Research Article

New periodic wave solutions of a (3+1)-dimensional Jimbo–Miwa equation

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Abstract
A new three-wave method is efficient and well-developed approach to solve nonlinear partial differential equation. In this paper, a (3+1)-dimensional Jimbo–Miwa equation is investigated by using this approach. Some periodic wave solutions and kink solutions are obtained through the Hirota bilinear form. Furthermore, figures of some special periodic wave solutions and kink solutions are presented to illustrate the dynamical features of these solutions.

Keywords (3+1)-Dimensional Jimbo–Miwa equation · New three-wave method · Periodic wave solutions

Mathematics Subject Classification 35B10 · 35Q51 · 35Q53 · 35C15 · 33F10 · 68M07

1 Introduction

As we all know, there are many nonlinear partial differential equation (NLPDEs) especially soliton equations in the fields of physics, chemistry, biology and mechanics. We explain the phenomena and dynamic processes in these fields by solving the exact solutions of NLPDEs. Naturally, searching exact solutions of NLPDEs becomes an important work. In recent years, many methods have been proposed to find the exact solutions of the NLPDEs, such as Hirota bilinear method [1–4], homogeneous balance method [5], multiple exp-function method [6, 7], the Bäcklund transformation [8]. The Hirota bilinear method is considered as a useful method to obtain exact solutions of nonlinear evolution equations. We find that a lot of integrable equations can be transformed into Hirota bilinear forms by different dependent variable transformations such as the KP equation and BKP equation, subsequently, different types of solutions can be successfully achieved, such as soliton solutions, compacton solutions, Wronskian forms of N-soliton soultions, rational solutions (which usually contain rogue waves and lump solutions) and periodic solutions [9–16]. Among them, the periodic solution is the one of more important solutions to understand some of the natural phenomena. A lot of scholars have constructed periodic soliton solutions by using Hirota bilinear forms with the aid of symbolic computation [17–19]. Furthermore, some double periodic solutions and quasi-periodic wave solutions have been also investigated [20, 21].

In this paper, we will consider the (3+1)-dimensional Jimbo–Miwa equation [22], which reads

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0, \tag{1.1}$$

under the dependent variable transformation

$$u = 2(lnf)_x. \tag{1.2}$$

Equation (1.2) can be transformed into the following bilinear form

$$(D_x^3 D_y + 2D_t D_y - 3D_x D_z) f \cdot f = 0, \tag{1.3}$$

or, equivalently

$$\begin{aligned} &ff_{xxxy} - f_y f_{xxx} - 3f_x f_{xxy} + 3f_{xx} f_{xy} + 2ff_{yt} \\ &\quad - 2f_y f_t - 3ff_{xz} + 3f_x f_z = 0, \end{aligned} \tag{1.4}$$

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where $f=f(x,y,z,t)$ is also real function with respect to variables x, y, z and t. $D_x^3D_y, D_tD_y$ and D_xD_z are called Hirota bilinear operators [23] defined by

$$D_x^aD_t^b(f\cdot g)=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^a\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^b\times f(x,t)g(x,t)|_{x'=x,t'=t}. \tag{1.5}$$

Many papers focus on analyzing the exact solutions of Eq. (1.1). For example, its exact solutions were explicitly given by transformed rational function method [24], the soliton solutions and Wronskian determinant solutions have been obtained by Hirota bilinear forms and Wronskian technique [25–27], the Bäcklund transformations and Lax system were discussed by Bell-polynomials theory [28], the lump and lump-type solutions have been derived by Hirota bilinear method [29–36] and some periodic wave solutions also have been found in Refs. [37–39].

This paper mainly aims at studying periodic wave solutions of (3+1)-dimensional Jimbo–Miwa equation by using the Hirota bilinear form and new three-wave method. Through various forms of graphic illustration, we would give a better understanding on the evolution of solutions of waves.

2 Describe the new three-wave method

Here, we briefly show the new three-wave method [40, 41].

Step 1: Taking the Cole–Hopf transformation $u=2(lnf)_x$, the Eq. (1.1) is transformed into the proper bilinear form

$$H(D_x,D_y,D_z,D_t,\cdots)ff=0, \tag{2.1}$$

where $f=f(x,y,z,t)$ and the derivatives D_x, D_y, D_z, D_t are the Hirota operators.

