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ORIGINAL ARTICLE

Vertex rough graphs

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Abstract
This article introduces the notion of vertex rough graph and discusses certain basic graph theoretic definitions and examples. Adjacency of vertices is used to create a matrix corresponding to a vertex rough graph. Also, the membership function of a vertex rough graph is introduced with the help of Pawlak's Rough set theory, and using this certain results are obtained. The concepts of rough precision and rough similarity degree are extended to vertex rough graphs.

Keywords Rough set · Edge rough graph · Vertex rough graph · Rough membership function

Introduction

Uncertainty and imprecision occurring in the form of vagueness and ambiguity make many of the naturally occurring situations complex and complicated. Classical mathematical techniques often fail to prosper in situations like this. Further, most of these techniques are crisp, precise and deterministic. The classical technique of probability theory has the limitation that the happening of an event is strictly determined by chance. Zadeh [1] has defined fuzzy sets which can mathematically model situations which are imprecise and vague. Pawlak [2] introduced the concept of rough sets which is an excellent mathematical tool to handle ambiguity and equivocalness associated with the given information. The main advantage of rough set theory is that it does not need any additional information about the data, like membership values in fuzzy sets. In classical set theory, Crisp sets are defined by a membership function, but in rough set theory, the primary concept to define a rough set is an indiscernibility relation. It employs indiscernibility relations to evaluate to what extent two objects were similar. Using this indiscernibility relation, one can construct lower and upper approximations of a set. Lower approximation consists of all instances which surely belongs to the concept, and upper approximation consists of all cases which possibly belongs to the concepts. One benefit of the rough set theory is that it does not require any additional parameter to extract information. Rough set theory has found main applications [3] in many branches like rough classification and logic [4,5], decision making [6,7], machine learning [8], data mining [9,10], banking [11], medicine [12], etc.

A Graph is a symmetric binary relation on a set. It is a fundamental tool in mathematical modelling and has applications in almost all branches of Science and Engineering. Many of the real life problems were solved through mathematical modelling with the help of graph theory. The theory of rough graphs is an attempt to unify rough set theory and graph theory. Graph theory, where objects are represented by vertices and relations by edges, is a convenient way of representing information involving relationship between objects. When there is ambiguity in the description of the objects or in its relationships or in both, it is quite natural that we need to design a structure supporting it, which is called a Rough Graph.

With the advent of World Wide Web, the amount of data need to be collected and stored has increased exponentially and a major part of this data can be represented as graphs which includes page link structures, social, professional and academic networks such as Facebook, Linkedin, DBLP, etc. Most of the times, the patterns of connection between entities in these, which represents non trivial topological features, which are neither purely crisp nor completely random, is called a Complex Network [13]. A major challenge nowa-

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days is to mine these complex networks and the abundance of data in these motivated a new area, called Graph mining, which focus on investigate, propose and develop new algorithms designed to mine complex networks. As ambiguity is naturally inherited in these networks, a suitable modelling can be achieved by utilizing the concept of Rough Graphs.

The notion of edge rough graph was introduced by He and Shi [14]. They have established the concept using a partition on the edge set of a graph. He et al. [15] extended this concept to weighted rough graph by enduing the edges of rough graph with weight attribute, and gave the algorithm of exploring the class optimal tree in weighted rough graph, which generalizes the classical Kruskal algorithm of exploring the optimal tree and presented an application in relationship analysis. Another application of Weighted Rough graph was discussed in [16]. Combining the edge rough graphs and cayley graphs, Liang et al. [17] studied an application of rough graph in data mining. Tong He introduced further rough theoretic properties of rough graphs [18] and representation forms of rough graphs [19]. Some other hybrid structures of rough graphs like soft rough graphs, neutrosophic soft rough graphs, intuitionistic fuzzy rough graphs are also introduced in [20–22].

In edge rough graph, there is no significance for vertex set. It is not possible to compare any two arbitrary rough graphs. They can be compared only if their vertex sets are same. If such a comparison is possible, then the real life applications of rough graph will have more flexibility. The main objective of this paper was to introduce the concept of vertex rough graph which is a more general concept than the edge rough graph. The vertex rough graph is constructed using a partition on the vertex set. Using a partition of vertex set, we define lower approximation and upper approximation of a graph. Hence, this paper is an introduction to the theory of vertex rough graph.

