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RESEARCH PAPER

# Increased accuracy of calculated fatigue resistance of welds through consideration of the statistical size effect within the notch stress concept

Andreas Deinböck[1] · Ann-Christin Hesse[2] · Michael Wächter[1] · Jonas Hensel[2] · Alfons Esderts[1] · Klaus Dilger[2]

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## Abstract

The notch stress concept has been established for fatigue life calculations of welded components. One of its advantages is that the *S-N* curve is not based on an *S-N* curve catalog in which the user has to identify a suitable FAT class; instead, a single *S-N* curve (based on modeling, failure location, and material) is used for different weld geometries. In return, however, the weld needs to be modeled in a detailed manner for finite element analysis. The evaluation of experimental fatigue results collected in a database shows a relatively high degree of scattering (low accuracy) of the strengths calculated according to the notch stress concept. With fatigue tests on different specimen geometries manufactured from the same welded base plates, a correlation between the highly stressed weld seam length and the experimentally determined notch stress strength can be observed. The fatigue strength decreases with the increasing length of the highly stressed weld seam area. The latter quantity can be calculated using the finite element simulations that are needed to determine the notch stress. The presented results are used to describe the statistical size effect as a qualitative influence and quantitative support factor that can be used within the notch stress concept to increase its accuracy.

**Keywords** Fatigue design · Welding · Steel · Statistical size effect · Notch stress concept · Highly stressed weld seam length · Support factor

## Nomenclature

| | |
|---|---|
| $A$ | Surface area |
| $A_{90\%}$ | Highly stressed surface area |
| $c$ | Proportionality factor |
| $F$ | External force |
| $\mathrm{FAT}_X$ | Classification reference to *S-N* curve, in which $x$ is the stress range in MPa at $2 \cdot 10^6$ cycles, according to IIW [1] |
| $\Delta F$ | Force range |
| $\Delta F(N)$ | Force range for a certain number of cycles $N$ |
| $m$ | Exponent of *S-N* curve |
| $K_t$ | Stress concentration factor |
| $k_{st}$ | Weibull's exponent |
| $L_{ref}$ | Reference length |
| $L_{90\%}$ | Highly stressed length |
| $M_\sigma$ | Mean stress sensitivity |
| $N$ | Fatigue life in numbers of cycles |
| $n_{st}$ | Statistical support factor |
| $t$ | Plate thickness |
| $V$ | Volume |
| $V_{90\%}$ | Highly stressed volume |
| $\sigma$ | Notch stress |
| $S$ | Nominal stress |
| $\Delta\sigma$ | Notch stress range |
| $\Delta\sigma_0$ | Modified notch stress range assessed with the statistical size effect |
| $\Delta\sigma(N)$ | Notch stress range for a certain number of cycles $N$ |
| $P_f$ | Probability of failure |
| $R$ | