Short Communication

Effect of relative movement between bearing races on load distribution on ball bearings

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Abstract
The current study presents the influence of relative motion between inner race and outer race in the presence of radial clearance. This relative motion alters the angular positions of any consecutive balls (balls) with respect to the bearing axis. Hence, the quasi-static analytical study shows the effects of alteration in angle of contact on the load distribution by balls. Based on the Hertz elastic contact theory the contact deformation between balls and races is determined. Further, a geometrical model of ball bearing is developed to establish the relation among the distributed loads, ball contact deformation, external applied load and internal radial clearance. The results show an improvement in load distribution by balls by considering two different positions of balls within the races. The proposed model is also compared at different angular displacement of ball and justifies the effect of relative motion between bearing races.

Keywords Relative motion · Angular shift · Radial force distribution · Radial clearance

Nomenclature

d	Bore diameter, mm
C	Constant clearance, mm
d_{ir}	Track diameter of inner ring, mm
d_{or}	Track diameter of outer ring, mm
d_b	Ball diameter, mm
F_r	External radial load, N
F_q	Load shared by qth ball, N
K	Contact stiffness, $N/mm^{3/2}$
N	Total no. of balls in a bearing
z	Active no. of balls
β	Angle between ball from fixed centre of outer race, degree
γ_q	Angular shift of qth ball, degree
J(ε)	Sjoväll's integral
δ_{inq}	Initial center shift of inner race w.r.t. outer race, mm
δ_q	Contact deformation of any qth ball, mm
δ_r	Total inner race center displacement, mm

Indexes

ir	Inner race
or	Outer race
q	Index of balls (boundary case of support onto even number of balls q = 1, 2, 3, …, z; boundary case of support onto odd number of balls q = 0, 1, 2, 3, … , z)

1 Introduction

Ball bearings play a crucial role in the design and stability of various mechanical systems. The load distribution and stiffness characteristics of the ball bearings can determine performances of the mechanical systems [1–5]. Radial clearance in all type ball bearings is another major aspect for optimal design. Therefore, all ball bearings should have slightly negative clearance or zero bearing internal clearance, which are major parameters for maximum bearing life and reliability. However, zero clearance or negative clearances allow the bearing to have high bearing contact

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stress between races and balls [6, 7]. Further, it increases the bearing friction, which makes the bearing vulnerable with the increase in temperature. Therefore, initial bearing clearances are generally chosen to overcome the problem. However, the design of ball bearings with initial radial clearance in both static and dynamic conditions has been progressing by leaps and bounds. Xiaoli et al. [6] developed a mathematical model based on Hertz elastic contact theory for determining the radial load distribution on ball and ball bearings with positive, negative and zero clearance. In this direction, Oswald et al. [7] have given the valuable contribution by showing the effect of positive and negative internal clearance on load distribution and fatigue life of radially loaded deep groove ball bearing. The life declines gradually with both increasing positive and negative clearance and maximum under small negative operating clearance. Further, in ball bearings, not all the balls are involved in load sharing but the ball below meridian plane [8] which is called as load region. The quasi-static analysis for the radial load distribution is based on the hypothesis of rigid body displacement of inner ring [9] due to the presence of radial clearance.

Many researchers have focused on the study of the behavior of ball bearing under some operating clearance. And their research shows that the bearing clearance not only effect the operating condition but also the load distribution. Among them Stribeck [10] was the first who calculated load distribution by the balls/ball in ball and suggested that the Stribeck coefficient with zero radial clearance is 4.37 and under some operating clearance it is 5 with some approximation. Later research reveals that the approximated value do not gives the satisfactory results for bearing supporting light loads. After few years Sjoväll [11] developed the integral model for load distribution with internal radial clearance by assuming that large number of ball of infinitely small diameter are lying in the load region. The Sjoväll concept proved significant and further used by many researchers. Harris [12] was one of them who used the concept of Sjoväll and developed the mathematical model for load distribution with zero and nonzero internal radial clearance and a load distribution factor (ε). However, there are some errors in the solution of equilibrium equation. Since the model used by Harris does not define the exact method to determine the active no. of balls, Tomović [13, 14] primarily has focused on determining the number of active balls that participate in external radial load transfer. While estimating load distribution by balls in a roller bearing, many researchers have also done research using other types of roller bearings. Such as, Hernot et al. [15] calculated the stiffness matrix of angular contact ball bearings in which bearing element loads are evaluated using Sjoväll integration. Ricci [16, 17] established a rapid numerical model for the estimation of internal load distribution for single-row, angular-contact ball bearings. Wang and Yuan [18] developed a computational model to determine the contact force distribution by first introducing constant clearance in the bearing. Furthermore, the static load carrying capacity of a double row four-point contact ball bearing subjected to combined radial, axial and overturning moment loadings is estimated. Tudose et al. [19] and Rusu et al. [20] analysis helps in finding the ball deflection that allows determining the number of active balls participate in the transfer of external radial load with radial clearance. They also show the influence of ball rotation angle on radial load transfer. Korolev et al. [21] discusses the mechanism of distribution of the external combined loads between balls. His mathematical model establishes the relationship between load on the balls, angle of contact and ratio of radial and axial external loads.

In all the above literature, analysis of static load distribution by ball/rollers in various operating/loading conditions is carried out. It is observed that researchers have been putting enormous efforts in optimizing the ball bearing design through estimation of internal load sharing by the balls of a bearing. However, in all those studies the effect of relative movement of bearing races on angle between consecutive balls are yet to be considered. This paper aims to show the effect of shift in angular positions of balls within the races due to the rigid body movement of inner race races on the distributed radial load by each ball. Therefore, a quasi-static analytical model for deep groove ball bearings with different radial clearances is proposed. The model comprises of two distinct orientations of balls. Finally, the model is used for SKF 6206 to illustrate the effects of angular shift on static load sharing.

2 Geometrical analysis of load distribution

Assumptions

- The cages of the rolling bearing keep the ball evenly distributed around the circumference of bearing race and prevent direct contact between neighboring balls.
- The translation motion of inner and balls is considered only in radial direction while the outer race is rigidly fixed.