In this paper, the basic idea of edge rough graph is extended to vertex rough graph. Section 2 discusses some basic definitions of graph theory, rough set and edge rough graph. In Sect. 3, the notion of vertex rough graph is introduced and some examples are given. Basic graph theoretic definitions of vertex rough graphs are defined and a counter example for a connected graph which is not surely connected is provided. Later, adjacency matrix of vertex rough graph is defined and some of its properties are discussed. In the last section, some rough theoretic ideas like membership functions and precisions of a vertex rough graphs are defined and related properties are derived.

Preliminaries

Some basic definitions from graph theory, Rough set theory and edge rough graph are given:

Definition 2.1 [23] A graph G is an ordered triple $(V(G), E(G), \psi_G)$ consisting of a non-empty set $V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$, of edges, and an incidence function ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G. If e is an edge and u and v are vertices such that $\psi_G(e) = uv$, then e is said to join u and v; the vertices u and v are called the ends of e. Two graphs G and H are identical (written $G = H$) if $V(G) = V(H)$, $E(G) = E(H)$, and $\psi_G = \psi_H$. Two graphs G and H are said to be isomorphic (written $G \cong H$) if there are bijections $\theta : V(G) \to V(H)$ and $\phi : E(G) \to E(H)$ such that $\psi_G(e) = uv$ if and only if $\psi_H(\phi(e)) = \theta(u)\theta(v)$; such a pair (θ, ϕ) of mappings is called an isomorphism between G and H.

Definition 2.2 [2] Suppose we are given a set of objects U called the universe and an indiscernibility relation $R \subseteq U \times U$, representing our lack of knowledge about elements of U. For the sake of simplicity we assume that R is an equivalence relation. Let X be a subset of U. We want to characterize the set X with respect to R:

- R-lower approximation of X

$$R(x)_* = \cup_{x \in X}\{R(x) : R(x) \subseteq X\}$$

- R-upper approximation of X

$$R(x)^* = \cup_{x \in X}\{R(x) : R(x) \cap X \neq \phi\}$$

- R-boundary region of X

$$RN_R(x) = R(x)^* - R(x)_*$$

The pair $(R(x)_*, R(x)^*)$ is called Rough Set. X is crisp (exact with respect to R), if the boundary region of X is empty. Set X is rough (inexact with respect to R), if the boundary region of X is non-empty.

Definition 2.3 [14] Given universe of discourse U, $V = \{v_1, v_2, \ldots, v_{|V|}\}$, $P = \{r_1, r_2, \ldots, r_{|P|}\}$ is attributes set on U, and P contains vertex attribute (v_i, v_j), where,$v_i \in V$, $v_j \in V$. Let $E = \cup e_k(v_i, v_j)$ is edge set on U, graph $U = (V, E)$ is called universe graph. For any attribute set $R \subseteq P$ on E, the elements (or be called edges) in E can be classified into different equivalence classes $[e]_R$. For any subgraph $T = (W, X)$, where $W \subseteq V$, $X \subseteq E$, graph T is called R-definable graph or R-exact graph if X is the union of some $[e]_R$. Conversely, graph T is called R-undefinable graph or R-rough graph. For R-rough graph, two exact graphs $R(T)_* = (W, R(X)_*)$ and $R(T)^* = (W, R(X)^*)$ can be used to define it approximately, where

$$R(X)_* = \{e \in E : [e]_R \subseteq X\}$$
$$R(X)^* = \{e \in E : [e]_R \cap X \neq \phi\}$$

The graphs $R(T)_*$ and $R(T)^*$ are called R-lower and R-upper approximate graphs of T. The pair of graph $(R(T)_*, R(T)^*)$ is called R-rough graph. The set $bn_R(X) = R(X)_* - R(X)^*$ is called the R-boundary of edges set X of T.

Vertex rough graph

In this section, Vertex rough graph of a graph with respect to a indiscernability relation on vertex set V is presented.

Definition 3.1 Let $G = (V, E)$ be a universe graph with $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. Let R be an equivalence relation defined on V. Then the elements in V can be divided into different equivalence classes $[v]_R$.

Definition 3.2 Let $T(W, X)$ be a subgraph of $G(V, E)$ where $W \subseteq V$,$X \subseteq E$, graph T is called R-definable graph or R-exact graph if W is the union of some $[v]_R$. Otherwise, the graph T is called R-undefinable graph or R-rough graph.

Definition 3.3 R-vertex rough graph is defined in terms of two exact graphs $R_*(T) = (R_*(W), R_*(X))$ and $R^*(T) = (R^*(W), R^*(X))$, where

$$R_*(W) = \{v \in V : [v]_R \subseteq W\}$$
$$R^*(W) = \{v \in V : [v]_R \cap W \neq \phi\}$$
$$R_*(X) = \{(v_i, v_j) \in X : v_i, v_j \in [v]_R \text{ for some } v \in s R_*(W)\}$$
$$R^*(X) = \begin{cases} (v_i, v_j) \in E : v_i \in [v_i]_R \ \& \ [v_i]_R \cap X \neq \phi \ \text{ and} \\ v_j \in [v_j]_R \ \& \ [v_j]_R \cap X \neq \phi\} \end{cases}$$

The graphs $R_*(T)$ and $R^*(T)$ are called R-lower approximate graph of T and R-upper approximate graph of T. The pair of graph $(R_*(T), R^*(T))$is called R-vertex rough graph.

Example 3.1 Consider $G(V, E)$ $V = \{v_1, v_2, v_3, v_4, v_5\}$
$V/R = \{\{v_3, v_4, v_5\}, \{v_1, v_2\}\}$
Consider $T = (W, X)$ be a subgraph of $G(V, E)$ (Fig. 1)
By using definition 3.3, we get the lower and upper approximations of vertex set and edge set as (Fig. 2):

$$R_*(W) = \{v_3, v_4, v_5\} \quad R^*(W) = \{v_1, v_2, v_3, v_4, v_5\}$$
$$R_*(X) = \{e_3, e_4\} \quad R^*(X) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$
$$R_*(T) = (R_*(W), R_*(X)), \quad R^*(T) = (R^*(W), R^*(X))$$

Proposition 3.1 *Lower and upper approximations of a graph have the following properties:*
For all $T, T_1, T_2 \subseteq G$,

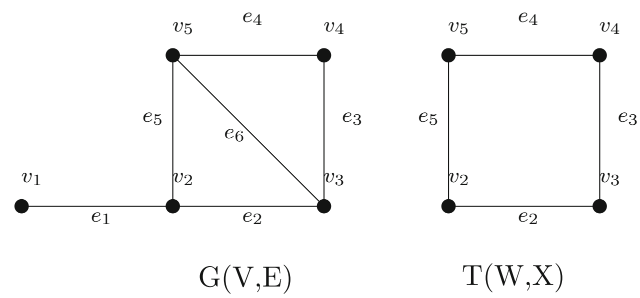

Fig. 1 Graph $G(V, E)$ and its sub-graph $T(W, X)$ in Example 3.1

1. $R_*(T) \subseteq T \subseteq R^*(T)$.
2. $R_*(K^c) = R^*(K^c) = K^c$, $R_*(G) = R^*(G) = G$ *where K is the Complete graph.*
3. $R_*(T_1 \cap T_2) = R_*(T_1) \cap R_*(T_2)$.
4. $R^*(T_1 \cup T_2) = R^*(T_1) \cup R^*(T_2)$.
5. $R_*(T_1 \cup T_2) \supseteq R_*(T_1) \cup R_*(T_2)$.
6. $R^*(T_1 \cap T_2) \subseteq R^*(T_1) \cap R^*(T_2)$.
7. $T_1 \subseteq T_2 \Rightarrow R_*(T_1) \subseteq R_*(T_2)$ & $R^*(T_1) \subseteq R^*(T_2)$.
8. $R_* R_*(T) = R^* R_*(T) = R_*(X)$,
   $R^* R^*(T) = R_* R^*(T) = R^*(T)$.

Definition 3.4 Let $T(W_1, X)$ and $S(W_2, Y)$ are subgraphs of $G(V, E)$ where $W_1 \subseteq V$, $W_2 \subseteq V$, $X \subseteq E$, $Y \subseteq E$, $T = (R_*(T), R^*(T))$ and $S = (R_*(S), R^*(S))$ be its rough graphs. S is said to be surely subgraph of T if $R_*(S) \subseteq R_*(T)$. Also S is said to be possibly subgraph of T if $R^*(S) \subseteq R^*(T)$ if S is both surely subgraph and possibly subgraph of T, then S is a rough subgraph of T.

Definition 3.5 A set of two or more edges of a rough graph T is said to be multiple or parallel edges if they have the same end vertices. An edge for which two ends are the same is called a loop at the common vertex. A rough graph $T = (R_*(T), R^*(T))$ is said to be surely simple if $R_*(T)$ contains no loops and parallel edges. A rough graph $T = (R_*(T), R^*(T))$ is said to be possibly simple if $R^*(T)$ contains no loops and parallel edges. A rough graph $T = (R_*(T), R^*(T))$ is said to be simple if it is both surely and possibly simple graphs.

Definition 3.6 Two rough graphs $T = (R_*(T), R^*(T))$ and $S = (R_*(S), R^*(S))$ are said to be surely isomorphic if there is a graph isomorphism between $R_*(T)$ and $R_*(S)$. Also it is said to be possibly isomorphic if there is a graph isomorphism between $R^*(T)$ and $R^*(S)$. Two rough graphs $T = (R_*(T), R^*(T))$ and $S = (R_*(S), R^*(S))$ are said to be isomorphic if they are both surely and possibly isomorphic.

Definition 3.7 Let $T = (R_*(T), R^*(T))$ be a Rough graph. The Complement T^c of T with respect to G is defined by taking $V(T^c) = V(T)$ and $T^c = (R_*(T)^c, R^*(T)^c)$ where adjacency of $R_*(T)^c$ is defined as two vertices of u and v are

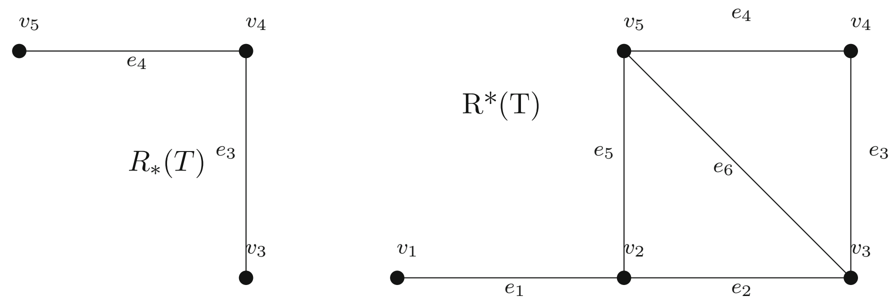

Fig. 2 Lower and upper approximations of the graph T in Example 3.1

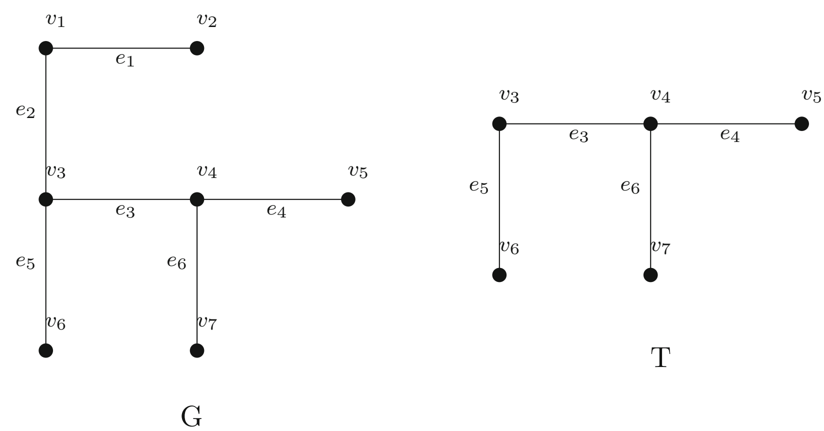

Fig. 3 Graph G and its sub-graph T in Example 3.2

adjacent if and only if they are non adjacent in $R_*(T)$. Also adjacency of $R^*(T)^c$ is defined as two vertices u and v are adjacent if and only if they are non adjacent in $R^*(T)$.

Remark 3.1 The connectedness of vertex rough graph is the same as the connectedness of edge rough graph.

Result 3.1 *If $T(W, X)$ is connected then it need not be surely connected. Similarly T is a tree then it need not be a sure tree.*

Example 3.2 Consider $G(V, E)$

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\},$$
$$V/R = \{\{v_7\}\{v_4, v_5\}, \{v_1, v_2\}, \{v_3, v_6\}\}$$

Consider $T = (W, X)$ be a subgraph (Fig. 3) of $G(V, E)$

Then, we get the lower approximation of vertex set and edge set as (Fig. 4)

$$R_*(W) = \{v_3, v_4, v_5, v_6\} R(X)_* = \{e_4, e_5\}$$
$$R_*(T) = (R_*(W), R_*(X))$$

Here $R_*(T)$ is disconnected. but $T = (W, X)$ is connected. Also T is a tree but $R_*(T)$ is not a tree. It is a forrest.

Matrix corresponding to a rough graph

Let $G=(V, E)$ be a universe graph with $V=\{v_1, v_2, \ldots, v_n\}$. R be an equivalent relation defined on V. Let $T(W, X)$ be a

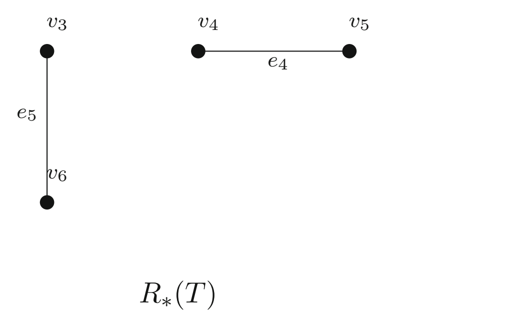

Fig. 4 Lower approximation of T in Example 3.2

subgraph of G. $T = (R_*(T), R^*(T))$ be the corresponding rough graph. Then we can define a nonzero ternary matrix $A_R(T)$ of T by

$$A_R(T) = (a_{ij}) = \begin{cases} 0\, if\ (v_i, v_j) \notin R^*(X) \\ 1\, if\ (v_i, v_j) \in R^*(X)\ \&\ (v_i, v_j) \notin R_*(X) \\ 2\, if\ (v_i, v_j) \in R_*(X) \end{cases}$$

Example 3.3 Matrix corresponding to the rough graph in Example 3.1 is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

Note: There is no one to one correspondence between set of all rough graph and set of all ternary matrices. But for every rough graph, there is a ternary matrix.

Remark 3.2 Let T be a rough graph and $A_R(T)$ be its corresponding matrix. Then

1. T is exact if all entries of $A_R(T)$ are 0 & 2.
2. T is Rough if atleast one entry of $A_R(T)$ is 1.

Rough properties of rough graph

In the same way of rough set theory, rough graph can be also defined employing instead of approximation, rough membership function.

Definition 4.1 The rough vertex membership function of a rough graph $T = (R_*(T), R^*(T))$ is a fuction $\mu_W^R : V \to [0, 1]$ is defined as $\mu_W^R(v) = \dfrac{|W \cap [v]_R|}{|[v]_R|}$.

Also, the rough edge membership function of a rough graph $T = (R_*(T), R^*(T))$ is a fuction $\delta_W^R : V \times V \to [0, 1]$ is defined as $\delta_W^R(v_i, v_j) = min\{\mu_W^R(v_i), \mu_W^R(v_j)\}$.

Definition 4.2 The vertex and edge membership function can be used to define the rough graph of a graph as shown

$$R_*(W) = \{v \in V : \mu_W^R(v) = 1\}$$
$$R^*(W) = \{v \in V : \mu_W^R(v) > 0\}$$
$$R_*(X) = \{(v_i, v_j) \in X : \delta_W^R(v_i, v_j) = 1\}$$
$$R^*(X) = \{(v_i, v_j) \in E : \delta_W^R(v_i, v_j) > 0\}$$

