.

The technical implementation of the methods used in examples evolved during the last few years. Most of them use basic (sometimes also additional developed during the project) functionalities of the `MCTester` package [45,46] and its native analysis function `MC_Analyze`. This allows to standardize not only production of benchmark histograms, but also validation of class reading event record `readParticlesFromTAUOLA_HepMC` method. This method is expected to be adapted or replaced by the user own, depending on the particular event record form.

Let us list some advantages of `MCTester`:

1. `MCTester` interface to `HepMC` is profoundly tested for many conventions on how to handle `HepMC` event record exceptions in the way it was filled in. `MCTester` offers thus debugging facility for communication too.
2. The only information exchanged between the `MCTester` and the `TauSpinner` modules is the event weight which is passed to `MC_Analyze` function together with the event itself
 

```
MC_Analyze(temp_event, WT);
```
3. Particular test specific plots, are defined in C scripts with names following `MCTester` analysis template script `UserTreeAnalysis.C`. These user-defined plots are then generated by `MC_Analyze` function.

The `TauSpinner` and the `MCTester` codes are independent, use of the latter is not mandatory, see Appendix B.8.

We are now ready to return to documentation of the `TauSpinner` application methods. There are three categories of methods prepared for the user: for initialization, evaluation of particular event and finally for gathering auxiliary information. Let us start from general methods. Methods for the specific applications will be discussed later, case after case in sub-appendices.

### Initialization

Before `TauSpinner` is activated and individual events analyzed general information about sample on which it is going to be used has to be provided. Initialization of `TauSpinner` has to be adjusted to the sample specific. The initialization is divided into two categories, the general one discussed now and the specific one for particular application which will be discussed in the sub-appendices. In every case the following initialization routine has to be invoked:

```
Tauola::initialize();
```

to initialize  $\tau$  decay library. Generation of decays will not be

performed but constants necessary for calculation of decay matrix elements have to be set. The library of PDFs has to be initialized as well,

```
string name="MSTW2008nnlo90cl.LHgrid";
LHAPDF::initPDFSetByName(name);
```

for every application and finally the basic initialization method of the TauSpinner can be invoked,

```
initialize_spinner(Ipp, Ipol, nonSM2, nonSMN,
CMSENE);
```

The input parameters of this method are as follows:

- bool Ipp = true; denote that the program is to be used for  $pp$  collisions. At present it is the only available option.
- int Ipol; states of the polarization in the input sample
  - 0 - events generated without spin effects
  - 1 - events generated with all spin effects
  - 2 - events generated with spin correlations only, no individual  $\tau$  lepton polarization.
  - 3 - events generated with spin correlations and polarization but missing angular dependence of individual  $\tau$  lepton polarization.
- int nonSM2 = 0 - Are we using nonSM calculations? y/n (1/0),
- int nonSMN = 0 - If we are using nonSM calculations we may consider corrections to shapes only: y/n (1/0), this option was introduced for  $2 \rightarrow 2$  algorithm only,
- double CMSENE - Center of mass system energy, used in reconstruction of  $x_1, x_2$ , the arguments for PDF distributions.

There are two random number generators used in typical applications. If re-decay of  $\tau$  leptons by Tauola is requested, then one may need to re-seed its native random generator, especially if parallel generation is performed. For this end, the sequence

```
int ijklin=..., int ntotin=..., int ntot2n=...;
Tauola::setSeed(ijklin,ntotin,ntot2n);
or Tauola::setSeed(time(NULL), 0, 0);
should be considered.
```

The TauSpinner algorithm for attribution of  $\tau$  lepton helicities use no-sophisticate random number generator. It can be replaced with the command

```
Tauola::setRandomGenerator( randomik );
```

Other initialization methods specific only for some applications will be presented in sub-appendices.

### Event loop

In a loop, for each consecutive event read from the data-file,<sup>19</sup> the four momenta and pdgID of the outgoing  $\tau$  leptons and

their<sup>20</sup> decay products are retrieved. The intermediate state boson  $X$  has to be known too (from the content of the event record or set manually).

For each event, information can be read from data-file with the help of `int status = readParticlesFromTAUOLA_HepMC(input_file, X, tau, tau2, tau_daughters, tau_daughters2);` method, present in the file `/TauSpinner/examples/read_particles_from_TAUOLA.cxx` but we assume that user will replace it with his own, matching the format of the data-file of the actual use.

### Single event analysis

On the basis of the intermediate boson  $X$  pdgID algorithm decides which matrix element version is used for calculating  $R_{i,j}$  (and production weight as well). The four momentum of  $X$  does not need to be equal to the sum of the outgoing  $\tau$  leptons (or  $\tau$  lepton plus  $\nu_\tau$  in case of  $W^\pm$  or  $H^\pm$  mediated processes). The non-conservation is expected, and will be attributed to the presence of final state emitted photons (prior to  $\tau$  lepton decays). Then, the weight defined as in Eq. (4) is calculated `wt_spin = calculateWeightFromParticlesH(X, tau, tau2, tau_daughters, tau_daughters2);`

If there is only a single  $\tau$  lepton among  $X$  decay products then it is assumed that `tau2=neutrino wt_spin = calculateWeightFromParticlesWorHpn(X, tau, tau2, tau_daughters);`

The formula Eq. (4) is used in both cases, for the second one where instead of  $\tau$  lepton, the  $\nu_\tau$  is present, a polarimetric vector (1, 0, 0, 0) is used.

The methods prepared for  $2 \rightarrow 4$  processes have similar input parameters, only the four momenta corresponding to outgoing partons (jets) are added. The event can be read from data-file, e.g. with the help of `int status = read_particles_for_VBF(input_file,p1,p2,X,p3,p4, tau1,tau2,tau1_daughters,tau2_daughters);` and later the weight is calculated `wt_spin = calculateWeightFromParticlesVBF(p3, p4, X, tau1, tau2, tau1_daughters, tau2_daughters);`

### Auxiliary information

As a by-product of preparing input for calculation of spin weight, Eq. (4), an auxiliary physics information is available and can be retrieved. This information is available through the following `get` functions that can be called after each `calculateWeightFrom...` function call.

```
double wtnonSM = getWtNonSM() - returns production weight  $wt_{prod}$  given by Eq. (11),
double WTp = getWtAmplitP();
double Wtm = getWtAmplitM();
return weights  $wt_{decay}^{\tau^+}, wt_{decay}^{\tau^-}$  of Eq. (12), corresponding
```

<sup>19</sup> In most cases event record format of HepMC is used as an input.

<sup>20</sup> Lists named `tau1_daughters, tau2_daughters` are needed.

to variation of  $\tau^\pm$  lepton decay matrix elements. Finally `double getTauSpin()` returns the  $\tau$  lepton helicity, attributed stochastically from the  $p_\tau^z$  probability given by Eq. (20), which is calculated from production and  $\tau$  lepton decay products kinematics. Note that such attribution is often subject of a further approximation than used for spin weight calculation: the transverse spin effects are then not included, and 100% spin correlation is assumed. That is why, providing helicity only for the first  $\tau$  lepton is sufficient.

The `getEWwt()`, `getEWwt0()` methods are described in Appendix B.4.

Before we explain further details of examples, let us list in the Table 4, all weights and their methods, and where they are addressed in Appendix.

### B.1 Longitudinal spin effects in $2 \rightarrow 2$ processes

From the first version of the `TauSpinner` [11], the purpose of calculating spin weight  $w_{t_{spin}}$  given by formula (4) was to evaluate how spin correlations between produced  $\tau^+$  and  $\tau^-$  leptons affect different observables. Applications for the cases of  $Z, \gamma^*, W^\pm, H, H^\pm$  mediated processes were prepared. Longitudinal spin effects were studied only. Other than  $t, z$  components of the  $R_{i,j}$  matrix, Eq. (4), were set to zero. Source of the paper [11] is available in the directory `Tauola/TauSpinner/paper`.

Methods introduced at that time form the core of the `TauSpinner` code. Later, more refined matrix elements and accompanying methods were introduced. General principles remained unchanged. The parameters of the Born amplitudes in case of the Higgs mediated process can be modified with: `void setHiggsParameters(int jak, double mass, double width, double normalization)`. At the time, these parameters were still unknown experimentally. For the Drell–Yan process the defaults of `Tauola` [25] are used.

A detailed discussion of examples is presented with multi-plot of Ref. [13]. The option depicted by the initialization parameter `Ipol` necessary to adopt to the level of spin effects embedded in to be re-weighted events is discussed. The figures for this paper and the paper itself are stored in the `TAUOLA/TauSpinner/examples/applications/`.

### B.2 Beyond standard model Higgs and Drell–Yan $2 \rightarrow 2$ processes

With Ref. [12] re-weighting with the help of the production matrix element was introduced and  $w_{t_{prod}}$  of Eq. (11) became available. The methods

```
setNonSMkey(1);
double getWtNonSM();
```

were introduced. The facility was prepared for handling the ‘new’ matrix element introduced by the user. The methods `set_nonSM_born(nonSM_adopt)`; `set_nonSM_bornH(nonSM_adoptH)`; were provided. The numerical values of the coupling constants for the default matrix elements of the `TauSpinner` can be changed in that way as well. We will exploit that point in context of  $2 \rightarrow 4$  processes.

The user provided matrix elements can be defined for the Higgs mediated processes or for the Drell–Yan ones. The algorithm will make a choice which one to use, on the basis of `pdgID` of the  $X$ . With  $w_{t_{prod}}$  one can change the initialization parameters of the Standard Model amplitudes as well.

If no user provided matrix elements are attributed, program will print warning and `exit(-1)` command will be activated.

### B.3 Examples for studies of Higgs parity observables.

The example program and reference numerical results for CP sensitive observables as of Ref. [14] are given in the directory `Tauola/TauSpinner/examples/CP-tests`.

The  $w_{t_{spin}}$  is used. The sub-directories with names starting with `H-` and `Z-`, are respectively for cases where the signal (Higgs boson) or background (Drell–Yan,  $Z$ -production) distributions are studied. The second part of the directory name denotes decay channels of  $\tau$  leptons, `pi` for  $\tau \rightarrow \pi\nu$  and `rho` for  $\tau \rightarrow \pi\pi^0\nu$ .

The sub-directory `generate-datafiles` includes the program to read and generate  $\tau$  lepton decays for events stored in input `Pythia HepMC` files. The `README` file with the instructions is prepared but was not updated now in every detail. The source of the paper [14] is in the directory: `TAUOLA/TauSpinner/paper-CP`.

#### B.3.1 CP Higgs

From the algorithmic point of view the introduction of CP-sensitive transverse spin correlations into  $R_{i,j}$  matrix was straightforward and completed in Ref. [14]. The task was simple because due to the zero spin of the Higgs there is no dependence on the Higgs production process. To activate the transverse spin correlations, one should initialize one of the below:

```
- setHiggsParametersTR(-1.0, 1.0, 0.0,
  0.0); for scalar Higgs,
- setHiggsParametersTR( 1.0,-1.0, 0.0,
  0.0); for pseudo-scalar Higgs,
- double theta = 0.2; for mixed parity state
  choose theta for mixing angle and then
  setHiggsParametersTR(-cos(2*theta), cos
  (2*theta), -sin(2*theta), -sin(2*theta));
```

**Table 4** Weights of the TauSpinner methods of calculation and Appendices explaining examples of their use

Weight	Method	Equations	Sections
$w_{tspin}$	calculateWeightFromParticlesH		
	calculateWeightFromParticlesVBF	(4)	B.5
	calculateWeightFromParticlesHpn		
	getTauSpin	(20)	4.2
$w_{tprod}$	getWtNonSM()	(11)	B.2
$w_{tdecay}$	getWtAmplidP, getWtAmplidM	(12)	B.7
$w_{tEW}$	getEWwt(), getEWwt0()		B.4

For the code, see the TAUOLA/TauSpinner/examples/CP-tests and its sub-directories H-pi and H-rho and for the example of results obtained with the methods Refs. [42,47].

### B.3.2 Transverse spin correlations in the background; $2 \rightarrow 2$ Drell–Yan process

With the help of the tables of EW corrections, the transverse spin effects of  $Z/\gamma^*$  mediated processes can be introduced into the sample. As default those terms are switched off. To activate initialization method as explained in Ref. [14], setZgamMultipliersTR(1., 1., 1., 1.); has to be used.

User may be interested to introduce transverse spin correlations in part, then some of these coefficients can be set to zero or even one can reverse their sign, by setting multipliers to  $-1$ . If any component of transverse density matrix for Drell–Yan has been turned on using setZgamMultipliersTR, spin weight  $w_{tspin}$  will also take into account transverse spin correlations  $R_{i,j}$ ,  $i, j = x, y$  (calculated from tables table1-1.txt, table2-2.txt which must be stored in the same directory as the program) by calling<sup>21</sup> sequence Tauolapp::TauolaParticlePair pp; pp.recalculateRij(ID, tau\_pdgid, S, cost); For the code, see TAUOLA/TauSpinner/examples/CP-tests and its sub-directories Z-pi and Z-rho.

### B.4 Electroweak corrections

In Ref. [14] it is explained how tables featuring EW one loop effects can be installed into weight calculations. At present, only EW corrections embedded as in the tables of TauSpinner:

```
/examples/CP-tests/Z-pi/table1-1.txt,
/examples/CP-tests/Z-pi/table2-2.txt
```

<sup>21</sup> The  $S, cost$  denote invariant mass squared of the  $\tau$  lepton pair as well as  $\cos \theta$  (of the  $2 \rightarrow 2$  scattering angle) and ID the identifier for the incoming parton flavour, all for the parton level kinematic configuration prepared by TauSpinner. To get the actual values interpolations between entries of tables is used.

are available. These results do not include line-shape corrections, that is multiloop corrections to vacuum polarization diagrams for quarks. That is why they are of the restricted practical use, except maybe for the forward-backward asymmetry simulation, unless replaced with the better ones originating e.g. from the DIZET library of KKMC program [26] of the LEP time.

If tables are present in the directory of the executable, then they will be read in and used. Then, if double wt1=Tauola::getEWwt(); double wt0=Tauola::getEWwt0(); is executed, one can also improve the distribution and normalization of the cross section with the wt1/wt0 factor added in the  $w_{tprod}$  event weight and not only take the transverse spin effects discussed above.

### B.5 Drell–Yan $\tau$ pair production with two jets

Necessary methods were introduced/discussed in Ref. [15]: it turned out of interest to extend calculation of matrix elements to the case where four-momenta of two jets accompany outgoing  $\tau$  lepton pair, are also used explicitly. This option is invoked,

```
see the TAUOLA/TauSpinner/examples/example-
VBF by call to WT = calculateWeightFromParticlesVBF
(p3, p4, X, tau1, tau2, tau1_daughters, tau2_
daughters); instead of discussed previously WT = calcul
ateWeightFromParticlesH(...).
```

The matrix elements replacement necessary for the calculation of the corresponding  $w_{tprod}$  is set by TauSpinner::set\_vbfdistrModif(vbfdistrModif); and then initialized with vbfini\_(&ref, &variant); For the non-standard calculation it is then available with setNonSMkey(1);.