Step 2: Based on the Hirota bilinear of Eq. (2.1), we assume that the solution can be expressed in the form

$$f=a_1\cosh\left(c_1t+k_1x+l_1y+b_1z\right)+a_2\cos\left(c_2t+k_2x+l_2y+b_2z\right)+a_3\cosh\left(c_3t+k_3x+l_3y+b_3z\right), \tag{2.2}$$

where $a_i, b_i, c_i, l_i, k_i, (i=1,2,3)$ are real parameters to be determined.

Step 3: Substituting (2.2) into (1.3), with the help of maple, collecting all the coefficients about $\cosh\left(c_1t+k_1x+l_1y+b_1z\right)$, $\cos\left(c_2t+k_2x+l_2y+b_2z\right)$, $\cosh\left(c_3t+k_3x+3y+b_3z\right)$, $\sinh\left(c_1t+k_1x+l_1y+b_1z\right)$, $\sin\left(c_2t+k_2x+l_2y+b_2z\right)$ and $\sinh\left(c_3t+k_3x+l_3y+b_3z\right)$, whose coefficients become zero, we can obtain a set of determining equations for $a_i, b_i, c_i, l_i, k_i, (i=1,2,3)$. By the transformation (1.2), we will find the solutions of Eq. (1.1).

3 Periodic wave solutions of the (3+1)-dimensional Jimbo–Miwa equation

According to the new three-wave method of the above stated in Sect. 2, we assume that Eq. (1.1) has the following type solutions. That is

$$f=a_1\cosh\left(c_1t+k_1x+l_1y+b_1z\right)+a_2\cos\left(c_2t+k_2x+l_2y+b_2z\right)+a_3\cosh\left(c_3t+k_3x+l_3y+b_3z\right), \tag{3.1}$$

where $a_i, b_i, c_i, l_i, k_i, (i=1,2,3)$ are arbitrary real constants.

In order to get the periodic wave solutions, substituting (3.1) into (1.3), collecting the coefficients about $\cosh\left(c_1t+k_1x+l_1y+b_1z\right)$, $\cos\left(c_2t+k_2x+l_2y+b_2z\right)$, $\cosh\left(c_3t+k_3x+l_3y+b_3z\right)$, $\sinh\left(c_1t+k_1x+l_1y+b_1z\right)$, $\sin\left(c_2t+k_2x+l_2y+b_2z\right)$ and $\sinh\left(c_3t+k_3x+l_3y+b_3z\right)$, we can obtain a system of algebraic system in $a_i, b_i, c_i, l_i, k_i, (i=1,2,3)$. Solving this system of equations with the help of symbolic computation, we get the following solutions of parameters:

Case 1.

$$\begin{aligned}
&a_1=0, a_2=a_2,\\
&a_3=a_3, b_2=-\frac{2k_3l_3\left(-2\,k_2{}^3+c_2\right)}{3k_2{}^2},\\
&b_3=\frac{l_3\left(-k_2{}^3+3\,k_2k_3{}^2+2\,c_2\right)}{3k_2},\\
&b_1=b_1, c_1=c_1, c_2=c_2,\\
&c_3=\frac{k_3\left(-k_2{}^3-k_2k_3{}^2+2\,c_2\right)}{2k_2}, k_1=k_1, k_2=k_2, k_3=k_3,\\
&l_1=l_1,\\
&l_2=-\frac{k_3l_3}{k_2}, l_3=l_3,
\end{aligned} \tag{3.2}$$

where $a_i(i=2,3), b_1, c_j(j=1,2), k_p(p=1,2,3)$ and $l_q(q=1,3)$ are arbitrary real constants.

Case 2.

$$\begin{aligned}
&a_1 = a_1, a_2 = a_2, a_3 = 0, b_1 = -\frac{2k_2 l_2 \left(2 k_1{}^3 + c_1\right)}{3k_1{}^2},\\
&b_2 = \frac{l_2 \left(k_1{}^3 - 3 k_1 k_2{}^2 + 2 c_1\right)}{3k_1},\\
&b_3 = b_3, c_1 = c_1,\\
&c_2 = \frac{k_2 \left(k_1{}^3 + k_1 k_2{}^2 + 2 c_1\right)}{2k_1},\\
&c_3 = c_3, k_1 = k_1, k_2 = k_2, k_3 = k_3,\\
&l_1 = -\frac{k_2 l_2}{k_1}, l_2 = l_2, l_3 = l_3,
\end{aligned} \tag{3.3}$$

where $a_i(i = 1, 2), b_3, c_j(j = 1, 3), k_p(p = 1, 2, 3)$ and $l_q(q = 2, 3)$ are arbitrary real constants.

Case 3.

$$\begin{aligned}
&a_1 = a_1, a_2 = a_2, a_3 = a_3, b_1 = -\frac{2k_2 l_2 (2k_1^3 + c_1)}{3k_1^2},\\
&b_2 = \frac{l_2 \left(k_1{}^3 - 3 k_1 k_2{}^2 + 2 c_1\right)}{3k_1},\\
&b_3 = \frac{k_1{}^3 l_3{}^2 + 3 k_1 k_2{}^2 l_2{}^2 + 2 c_1 l_3{}^2}{3k_1 l_3}, c_1 = c_1,\\
&c_2 = \frac{k_2 \left(k_1{}^3 + k_1 k_2{}^2 + 2 c_1\right)}{2k_1}, k_1 = k_1,\\
&k_2 = k_2,\\
&c_3 = -\frac{k_2 l_2 \left(k_1{}^3 l_3{}^2 - k_1 k_2{}^2 l_2{}^2 + 2 c_1 l_3{}^2\right)}{2 l_3{}^3 k_1}, k_3 = -\frac{k_2 l_2}{l_3}, l_1 = -\frac{k_2 l_2}{k_1},\\
&l_2 = l_2, l_3 = l_3,
\end{aligned} \tag{3.4}$$

where $a_i(i = 1, 2, 3), c_1, k_p(p = 1, 2)$ and $l_q(q = 2, 3)$ are arbitrary real constants.

Case 4.

$$\begin{aligned}
&a_1 = a_1, a_2 = 0, a_3 = a_3, b_1 = \frac{2k_3 l_3 \left(2 k_1{}^3 + c_1\right)}{3k_1{}^2},\\
&b_3 = \frac{l_3 \left(k_1{}^3 + 3 k_1 k_3{}^2 + 2 c_1\right)}{3k_1},\\
&b_2 = b_2, c_1 = c_1, c_2 = c_2,\\
&c_3 = \frac{k_3 \left(k_1{}^3 - k_1 k_3{}^2 + 2 c_1\right)}{2k_1}, k_1 = k_1, k_2 = k_2, k_3 = k_3,\\
&l_1 = \frac{k_3 l_3}{k_1}, l_2 = l_2, l_3 = l_3,
\end{aligned} \tag{3.5}$$

where $a_i(i = 1, 3), b_2, c_j(j = 1, 2), k_p(p = 1, 2, 3)$ and $l_q(q = 2, 3)$ are arbitrary real constants.

Thus, from Case 1, the f is given by

$$\begin{aligned}
f = {} & a_2 \cos\left(k_2 x - \frac{k_3 l_3}{k_2} y + c_2 t - \frac{2k_3 l_3 \left(-2 k_2{}^3 + c_2\right)}{3k_2{}^2} z\right)\\
& + a_3 \cosh\left(k_3 x + l_3 y + \frac{k_3 \left(-k_2{}^3 - k_2 k_3{}^2 + 2 c_2\right)}{2k_2} t\right.\\
& \left. + \frac{l_3 \left(-k_2{}^3 + 3 k_2 k_3{}^2 + 2 c_2\right)}{3k_2} z\right).
\end{aligned} \tag{3.6}$$

So we obtain one solution of Eq. (1.1), that is

$$u_1 = \frac{2f_x}{f} = \frac{-2a_2 k_2 \sin(k) + 2a_3 k_3 \sin(l)}{a_2 \cos(k) + a_3 \cosh(l)}, \tag{3.7}$$

where k, l is defined by

$$\begin{cases}
k = k_2 x - \dfrac{k_3 l_3}{k_2} y + c_2 t - \dfrac{2k_3 l_3 \left(-2 k_2{}^3 + c_2\right)}{3k_2{}^2} z,\\
l = k_3 x + l_3 y + \dfrac{k_3 \left(-k_2{}^3 - k_2 k_3{}^2 + 2 c_2\right)}{2k_2} t\\
\quad + \dfrac{l_3 \left(-k_2{}^3 + 3 k_2 k_3{}^2 + 2 c_2\right)}{3k_2} z.
\end{cases} \tag{3.8}$$

From Case 2, by the same calculation, we can obtain another solution, that is

$$u_2 = \frac{2a_1 k_1 \sinh(m) - 2a_2 k_2 \sin(n)}{a_1 \cosh(m) + a_2 \cos(n)}, \tag{3.9}$$

where f, m, n are defined by

$$\begin{cases}
f = a_1 cosh(m) + a_2 cos(n),\\
m = c_1 t + k_1 x - \dfrac{k_2 l_2}{k_1} y - \dfrac{2k_2 l_2 \left(2 k_1{}^3 + c_1\right)}{3k_1{}^2} z,\\
n = \dfrac{k_2 \left(k_1{}^3 + k_1 k_2{}^2 + 2 c_1\right)}{2k_1} t\\
\quad + k_2 x + l_2 y + \dfrac{l_2 \left(k_1{}^3 - 3 k_1 k_2{}^2 + 2 c_1\right)}{3k_1} z.
\end{cases} \tag{3.10}$$

From Case 3, similarly, we can get the third solution, that is

$$u_3 = \frac{2 a_1 k_1 \sinh(h) - 2 a_2 k_2 \sin(r) + 2 \frac{a_3 k_2 l_2}{l_3} \sinh(s)}{a_1 \cosh(h) + a_2 \cos(r) + a_3 \cosh(s)}, \tag{3.11}$$

where f, h, r, s are given by

$$\begin{cases} f = a_1 \cosh(h) + a_2 \cos(r) + a_3 \cosh(s), \\ h = k_1 x - \dfrac{k_2 l_2}{k_1} y + c_1 t - \dfrac{2k_2 l_2 (2k_1^3 + c_1)}{3k_1^2} z, \\ r = k_2 x + l_2 y + \dfrac{k_2 (k_1^3 + k_1 k_2^2 + 2c_1)}{2k_1} t \\ \quad + \dfrac{l_2 (k_1^3 - 3k_1 k_2^2 + 2c_1)}{3k_1} z, \\ s = \dfrac{k_2 l_2}{l_3} x - l_3 y + \dfrac{k_2 l_2 (k_1^3 3 l_3^2 - k_1 k_2^2 l_2^2 + 2c_1 l_3^2)}{l_3^3 k_1} t \\ \quad - \dfrac{(k_1^3 3 l_3^2 + 3k_1 k_2^2 l_2^2 + 2c_1 l_3^2)}{k_1 l_3} z. \end{cases} \tag{3.12}$$

From Case 4, similarly, we can get the fourth solution, that is

$$u_4 = \frac{2a_1 k_1 \sinh(v) + 2a_3 k_3 \sinh(w)}{a_1 \cosh(v) + a_3 \cosh(w)} \tag{3.13}$$

where f, v, w are given by

$$\begin{cases} f = f = a_1 \cosh(v) + a_3 \cosh(w), \\ v = k_1 x + \dfrac{k_3 l_3}{k_1} y + c_1 t + \dfrac{2k_3 l_3 (2k_1^3 + c_1)}{3k_1^2} z, \\ w = k_3 x + l_3 y + \dfrac{k_3 (2k_1^3 - k_1 k_3^2 + 2c_1)}{k_1} t \\ \quad + \dfrac{l_3 (k_1^3 + 3k_1 k_3^2 + 2c_1)}{3k_1} z. \end{cases} \tag{3.14}$$

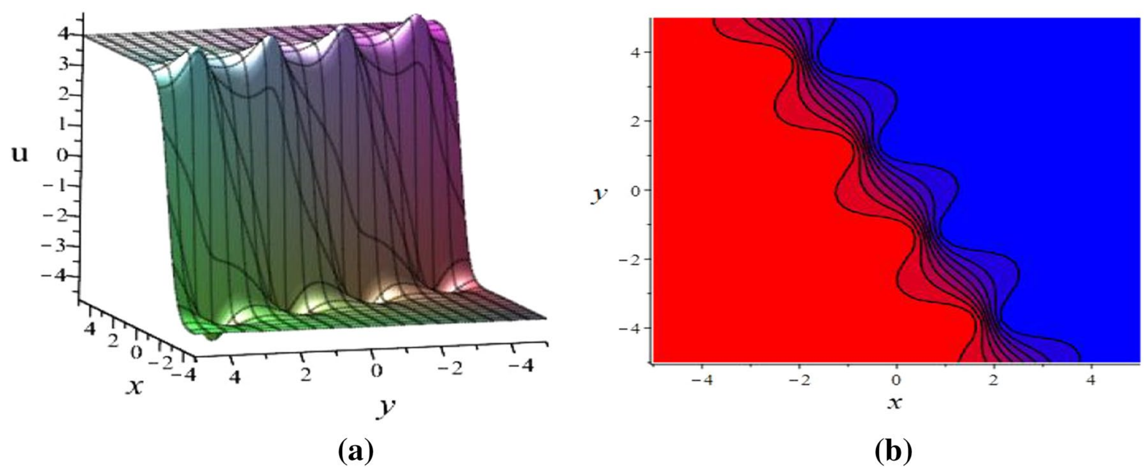

Fig. 1 The periodic wave solution of Eq. (3.16) with $z = 0, t = 0$. **a** Perspective view of the wave. **b** Contour plot

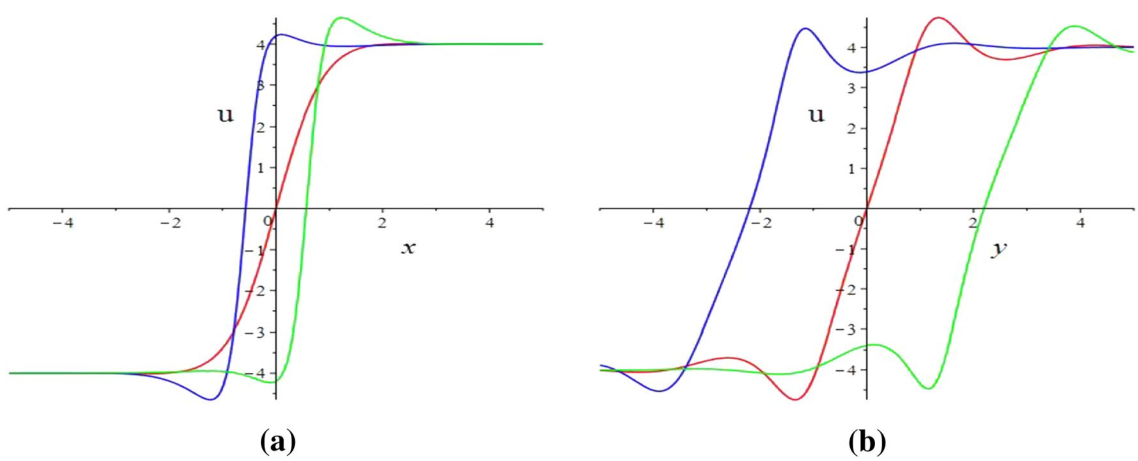

Fig. 2 The periodic wave solution of Eq. (3.16) with $z = 0, t = 0$. **a** x-curve with $y = 0$ (red), $y = 1$ (blue) and $y = -1$ (green). **b** y-curve with $x = 0$ (red), $x = 1$ (blue) and $x = -1$ (green)

Now we depict the dynamic behaviors of some special periodic wave solutions in each Case.

In Case 1, we take the parameters as

$$a_2 = 1, a_3 = 2, b_1 = 1, c_1 = 1, c_2 = 1, k_1 = 2, k_2 = 1, \\ k_3 = 2, l_1 = 1, l_3 = 1, \tag{3.15}$$

get

$$u_1 = \frac{-2\sin\left(t + x - 2y + \frac{4}{3}z\right) - 8\sinh\left(3t - 2x - y - \frac{13}{3}z\right)}{\cos\left(t + x - 2y + \frac{4}{3}z\right) + 2\cosh\left(3t - 2x - y - \frac{13}{3}z\right)}. \tag{3.16}$$

Its plot when $z = 0, t = 0$ is showed in Fig. 1 and the wave along different axis is depicted in Fig. 2.

In Case 2, we take the parameters as

$$a_1 = 1, a_2 = 2, b_3 = 1, c_1 = 1, c_3 = 1, k_1 = 1, \\ k_2 = 2, k_3 = 2, l_2 = 1, l_3 = 1, \tag{3.17}$$

get

$$u_2 = \frac{2\sinh(t + x - 2y - 4z) - 8\sin(7t + 2x + y - 3z)}{\cosh(t + x - 2y - 4z) + 2\cos(7t + 2x + y - 3z)}. \tag{3.18}$$

Their plots when $z = 0$ and $t = 10, 30$ are respectively depicted in Figs. 3 and 4.

In Case 3, we take the parameters as

$$a_1 = 3, a_2 = 12, a_3 = 2, c_1 = 3, k_1 = 1, k_2 = -1, l_2 = -2, l_3 = 1, \tag{3.19}$$

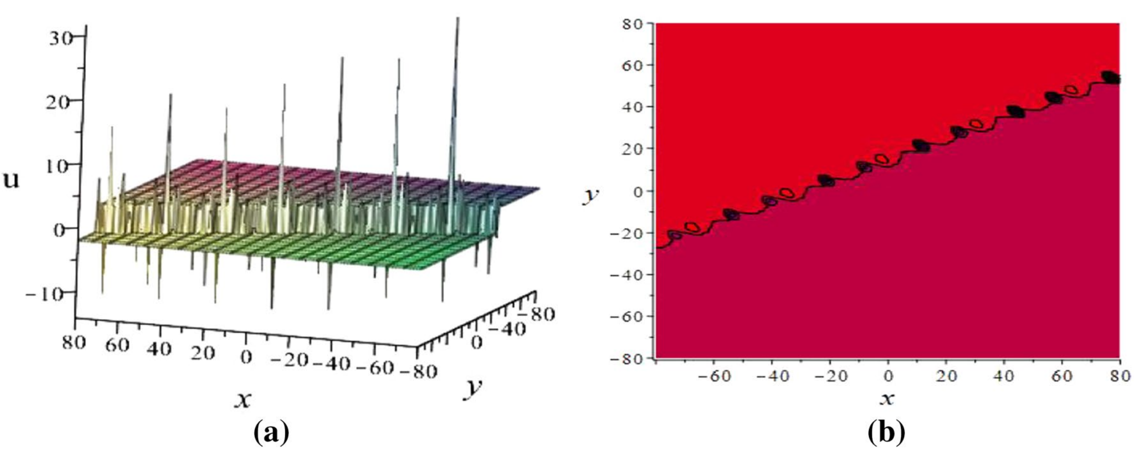

Fig. 3 The periodic wave solution of Eq. (3.18) with $z = 0, t = 10$. **a** Perspective view of the wave. **b** Contour plot

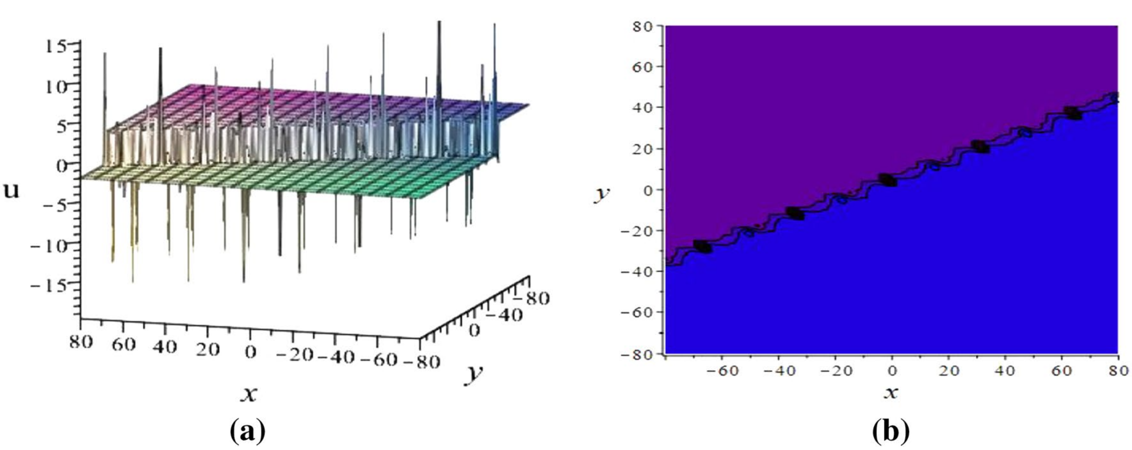

Fig. 4 The periodic wave solution of Eq. (3.18) with $z = 0, t = 30$. **a** Perspective view of the wave. **b** Contour plot

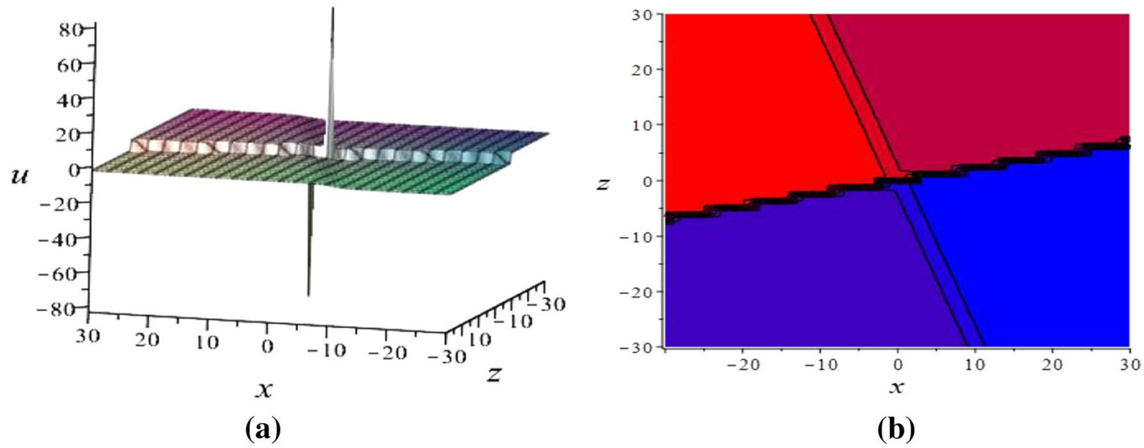

Fig. 5 The periodic wave solution of Eq. (3.20) with $y = 0, t = 0$. **a** Perspective view of the wave. **b** Contour plot

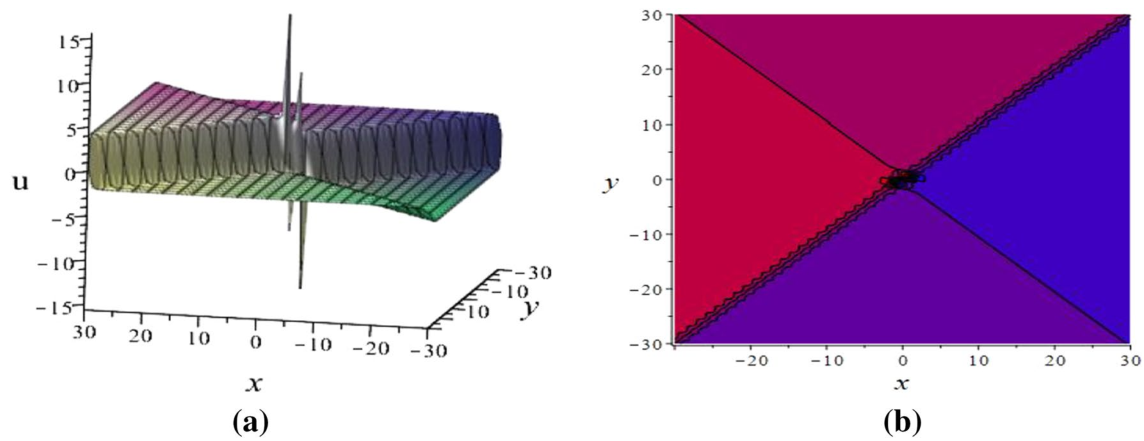

Fig. 6 The periodic wave solution of Eq. (3.20) with $z = 0, t = 0$. **a** Perspective view of the wave. **b** Contour plot

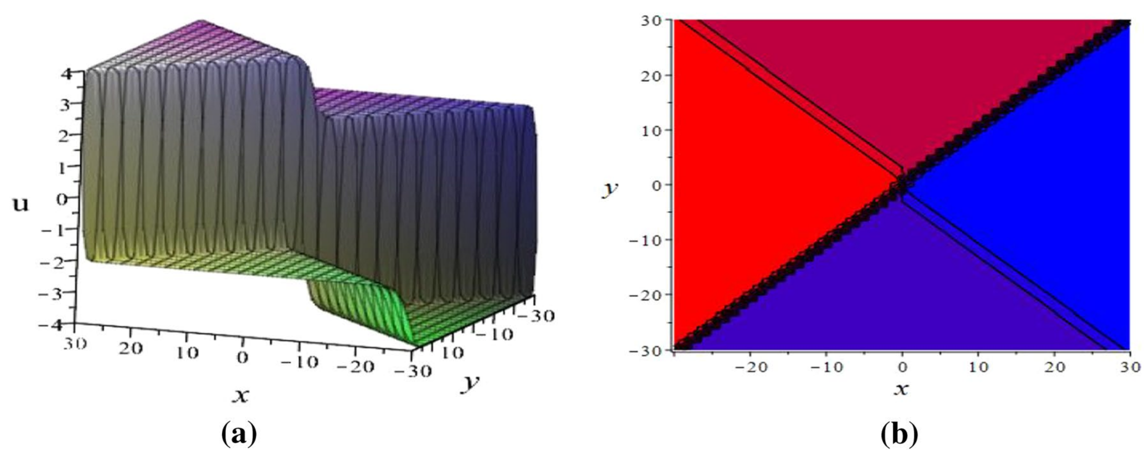

Fig. 7 The kink solution of Eq. (3.22) with $z = 0, t = 0$. **a** Perspective view of the wave. **b** Contour plot