2.1 Relative displacement of inner race center in relation to outer race center under no load condition

The constant radial clearance in ball bearings allows the rigid body motion of inner race in the direction of applied radial load relative to the outer race. This initial shift of inner race brings the balls of the load region into contact through which load transfer takes place. However, a distinct initial shift can be observed with two different orientations or boundary conditions of balls in a ball bearing, which is shown in Fig. 1. These two orientations are presented as

- Case-I, where the first contacting ball is placed at an angle from the line of applied radial load (Fig. 1a). There always an even number of balls lie in the load region.
- Case-II, where the first contacting balls lies along the line of applied radial load (Fig. 1b). Hence, there always an odd numbers of balls lie in the load region.

In both the case shown in Fig. 1a, b, 'A' is the fixed center (center of outer race) and 'A_1' is the center of shifted inner race. The distance AA_1 is the initial shift of inner race. This initial shifts changes the angular positions of the balls with respect to the center of inner race through which applied radial load gets transferred through balls. The current section presents the evaluation of this angular shift through geometrical modeling.

Case-I *First contacting ball is positioned at a certain angle from the line of action of applied load.*

In this case, the balls in the load region participate in load transfer in pairs from the very beginning of load distribution. In order to solve the initial radial shift as shown in extended geometrical view Fig. 1a, the initial radial shift of inner race center to have contact with 1st pair of ball can be estimated as

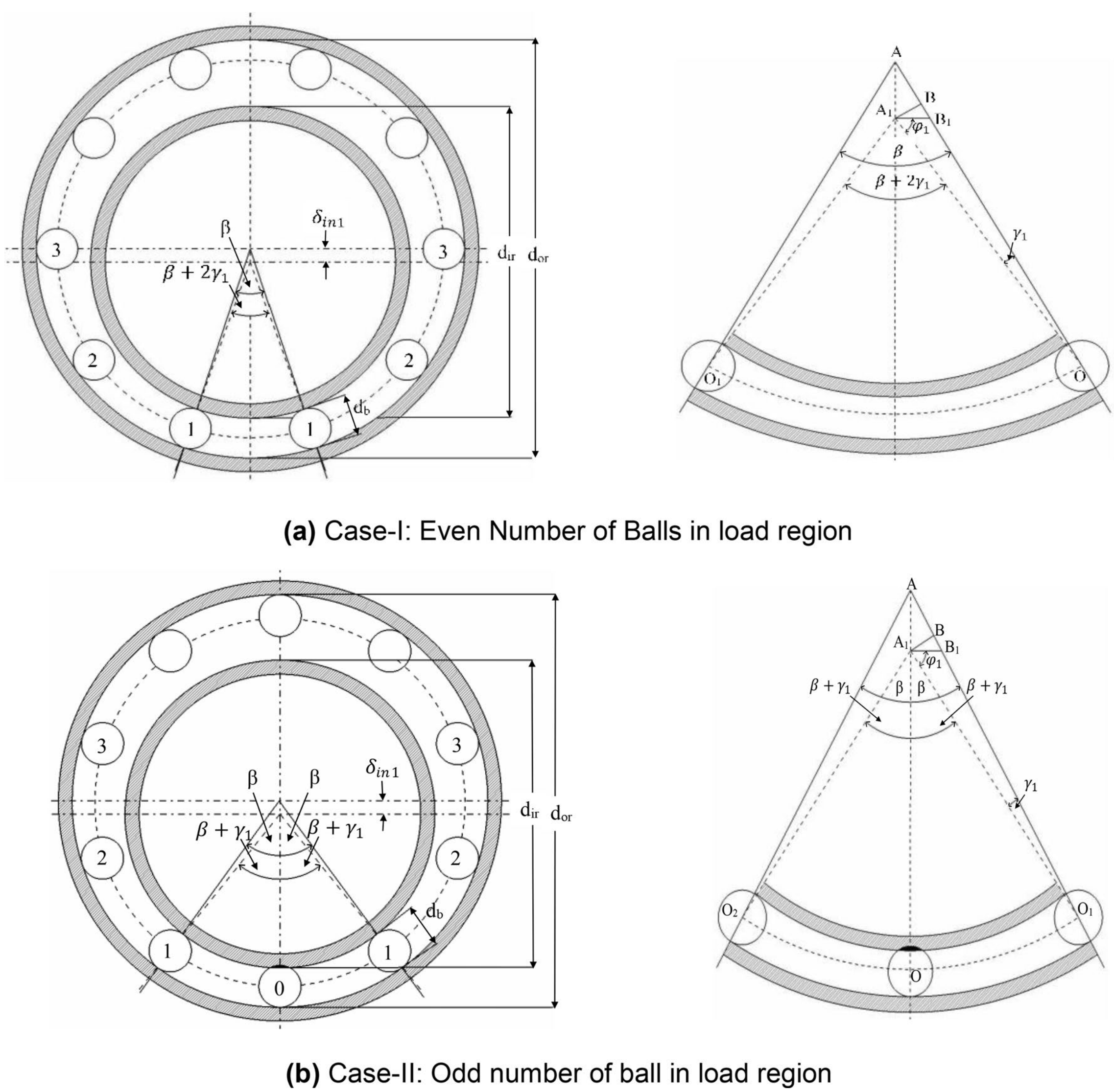


Fig. 1 Centre displacement of inner ring to bring first pair of ball into load region

$$\varphi_1 = \cos^{-1}\left[\frac{\frac{C}{2} + \frac{d_{ir}+d_b}{2}}{\frac{d_{ir}+d_b}{2}} \sin\frac{\beta}{2}\right] \tag{1}$$

$$\delta_{in1} = \frac{\frac{C}{2} + \frac{d_{ir}+d_b}{2} - \frac{d_{ir}+d_b}{2}\sin\left(\frac{\beta}{2} + \varphi_1\right)}{\cos\frac{\beta}{2}} \tag{2}$$

And corresponding angular shift, is given by

$$\gamma_1 = \sin^{-1}\left(2\frac{\delta_{in1}}{d_{ir} + d_b}\sin\frac{\beta}{2}\right) \tag{3}$$

Similarly, initial radial shift and angular shift required to have contact with any zth pair of balls can be expressed as,

$$\varphi_z = \cos^{-1}\left[\frac{\frac{C}{2} + \frac{d_{ir}+d_b}{2}}{\frac{d_{ir}+d_b}{2}} \sin\frac{(2z-1)\beta}{2}\right] \tag{4}$$

$$\delta_{in\,z} = \frac{\frac{C}{2} + \frac{d_{ir}+d_b}{2} - \frac{d_{ir}+d_b}{2}\sin\left(\frac{(2z-1)\beta}{2} + \varphi_Z\right)}{\cos\frac{(2z-1)\beta}{2}} \tag{5}$$

$$\gamma_z = \sin^{-1}\left(2\frac{\delta_{in\,z}}{d_{ir} + d_b}\sin\frac{(2z-1)\beta}{2}\right) \tag{6}$$

Case-II *One ball is lying along the line of action of applied load in load region.*

In this case, first, the 0th ball comes in contact with inner race. Then balls in the load region participate in the load transfer in pairs. Hence, the initial radial shift of inner race center required to have contact with 0th ball is equal to,

$$\delta_{in0} = \frac{2d_{or} - d_{ir} - 2d_b}{2} = \frac{C}{2} \tag{7}$$

This is the initial shift of the inner race at no applied load. Thereafter, the intensity of external load increases and more pairs of ball enter the load region. Hence, to bring first pair of ball in the load region besides 0th ball, the center of inner race shifts as

$$\varphi_1 = \cos^{-1}\left[\frac{\frac{C}{2} + \frac{d_{ir}+d_b}{2}}{\frac{d_{ir}+d_b}{2}} \sin\beta\right] \tag{8}$$

$$\delta_{in1} = \frac{\frac{C}{2} + \frac{d_{ir}+d_b}{2} - \frac{d_{ir}+d_b}{2}\sin\left(\beta + \varphi_1\right)}{\cos\beta} \tag{9}$$

and the angular shift (γ_1) can be calculated as shown in the extended geometrical view Fig. 1b

$$\gamma_1 = \sin^{-1}\left(2\frac{\delta_{in1}}{d_{ir} + d_b}\sin\beta\right) \tag{10}$$

The values of this internal radial shift and angular shift change when more pairs of ball support the inner race. Therefore, a general mathematical formulation can be written to bring zth pair of ball in contact/load region

$$\varphi_z = \cos^{-1}\left[\frac{\frac{C}{2} + \frac{d_i+d_b}{2}}{\frac{d_i+d_b}{2}} \sin(z\beta)\right] \tag{11}$$

$$\delta_{inz} = \frac{\frac{C}{2} + \frac{d_i+d_b}{2} - \frac{d_i+d_b}{2}\sin\left[(z\beta) + \varphi_Z\right]}{\cos(z\beta)} \tag{12}$$

$$\gamma_z = \sin^{-1}\left(2\frac{\delta_{inZ}}{d_i + d_b}\sin(z\beta)\right) \tag{13}$$

2.2 Radial load distribution

The radial load distribution by balls lying in load region by developing the series of static equilibrium equation for ball deformation along bearing axis as Tomović [4, 5] is estimated. Moreover, unlike Tomović's formulation, the mathematical expression is developed by considering the effect of relative movement of bearing races on ball separation angle.

Based on static equilibrium condition, the expression for external radial force (F_r) can be expressed as a summation of distributed load on each qth ball in the load region:

- For support an even no. of ball from Fig. 2a

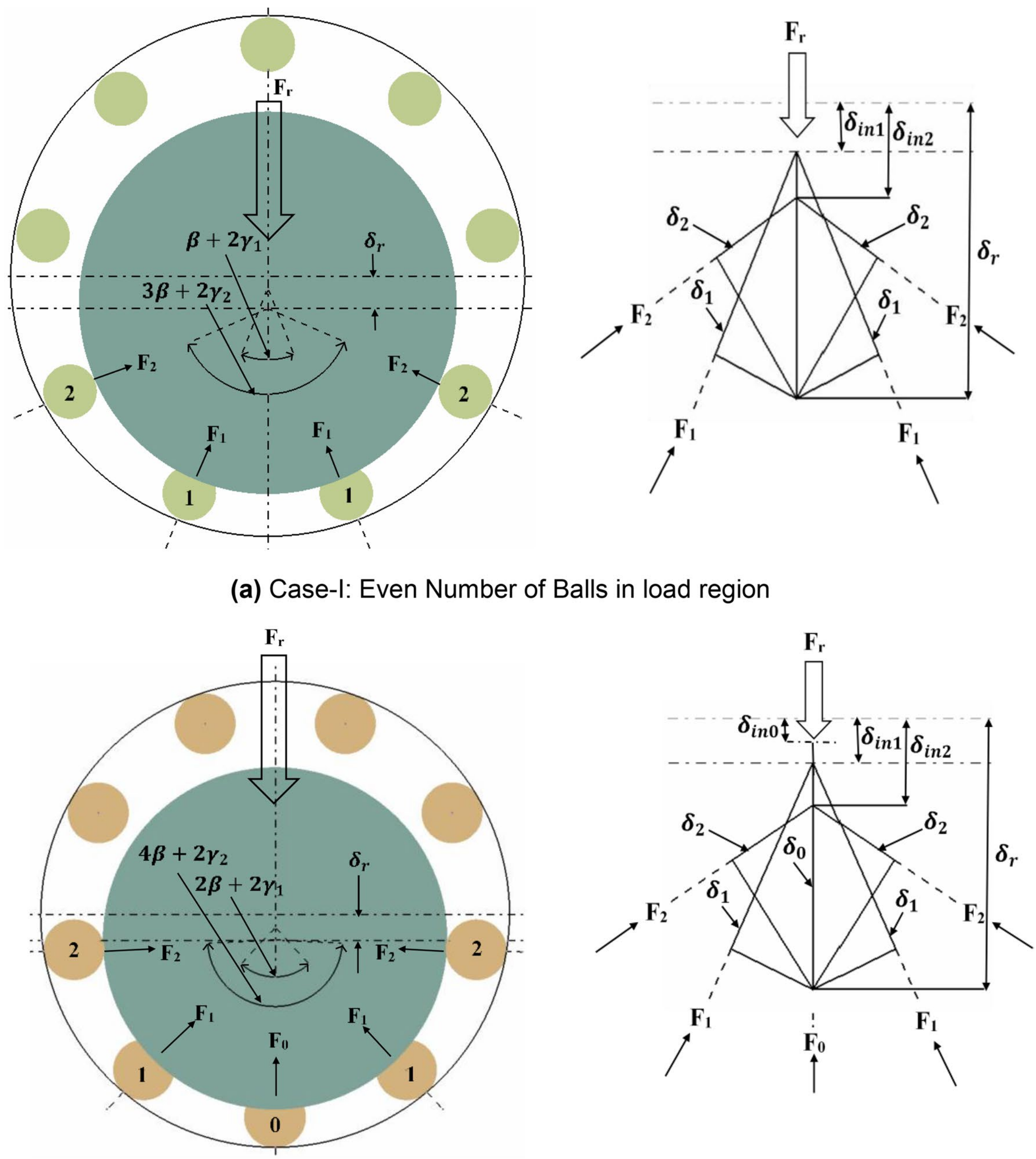
$$F_r = 2\sum_{q=1}^{z} F_q \times \cos\left[\frac{\beta(2q-1) + 2\gamma_q}{2}\right] \tag{14}$$