Proposition 4.1 *The membership function has the following properties:*

1. $\mu_W^R(v) = 1$ *iff* $v \in R_*(W)$ *and* $\delta_W^R(v_i, v_j) = 1$ *iff*$(v_i, v_j) \in R_*(X)$
2. $\mu_W^R(v) = 0$ *iff* $v \in V - R^*(W)$ *and* $\delta_W^R(v_i, v_j) = 0$ *iff* $(v_i, v_j) \in E - R^*(X)$
3. $\mu_{V-W}^R(v) = 1 - \mu_W^R(v)$ *and* $\delta_{V-W}^R(v_i, v_j) \leq 1 - \delta_X^R(v_i, v_j)$
4. $\mu_{W_1 \cup W_2}^R(v) = \mu_{W_1}^R(v) + \mu_{W_2}^R(v) - \mu_{W_1 \cap W_2}^R(v))$

Proof 1.

$$\begin{aligned}
\mu_W^R(v) = 1 &\Leftrightarrow \frac{|W \cap [v]_R|}{|[v]_R|} = 1 \\
&\Leftrightarrow |W \cap [v]_R| = |[v]_R| \\
&\Leftrightarrow [v]_R| \subseteq W \\
&\Leftrightarrow v \in R_*(W) \\
\delta_W^R(v_i, v_j) = 1 &\Leftrightarrow min\{\mu_W^R(v_i), \mu_W^R(v_j)\} = 1 \\
&\Leftrightarrow \mu_W^R(v_i) = 1 \; \& \; \mu_W^R(v_j) = 1 \\
&\Leftrightarrow v_i \in R_*(W) \; \& \; v_j \in R_*(W) \\
&\Leftrightarrow (v_i, v_j) \in R_*(X)
\end{aligned}$$

2.

$$\begin{aligned}
\mu_W^R(v) = 0 &\Leftrightarrow \frac{|W \cap [v]_R|}{|[v]_R|} = 0 \\
&\Leftrightarrow |W \cap [v]_R| = 0 \\
&\Leftrightarrow W \cap [v]_R = \phi \\
&\Leftrightarrow [v]_R| \subseteq V - W \\
&\Leftrightarrow v \in V - R^*(W) \\
\delta_W^R(v_i, v_j) = 0 &\Leftrightarrow min\{\mu_W^R(v_i), \mu_W^R(v_j)\} = 0 \\
&\Leftrightarrow \mu_W^R(v_i) = 0 \; or \; \mu_W^R(v_j) = 0 \\
&\Leftrightarrow v_i \in V - R^*(W) \; or \; v_j \in V - R^*(W) \\
&\Leftrightarrow (v_i, v_j) \in E - R^*(X)
\end{aligned}$$

3.

$$\begin{aligned}
\mu_{V-W}^R(v) &= \frac{|(V - W) \cap [v]_R|}{|[v]_R|} \\
&= 1 - \frac{|W \cap [v]_R|}{|[v]_R|} \\
&= 1 - \mu_W^R(v) \\
\delta_{V-W}^R(v_i, v_j) &= min\{\mu_{V-W}^R(v_i), \mu_{V-W}^R(v_j)\} \\
&= min\{1 - \mu_W^R(v_i), 1 - \mu_W^R(v_j)\} \\
&\leq 1 - min\{\mu_W^R(v_i), \mu_W^R(v_j)\} \\
&\leq 1 - \delta_W^R(v)
\end{aligned}$$

4.

$$\begin{aligned}
\mu_{W_1 \cup W_2}^R(v) &= \frac{|(W_1 \cup W_2) \cap [v]_R|}{[v]_R} \\
&= |W_1 \cap [v]_R| + |W_2 \cap [v]_R| \\
&\quad - |(W_1 \cap W_2) \cap [v]_R| \\
&= \mu_{W_1}^R(v) + \mu_{W_2}^R(v) - \mu_{W_1 \cap W_2}^R(v)
\end{aligned}$$

Next we extend the definition of Edge precision $\alpha_R(T)$ [18] of a edge rough graph to vertex rough graph:

Definition 4.3 A vertex rough graph $T = (R_*(T), R^*(T))$ where $R^*(T) = (R^*(W), R^*(X))$ and $R_*(T) = (R_*(W), R_*(X))$. $\alpha_R(T)$ is the R-vertex precision of T and $\beta_R(T)$ is the R-edge precision of T defined by $\alpha_R(T) = \dfrac{|R_*(W)|}{|R^*(W)|}$ and $\beta_R(T) = \dfrac{|R_*(X)|}{|R^*(X)|}$ where $W \neq \phi, X \neq \phi$.