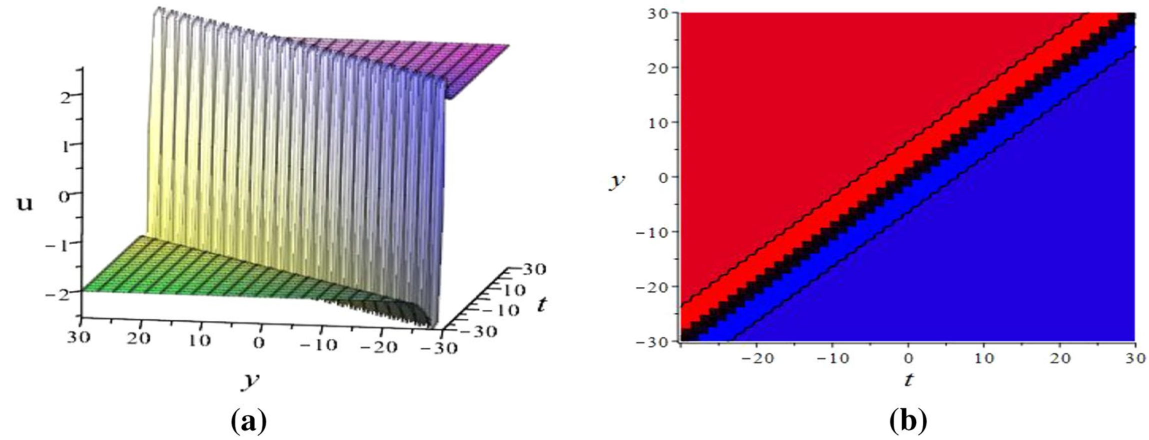

Fig. 8 The kink solution of Eq. (3.22) with $x = 0, z = 0$. **a** Perspective view of the wave. **b** Contour plot

get

$$u_3 = \frac{6\sinh\left(3t + x - 2y - \frac{20}{3}z\right) - 24\sin\left(4t + x + 2y + \frac{8}{3}z\right) + 8\sinh\left(3t + 2x - y - \frac{19}{3}z\right)}{3\cosh\left(3t + x - 2y - \frac{20}{3}z\right) + 12\cos\left(4t + x + 2y + \frac{8}{3}z\right) + 2\cosh\left(3t + 2x - y - \frac{19}{3}z\right)}. \tag{3.20}$$

Their plots when $t = 0$ and $y = 0, z = 0$ are showed in Figs. 5 and 6, respectively.

In Case 4, we take the parameters as

$$\begin{aligned} a_1 &= 2, a_3 = 5, b_2 = 6, c_1 = 2, c_2 = -4, k_1 = 1, \\ k_2 &= 7, k_3 = 2, l_2 = -2, l_3 = -1, \end{aligned} \tag{3.21}$$

get

$$u_4 = \frac{4\sinh\left(2t + x - 2y - \frac{16}{3}z\right) + 20\sinh\left(t + 2x - y - \frac{17}{3}z\right)}{2\cosh\left(2t + x - 2y - \frac{16}{3}z\right) + 5\cosh\left(t + 2x - y - \frac{17}{3}z\right)} \tag{3.22}$$

Their plots when $z = 0$ and $t = 0, z = 0$ are showed in Figs. 7 and 8, respectively.

4 Conclusions

In this paper, associating with the Hirota bilinear form of the (3+1)-dimensional Jimob-Miwa equation, we construct the periodic wave solutions and kink solutions through the new three-wave method. Some special solutions are shown graphically in order to demonstrate that the method is quite effective for handling nonlinear evolution equations. Meanwhile, the solutions obtained in this paper can effectively explain more phenomena in fluid or plasma mechanics.

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Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflicts of interest. Disclosure of potential conflicts of interest Research involving Human Participants.

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