In [15] it was found interesting to use this interface to redefine initialization of the parameters in the default matrix elements. Distinct EW schemes can be initialized for standard and non-standard calculations. Variants for running  $\alpha_S$  can also be distinct for standard and non-standard calculation. Note that the default for non-standard calculation is to use the same set of matrix elements as for the Standard

Model calculation. The EW scheme can be nonetheless different (the one corresponding to `variant`). Also option for distinct scale in calculation of PDFs is prepared. It can be set with `setPDFOpt(QCDdefault,QCDvariant)`; respectively for the default and non-standard calculation. Also a new parametrization of running strong coupling can be introduced with this interface. The default function can be replaced with the user own:

```
TauSpinner::set_alphasModif(alphas
Modif);
```

These options were already explored in Ref. [15].

## B.6 Spin2

So far we have discussed the production weight  $wt_{prod}$  as a tool to re-weight to slightly modified initialization parameters of the Standard Model production model. As demonstrated in Ref. [48], matrix elements calculated for the non-standard type (e.g. for non-standard resonance) may differ substantially from the default-standard mode ones. Invoking `TauSpinner::set_vbfDistrModif(SPIN2::spin2distr)`; will activate user function, as of our example, the `SPIN2::spin2distr(...,KEY)`. In this case, user matrix element prototype invokes a whole library which need initialization, see the `TAUOLA/TauSpinner/examples/example-VBF/SPIN2`. Its organization and initialization is quite similar to the code of the `TauSpinner` library of  $\tau\tau jj$  processes, but it is nonetheless completely independent. The `spin2init_(&ref,&variant)`; has the same parameters as the `vbfinit_`, but not all of its functionality is at this moment active. The parameter `ref` is dummy and all constants and coupling are independently initialized, in particular hard coded own value for not-running  $\alpha_s$  is used. The parameter `variant` can be used to vary EW scheme though.

One should bear in mind that re-weighting of matrix element require spin weight factor  $WTME$  to be included with the  $wt_{prod}$ , it has to be calculated as follows:

```
setNonSMkey(0); // to calculate spin weight
                // of Standard Model
double WT0 = calculateWeightFromParticlesVBF
(p3, p4, X, tau1,tau2,\pg{\break}tau1_daughters,
tau2_daughters);
setNonSMkey(1); // e.g. for use of SPIN2 ME and
                // calculate all weights
WT = calculateWeightFromParticlesVBF(p3, p4, X,
tau1, tau2, tau1_daughters, tau2_daughters);
double WTME=getWtNonSM(); // matrix element wt
WTME=WTME*WT/WT0; // factor to take in
                // spin correlations
                // of tau-tau decays
```

## B.7 Changing matrix elements of $\tau$ lepton decays

Ambiguity of choice for the hadronic currents used in simulations may be a source of systematic uncertainties, which is of the interest to be estimated. An attempt for such estimation is shown in the example `TAUOLA/TauSpinner/examples/CLEO-to-RCHL/tau-reweight-CLEO-to-RCHL.cxx`

The calculation with the two versions of hadronic currents is possible without any intervention into the code of libraries and interchanged thanks to the method `Tauola::setNewCurrents(int key)`; where `key=0` or `key=1` is possible. Once the hadronic current model is set, the usual weight calculation can be performed and `double getWtAmplitP()`; `double getWtAmplitM()`; can be used to obtain matrix elements squared respectively for  $\tau^+$  and  $\tau^-$  leptons. The ratio of the results for the two versions of hadronic currents will constitute the  $wt_{decay}^{\tau^+}$  and  $wt_{decay}^{\tau^-}$  weights.

## B.8 Not published tests programs

The set of selected test programs, coded without `MCTester` user analysis helper, are collected in directory `TAUOLA/TauSpinner/examples/testSERW`.

## B.9 Function sigborn

The input parameters are `pdgID` of incoming partons, virtuality squared of the resonance  $Q^2$  and  $\cos\theta$  of the outgoing lepton, the intermediate particle identifier is not passed as an argument. Appropriate choice of the used function is performed by the `TauSpinner` method calculating spin and production weight.