- For support an odd no. of ball from Fig. 2b

$$F_r = F_0 + 2\sum_{q=1}^{z} F_q \times \cos\left[\left(q\beta + \gamma_q\right)\right] \tag{15}$$

3 Results and discussion

In this section, the radial distribution by balls in a ball bearing is calculated using quasi-static analytical method. The method considers the effect of relative displacement of inner race to encounter the constant radial



(a) Case-I: Even Number of Balls in load region

(b) Case-II: Odd number of ball in load region

Fig. 2 Boundary condition of load distribution and ball deflection for zth pair of ball in the load region

clearance such that the balls in the load region could participate in the load transfer simultaneously. Throughout the analysis, the outer ring is fixed in all directional movement, and the deformation is evaluated locally at the contact locations of the balls and races. A radial load of 9000 N is applied gradually.

3.1 Influence of radial clearances on angular shift

Equations (1)–(13) are presented to estimate the initial radial shift and corresponding angular shift to bring zth pairs of ball. However, to understand the effect of such relative motion of inner race in the existence of internal radial clearance, single row deep groove radial ball bearing SKF 6206 with various possible clearances is considered for the analysis as mentioned Table 1.

The relative motion of inner race changes the relative angular positions of ball with respect to center of bearing inner race. As per Eqs. (6) and (13), it can be seen that the change in angular positions i.e. angular shift depends on number of pairs of ball (z) participating in the load distribution and the angular position of any balls with respect to the line of applied radial load (*β*) at fixed center. Hence, Figs. 3 and 4 show the significance of various radial clearance values on angular shift with

Table 1 SKF6206 radial ball bearing radial clearances adapted from ABMA [22]

Bore size	Group 2		Group N		Group 3		Group 4		Group 5	
	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
30	0.001	0.011	0.005	0.020	0.013	0.028	0.023	0.041	0.030	0.053

All dimensions are in 'mm'

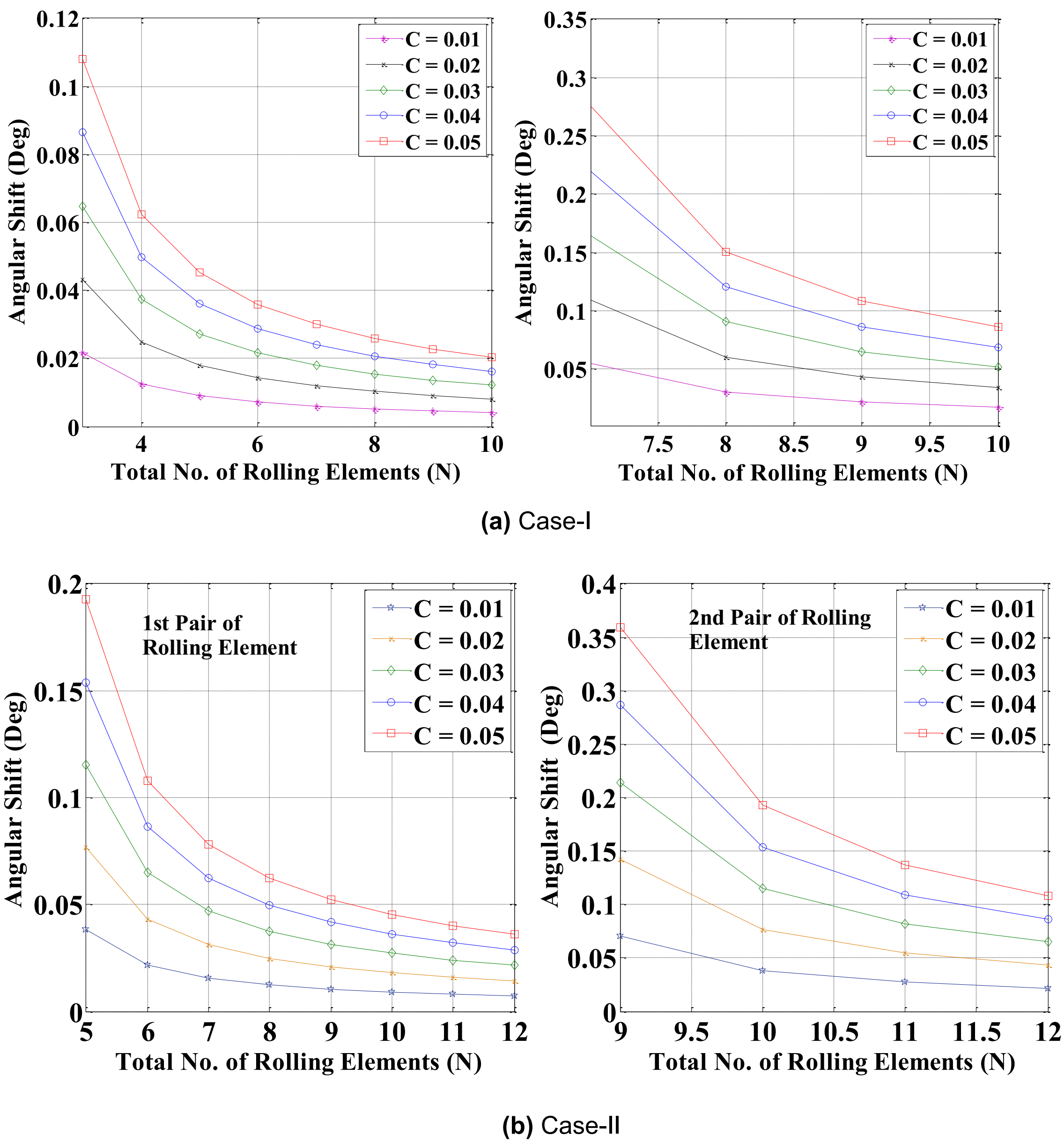


Fig. 3 Effect of internal radial clearance on angular shift. (**a**) Case-I and (**b**) Case-II

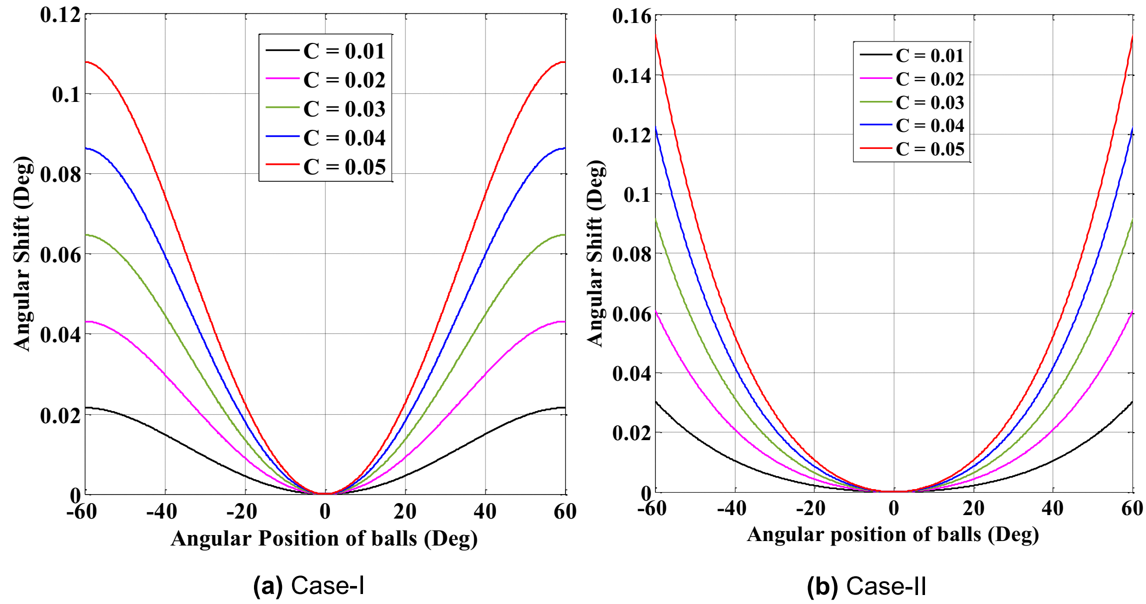


Fig. 4 Effect of angular shift on angle of rotation considering $N = 9$. (**a**) Case-I and (**b**) Case-II

respect to number of pairs of ball (z) and angular position (β).

Figure 3 shows the variation of angular shift is higher for greater clearance and decreases sharply when the number of balls in a bearing increases. It also suggests that under small clearance and high number of balls the value of angular shift can be neglected. However, under higher clearance and small number of balls the effect is quite evident and hence cannot be ignored. Moreover, Fig. 4 suggest that the pair of balls lying in load region but positioned away from bearing axis having higher angular shift and this shift will decreases for the balls positioned closer and become zero for the ball placed along the bearing axis.

3.2 Radial load distributions

As per Eqs. (14) and (15), the radial load distribution by the ball in the load region is calculated. Considering a total number of nine balls in SKF6206, the load distribution by each pairs of ball is estimated. The bearing parameters are shown in Table 2.

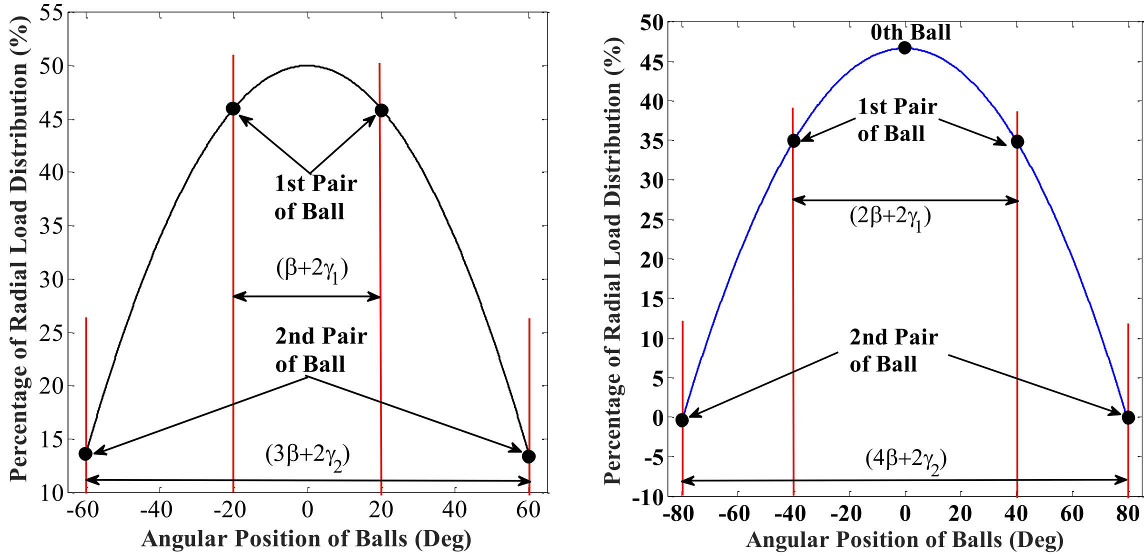
For a total number of nine ball in the bearing, two pairs of ball support the entire radial load for Case-I whereas a total of five balls support the load for Case-II. Considering the sum of load shared by the balls is equal to the total applied radial load, the percentage of load shared by the ball in both Case-I and II is presented in Fig. 5. The results shown comprise the effect of angular shift of the balls due to the relative motion of inner race.