Result 4.1 *Let M be the set of all vertex rough graphs. For any vertex attribute set W and edge attribute set X and $t \subseteq M$ then, $0 \leq \alpha_R(T) \leq 1$ & $0 \leq \beta_R(T) \leq 1$. If T is exact iff $\alpha_R(T) = 1$ & $\beta_R(T) = 1$.*

Proof Since $R_*(W) \subseteq R^*(W)$ and $R_*(X) \subseteq R^*(X)$.
Therefore, $0 \leq \alpha_R(T) \leq 1$ & $0 \leq \beta_R(T) \leq 1$.

$$\text{If T is exact} \Leftrightarrow R_*(W) = R^*(W) \ \& \ R_*(X) = R^*(X)$$
$$\Leftrightarrow \alpha_R(T) = 1 \ \& \ \beta_R(T) = 1$$

Result 4.2 *If T and S are two vertex rough graphs, where $T = (W_1, X_1)$ and $S = (W_2, X_2)$. S is a vertex rough subgraph of T, then $\alpha_R(S) \leq \alpha_R(T)$ and $\beta_R(S) \leq \beta_R(T)$.*

To compare two rough graphs, rough similarity degree [18] is an important measure. We can extend it to vertex rough graph.

Definition 4.4 Given vertex rough graph set M, attribute set R. $K = (M, R)$ is a knowledge system. Let $H, J \subseteq M$ where $H = (W_1, X)$, $J = (W_2, Y)$ and

$$R_*(H) = (R_*(W_1), R_*(X)),\ R^*(H) = (R^*(W_1), R^*(X))$$
$$R_*(J) = (R_*(W_2), R_*(Y)),\ R^*(J) = (R^*(W_2), R^*(Y)).$$

1. Rough vertex similarity degree ($\langle H, J\rangle_R$) and rough edge similarity degree($[H, J]_R$) between H and J are defined by
$$\langle H, J\rangle_R = \min\left\{\frac{|R_*(W_1)\cap R_*(W_2)|}{|R_*(W_1)\cup R_*(W_2)|}, \frac{|R^*(W_1)\cap R^*(W_2)|}{|R^*(W_1)\cup R^*(W_2)|}\right\}$$
$$[H, J]_R = \min\left\{\frac{|R_*(X)\cap R_*(Y)|}{|R_*(X)\cup R_*(Y)|}, \frac{|R^*(X)\cap R^*(Y)|}{|R^*(X)\cup R^*(Y)|}\right\}.$$
2. Lower rough vertex similarity degree($\langle H, J\rangle_{R_*}$) and lower rough edge similarity degree($[H, J]_{R_*}$) between H and J are defined by $\langle H, J\rangle_{R_*} = \dfrac{|R_*(W_1)\cap R_*(W_2)|}{|R_*(W_1)\cup R_*(W_2)|}$ and $[H, J]_{R_*} = \dfrac{|R_*(X)\cap R_*(Y)|}{|R_*(X)\cup R_*(Y)|}$
3. Upper rough vertex similarity degree ($< H, J >_{R^*}$) and upper rough edge similarity degree ($[H, J]_{R^*}$) between H and J are defined by $\langle H, J\rangle_{R^*} = \dfrac{|R^*(W_1)\cap R^*(W_2)|}{|R^*(W_1)\cup R^*(W_2)|}$ and $[H, J]_{R^*} = \dfrac{|R^*(X)\cap R^*(Y)|}{|R^*(X)\cup R^*(Y)|}$

Proposition 4.2 *Given vertex rough graph set M, R to be the attribute set. $K = (M, R)$ to be the knowledge system. $H, J \subseteq M$. Then*

1. *H and J are R-rough equal iff $\langle H, J\rangle_R = [H, J]_R = 1$*
2. *H and J are R-lower rough equal iff $\langle H, J\rangle_{R_*} = [H, J]_{R_*} = 1$*
3. *H and J are R-upper rough equal iff $\langle H, J\rangle_{R^*} = [H, J]_{R^*} = 1$*

Proof 1.