In case of the SM Higgs boson production, assumed is  $gg \rightarrow H$  production process only, the formula for the lowest-order cross-section summed over all  $\tau$  lepton helicity configurations using wrapper to `dsth` function from the `Tauola` code read as follows

$$\sigma_{Born} = \frac{1}{\pi} \frac{m_h^3 \cdot \Gamma_h}{(Q^2 - m_h^2)^2 + m_h^2 \cdot \Gamma_h^2} (dsth(Q^2, \cos\theta, 1, 1) + dsth(Q^2, \cos\theta, 1, -1) + dsth(Q^2, \cos\theta, -1, 1) + dsth(Q^2, \cos\theta, -1, -1)). \quad (39)$$

In case of BSM Higgs boson following formula is coded

$$\sigma_{Born} = \frac{1}{\pi} X_{norm} \frac{m_h \cdot \Gamma_h}{(Q^2 - m_h^2)^2 + m_h^2 \cdot \Gamma_h^2} (dsth(Q^2, \cos\theta, 1, 1) + dsth(Q^2, \cos\theta, 1, -1) + dsth(Q^2, \cos\theta, -1, 1) + dsth(Q^2, \cos\theta, -1, -1)) \quad (40)$$

**Table 5** Setting of EW parameters in `initwkw` routine for effective Born of `Tauola` and in routine `DISTJKWK` of effective Born used in Ref. [12]. Note that the parameters do not match. The purpose of EW parameters in `DISTJKWK` was to relate New Physics amplitudes with the Standard Model ones, its own calculation scheme of quite rough Standard Model initialization was used

Parameters	<code>initwkw</code>	<code>DISTJKWK</code>
$\sin \theta_W^2$	0.23147	0.2315
$m_e$	0.511 MeV	–
$m_Z$	91.1882 GeV	91.187 GeV
$\Gamma_Z$	2.4952 GeV	2 GeV
$m_\tau$	1.77703 GeV	–
$\alpha_{QED}(m_Z)$		1/128
$G_F$		$1.1667 \cdot 10^{-5}$

where  $X_{norm}$  (in units of  $\text{GeV}^2$ ) stands for the normalization factor, see Ref. [12]. In case of Drell–Yan process, respective formula reads, using wrapper to `t_born` function from the `Tauola` code.

$$\sigma_{Born} = \frac{1}{Q^2 \cdot \mathcal{N}} \left( t\_born(0, Q^2, \cos \theta, 1, 1) + t\_born(0, Q^2, \cos \theta, 1, -1) + t\_born(0, Q^2, \cos \theta, -1, 1) + t\_born(0, Q^2, \cos \theta, -1, -1) \right) \quad (41)$$

where  $Q^2$  is the virtuality of the resonance,  $\cos \theta$  is the cosine of angle between  $\tau^+$  lepton and the first beam and last two arguments are helicity states of  $\tau^+$  and  $\tau^-$  leptons respectively. The  $\mathcal{N} = 123231 \approx 2 \cdot \pi / \alpha_{QED}^2$  represent normalization factor. Details of normalization were not studied so far, this was out of scope for the studied applications, as in the  $wt_{spin}$  ratios of  $\sigma_{Born}$  were used and for  $wt_{prod}$  required precision for  $\mathcal{N}$  was sufficient, in cases when required precision was higher the ratio of  $\sigma_{Born}$  was used for  $wt_{prod}$  as well.

The EW parameters are initialized with the setting as in Table 5, using wrappers to `initwkw_` function from the `Tauola` library code. In Appendix B.4, it is explained how to activate EW one loop corrections, of physics content as described in Refs. [35,49] into effective Born angular distributions. Appendices C.7 and D of Ref. [25] document how the scheme of these EW effects calculation can be changed; the example code is stored in `TAUOLA/TauSpinner/examples/testsERW/ex4`.

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