3.3 A case study for the comparison of results of radial load distributions

In this section, the results of developed analytical expressions (Eqs. 14 and 15) for the estimation of radial load distribution by balls are compared with established Tomović's results. Tomović [14] represents the same static equilibrium model for estimation of radial load distribution without considering the effect of angular shift. Therefore, the load shared by 0th ball (for Case-II) is estimated based on the proposed and Tomović's formulation. For this, SKF 6206 with a total nine balls is employed under five group of clearance as mentioned in Table 1 to demonstrate the influence of angular shift (γ_z) in load shared by z balls. Figure 6 shows the comparison of the external radial load distributions on maximum loaded ball for odd number of ball supports the inner race obtained by Tomović model and the proposed model. An external radial load upto 9000 N is applied gradually for the comparison study. The comparison result shows

Table 2 Ball bearing parameters considered for numerical examples

Ball bearing type	Bearing stiffness (K) (N/mm$^{3/2}$)	Total no. of balls (N)	Bore diameter (mm)	d_b (mm)	d_{ir} (mm)	d_{or} (mm)
6206	3.31×10^5	9	30	9.525	36.48	55.53



(a) Case-I **(b)** Case-II

Fig. 5 Percentage of load distributions with respect to different angular positions of balls considering a total of nine ball in the bearing

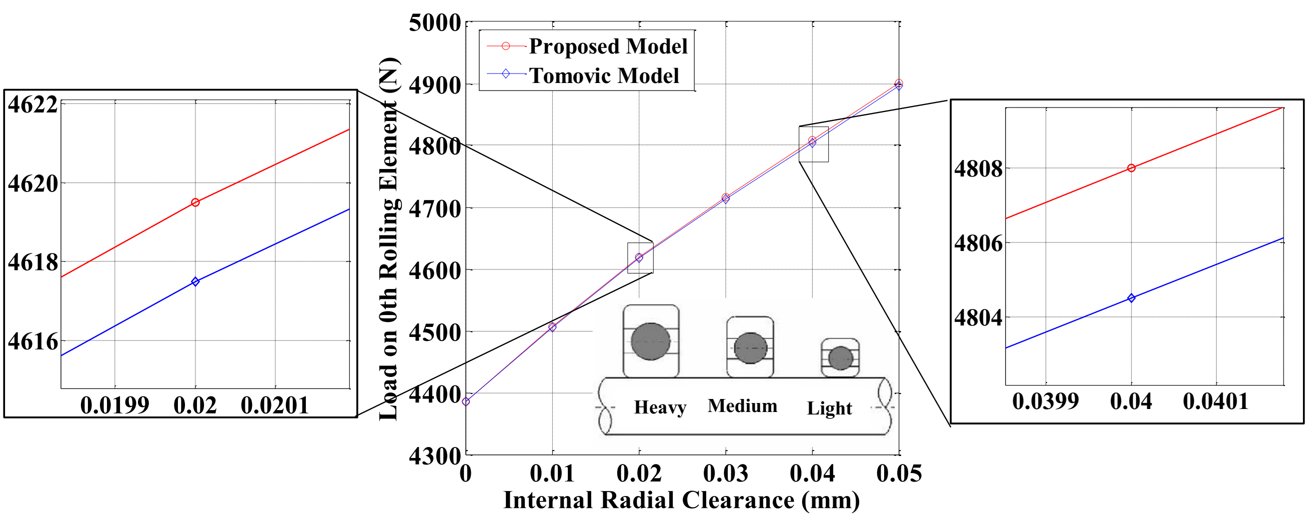


Fig. 6 Comparison of load distribution on maximum loaded ball in the load region

that the variation in the load distribution is higher at higher clearance (lighter loads) and decreases with decreasing value of clearances (medium loads) and seize to zero at zero clearance (heavy loads). The result obtained by the proposed model is higher than the Tomović model this is due to the fact that Tomović has neglected the effect of relative movement on ball separation angle. Since the ball bearings are high precision rotary machine elements therefore under high clearance the assumption made by Tomović is not satisfactory. Since proposed model has considered the angular shift which is neglected by Tomovic hence degree of participation of most loaded ball is higher for the proposed model than compared model under same external radial load. Therefore the proposed model can help in reducing the error for the calculation of bearing life since the effect of relative movement of bearing races on separation angle is taken into consideration.

4 Conclusion

The study presents a quasi-static analytical model using static-equilibrium equations for the estimation of static radial load distribution by the balls of a ball bearing. The estimated load distribution embodies the effect of relative motion of inner race through geometrical analysis. The analysis in this paper is presented by considering two boundary positions of inner ring support (even and odd no. of balls). The initial radial shift and angular shift for zth pair of ball due to relative movement of the inner ring is calculated. The equations show that the value of this shift depends on the basic internal geometry of bearing, internal radial clearances, ball separation angle, number of balls and relative movement between races. The developed analytical model is used to calculate the load distribution for a particular ball bearing under 9000 N load and a comparison study is made with Tomović model. It can be observed that the proposed model gives distinctly improved results for bearing with high radial clearance. Further, the results show that for higher radial load and higher radial clearance, the proposed model gives significant improvement in bearing life and load-carrying capacity of a particular bearing. However, the proposed model may not be very much useful for high axial load and less radial load applied to the bearing, which is very uncommon.

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Compliance with ethical standard

Conflict of interest The authors declare that they have no conflict of interest.

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Appendix 1

Calculation of ball bearing contact stiffness

According to Hamrock and Anderson [23] the contact stiffness of radial ball bearing can be expressed by the relation:

$$K_{ir} = \pi.\kappa_{ir}.E^{I}.\sqrt{\frac{2\varepsilon_{ir}R_{ir}}{9\xi_{ir}{}^{3}}}$$

$$K_{or} = \pi.\kappa_{or}.E^{I}.\sqrt{\frac{2\varepsilon_{or}R_{or}}{9\xi_{or}{}^{3}}}$$

where κ ellipticity parameter, ε is the elliptic integral of the second kind, R is the effective radius of curvature, ξ is the elliptic integral of the first kind and E^{I} is the effective elastic modulus.

The combined stiffness is given by

$$K = \left[\left(1/K_{ir}\right)^{1/n} + \left(1/K_{or}\right)^{1/n}\right]^{-n}$$

where index 'ir' and 'or' denotes inner and outer raceways (Table 3).

The effective radii of curvature R_{ir} and R_{or}, for the inner and outer ball bearing races are given by

$$R_{ir} = \left[\frac{2D_p}{d_b\left(D_p - d_b\right)} + \frac{2f_{ir} - 1}{f_{ir}d_b}\right]^{-1} = 3.6647mm$$

and,

$$R_{or} = \left[\frac{2D_p}{d_b\left(D_p + d_b\right)} + \frac{2f_{or} - 1}{f_{or}d_b}\right]^{-1} = 5.4935mm$$

where f = race conformity (ratio of race groove radius to bearing ball diameter) and D_p = pitch diameter.