$$\langle H, J\rangle_R = [H, J]_R = 1$$
$$\Leftrightarrow \min\left\{\frac{|R_*(W_1)\cap R_*(W_2)|}{|R_*(W_1)\cup R_*(W_2)|}, \frac{|R^*(W_1)\cap R^*(W_2)|}{|R^*(W_1)\cup R^*(W_2)|}\right\} = 1,$$
$$\min\left\{\frac{|R_*(X)\cap R_*(Y)|}{|R_*(X)\cup R_*(Y)|}, \frac{|R^*(X)\cap R^*(Y)|}{|R^*(X)\cup R^*(Y)|}\right\} = 1$$
$$\Leftrightarrow \frac{|R_*(W_1)\cap R_*(W_2)|}{|R_*(W_1)\cup R_*(W_2)|} = 1 \ \& \ \frac{|R^*(W_1)\cap R^*(W_2)|}{|R^*(W_1)\cup R^*(W_2)|} = 1,$$
$$\frac{|R_*(X)\cap R_*(Y)|}{|R_*(X)\cup R_*(Y)|} = 1 \ \& \ \frac{|R^*(X)\cap R^*(Y)|}{|R^*(X)\cup R^*(Y)|} = 1$$
$$\Leftrightarrow |R_*(W_1)\cap R_*(W_2)| = |R_*(W_1)\cup R_*(W_2)| \ \&$$
$$|R^*(W_1)\cap R^*(W_2)| = |R^*(W_1)\cup R^*(W_2)|,$$
$$|R_*(X)\cap R_*(Y)| = |R_*(X)\cup R_*(Y)| \ \&$$
$$|R^*(X)\cap R^*(Y)| = |R^*(X)\cup R^*(Y)|$$
$$\Leftrightarrow R_*(W_1) = R_*(W_2) \ \& \ R^*(W_1) = R^*(W_2),$$
$$R_*(X) = R_*(Y) \ \& \ R^*(X)\cap R^*(Y)$$
$$\Leftrightarrow H \ \& \ J \ are \ R-rough \ equal.$$

2.

$$\langle H, J\rangle_{R_*} = [H, J]_{R_*} = 1$$
$$\Leftrightarrow \frac{|R_*(W_1)\cap R_*(W_2)|}{|R_*(W_1)\cup R_*(W_2)|} = 1 \ \& \ \frac{|R_*(X)\cap R_*(Y)|}{|R_*(X)\cup R(Y)_*|} = 1$$
$$\Leftrightarrow |R_*(W_1)\cap R_*(W_2)| = |R_*(W_1)\cup R_*(W_2)| \ \&$$
$$|R_*(X)\cap R_*(Y)| = |R_*(X)\cup R_*(Y)|$$
$$\Leftrightarrow R_*(W_1) = R_*(W_2) \ \& \ R_*(X) = R_*(Y)$$
$$\Leftrightarrow H \text{ and } J \text{ are } R\text{-lower rough equal}$$

3.

$$\langle H, J\rangle_{R^*} = [H, J]_{R^*} = 1$$
$$\Leftrightarrow \frac{|R^*(W_1)\cap R^*(W_2)|}{|R^*(W_1)\cup R^*(W_2)|} = 1 \ \& \ \frac{|R^*(X)\cap R^*(Y)|}{|R^*(X)\cup R^*(Y)|} = 1$$
$$\Leftrightarrow |R^*(W_1)\cap R^*(W_2)| = |R^*(W_1)\cup R^*(W_2)| \ \&$$
$$|R^*(X)\cap R^*(Y)| = |R^*(X)\cup R^*(Y)|$$
$$\Leftrightarrow R^*(W_1) = R^*(W_2) \ \& \ R^*(X) = R^*(Y)$$
$$\Leftrightarrow H \text{ and } J \text{ are } R\text{-upper rough equal}$$

□

Conclusion

Both the Rough set theory and the graph theory have a variety of applications across different fields. This paper introduced the concept of vertex rough graph which combines rough set theory and the graph theory. Similar to rough set theory, the notion of vertex and edge rough membership function is introduced and using this membership functions, an alternative definition of vertex rough graph has been developed. Later, vertex precision and edge precision are defined and some properties are discussed. Since edge rough graph has lot of applications in various fields, like relationship analysis, data mining, etc., the vertex rough graphs also will have applications in these fields as well as many other fields. In future we will find out further rough properties and applications of vertex rough graphs.

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