Hamrock and Anderson [23] provided approximate expressions for the ellipticity parameter and elliptic integrals for a radially loaded ball bearing in terms of the radius ratios, α_{ir} and α_{or}.

$$\alpha_{ir} = \frac{2f_{ir}D_p}{\left(D_p - d_b\right)\left(2f_{ir} - 1\right)} = 32.7887$$

$$\alpha_{or} = \frac{2f_{or}D_p}{\left(D_p + d_b\right)\left(2f_{or} - 1\right)} = 21.5402$$

Now ellipticity parameter,

$$\kappa_{ir} = \alpha_{ir}{}^{2/\pi} = 9.2244$$
$$\kappa_{or} = \alpha_{or}{}^{2/\pi} = 7.0595$$

Elliptic integral of first kind,

Table 3 Bearing parameter for the calculation of bearing stiffness

Ball bearing type	Modulus of elasticity, E (N/mm^2)	Effective modulus of elasticity, E^{I} (N/mm^2)	Poisson's ratio, $\upsilon_{ir} = \upsilon_{or}$	Bore diameter (mm)	d_b (mm)	d_{ir} (mm)	r_{ir} (mm)	d_{or} (mm)	r_{or} (mm)
6206	212,000	232,970	0.3	30	9.525	36.48	4.953	55.53	4.953

$$\xi_{ir} = \frac{\pi}{2} + \left(\frac{\pi}{2} - 1\right) \ln\left(\alpha_{ir}\right) = 3.5629$$

$$\xi_{or} = \frac{\pi}{2} + \left(\frac{\pi}{2} - 1\right) \ln\left(\alpha_{or}\right) = 3.3231$$

Elliptic integral of second kind,

$$\xi_{or} = 1 + \left(\frac{\pi}{2} - 1\right) \Big/ \alpha_{or}\alpha_{or} = 1.0265$$

$$\xi_{or} = 1 + \left(\frac{\pi}{2} - 1\right) \Big/ \alpha_{or} = 1.0265$$

Now, contact stiffness of radial ball bearing

$$K_{ir} = \pi.\kappa_{ir}.E^{l}.\sqrt{\frac{2\varepsilon_{ir}R_{ir}}{9{\xi_{ir}}^{3}}} = 9.1378 \times 10^{5} N/mm^{1.5}$$

$$K_{or} = \pi.\kappa_{or}.E^{l}.\sqrt{\frac{2\varepsilon_{or}R_{or}}{9{\xi_{or}}^{3}}} = 9.5479 \times 10^{5} N/mm^{1.5}$$

At last the combined stiffness

$$K = \left[\left(1/K_{ir}\right)^{1/n} + \left(1/K_{or}\right)^{1/n}\right]^{-n} = 3.3019 \times 10^{5} N/mm^{1.5}$$

Note:

$$E^{l} = \frac{2}{\left(\frac{1-{\upsilon_{ir}}^{2}}{E} + \frac{1-{\upsilon_{or}}^{2}}{E}\right)} N/mm^{2}$$

Appendix 2

Comparison of proposed model with Tudose et al. [19] and Rusu et al. [20] model at varies clearance value

Tudose et al. [19] in his model defines the position of qth ball with respect to the direction of applied radial load as

$$\beta_q = (q-1)\beta + \theta \quad (q = 1, 2, 3 \ldots \quad \ldots \quad \ldots N)$$

where θ = angular displacement of most loaded ball.

And the load on any qth ball as

$$F_q = K\left(\frac{C}{2}\right)^{1.5}\left(\Delta_{\max}\cos\beta_q - 1\right)^{1.5}$$

Under static equilibrium position the sum of the radial components of these loads must be equal to the applied load F_r

$$F_r = K\left(\frac{C}{2}\right)^{1.5}\sum_{q=1}^{q=N}\left[\left(\Delta_{\max}\cos\beta_q - 1\right)^{1.5}\cos\beta_q\right]$$

The above equation can be solved for $\Delta_{\max}$ using numerical method.

The comparison study at different angular displacement of first and ninth ball under external radial load for radial clearance is shown in Fig. 7. As shown in the figure as the value of clearance increases the difference between two curves increases which shows the effect of relative displacement of bearing races on load distribution. Hence the present model conclude that at any angular position under higher radial clearance the influence of mutual displacement of bearing races on separation angle cannot be neglected.

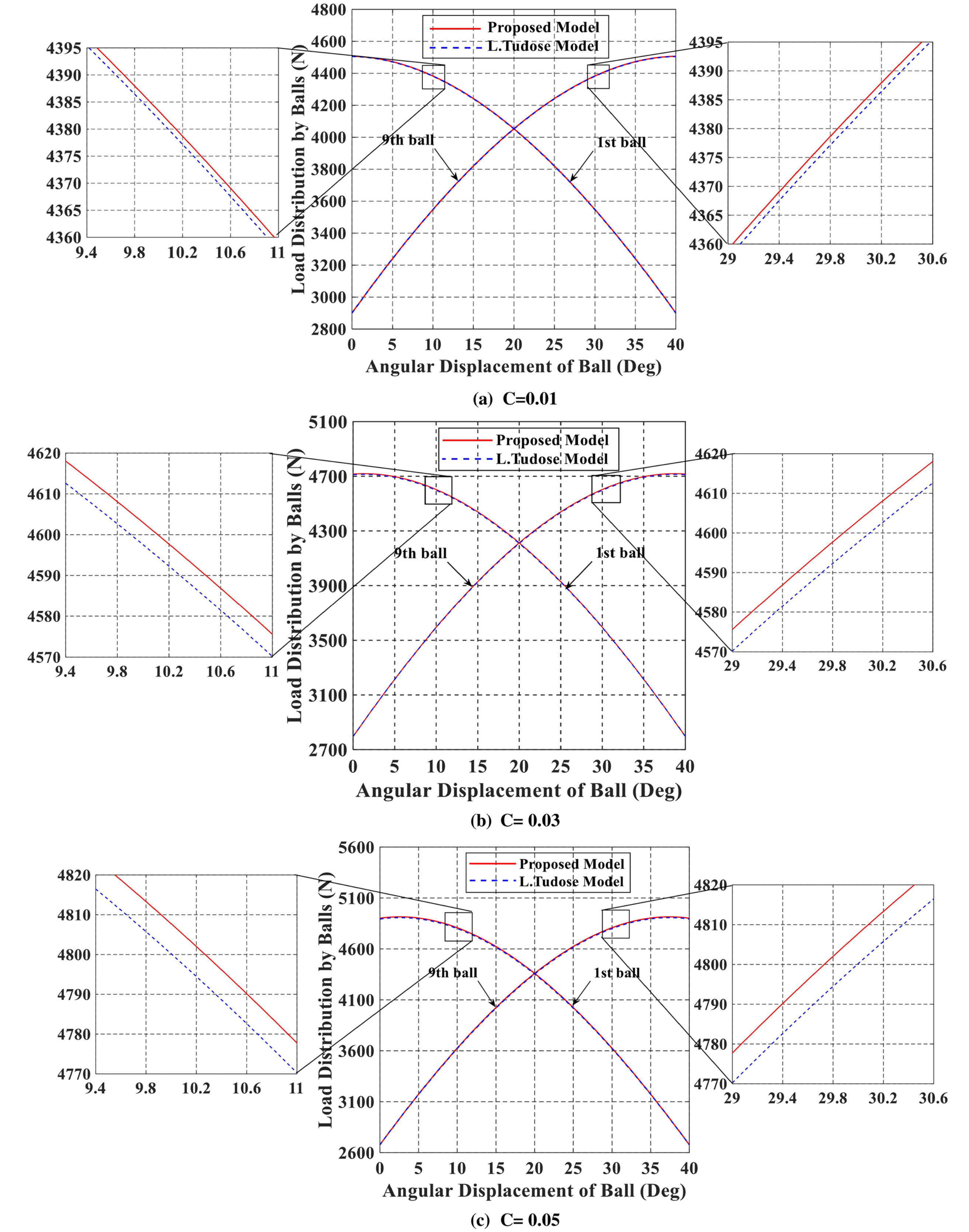

Fig. 7 Load on the first and ninth ball at different angular displacement of ball. (**a**) C = 0.01. (**b**) C = 0.03. (**c**) C = 0.05

References

1. Liu J, Shao Y (2017) An improved analytical model for a lubricated roller bearing including a localized defect with different edge shapes. J Vib Control 24:3894–3907. https://doi.org/10.1177/1077546317716315
2. Liu J, Shao Y (2017) Dynamic modeling for rigid rotor bearing systems with a localized defect considering additional deformations at the sharp edges. J Sound Vib 398:84–102. https://doi.org/10.1016/j.jsv.2017.03.007
3. Liu J, Shao Y, Lim TC (2012) Vibration analysis of ball bearings with a localized defect applying piecewise response function. Mech Mach Theory 56:156–169. https://doi.org/10.1016/j.mechmachtheory.2012.05.008
4. Liu J, Shi Z, Shao Y (2017) An analytical model to predict vibrations of a cylindrical roller bearing with a localized surface defect. Nonlinear Dyn 89:2085–2102. https://doi.org/10.1007/s11071-017-3571-5
5. Liu J, Xu Z, Zhou L, Yu W, Shao Y (2019) A statistical feature investigation of the spalling propagation assessment for a ball bearing. Mech Mach Theory 131:336–350. https://doi.org/10.1016/j.mechmachtheory.2018.10.007
6. Xiaoli R, Jia Z, Ge R (2017) Calculation of radial load distribution on ball and roller bearings with positive, negative and zero clearance. Int J Mech Sci 131–132:1–7. https://doi.org/10.1016/j.ijmecsci.2017.06.042
7. Oswald FB, Zaretsky EV, Poplawski JV (2012) Effect of internal clearance on load distribution and life of radially loaded ball and roller bearings. Tribol Trans 55:245–265. https://doi.org/10.1080/10402004.2011.639050
8. Sung K, Shirley P, Baer S (2008) Essentials of interactive computer graphics: concepts and implementation, 1st edn. CRC Press, Baton Rouge. https://doi.org/10.1201/b15723-9
9. Jones AB (1960) A general theory for elastically constrained ball and radial roller bearings under arbitrary load and speed conditions. J Basic Eng 82:309–320. https://doi.org/10.1115/1.3662587
10. Stribeck R (1907) Ball bearings for various loads. Trans ASME 29:420–463
11. Sjoväll H (1933) The load distribution within ball and roller bearings under given external radial and axial load. Tek Mek h:9
12. Harris TA, Kotzalas MN (2006) Advanced concepts of bearing technology, 5th edn. CRC Press, Boca Raton. https://doi.org/10.1201/9781420006582
13. Tomović R (2012) Calculation of the boundary values of rolling bearing deflection in relation to the number of active rolling elements. Mech Mach Theory 47:74–88. https://doi.org/10.1016/j.mechmachtheory.2011.08.006
14. Tomović R (2012) Calculation of the necessary level of external radial load for inner ring support on q rolling elements in a radial bearing with internal radial clearance. Int J Mech Sci 60:23–33. https://doi.org/10.1016/j.ijmecsci.2012.04.002
15. Hernot X, Sartor M, Guillot J (2000) Calculation of the stiffness matrix of angular contact ball bearings by using the analytical approach. J Mech Des 122:83–90. https://doi.org/10.1115/1.533548
16. Ricci MC (2009) Internal loading distribution in statically loaded ball bearings subjected to an eccentric thrust load. Math Probl Eng 2009:471804. https://doi.org/10.1155/2009/471804
17. Ricci M (2010) Static load distribution in ball bearings. In: Proc. 40th aerosp. mech. symp., pp 1–6
18. Wang Y, Yuan Q (2013) Contact force distribution and static load-carrying capacity of large size double row four-point contact ball bearing. Defence Technol 9:229–236. https://doi.org/10.1016/j.dt.2013.12.003
19. Tudose C, Rusu F, Tudose L (2013) Influence of rotation angle on bearing rolling bodies load distribution. Part 1: mathematical model, Acta Tech. Napocensis Ser. Appl Math Mech 56:469–474
20. Rusu F, Tudose C, Tudose L (2013) Influence of rotation angle on bearing rolling bodies load distribution. Part 2: simulation results, Acta Tech. Napocensis Ser. Appl Math Mech 56:795–800
21. Korolev AV, Korolev AA, Neugebauer KS (2014) Mechanism ball rolling ring bearings. Russ Internet J Ind Eng 2:75–80. https://doi.org/10.24892/rijie/20140313
22. A.B.M. Association (2011) Radial bearings of ball, cylindrical roller and spherical roller types—metric design, p 78
23. Hamrock BJ, Anderson WJ (1983) Rolling-element bearings. NASA Ref. Publ.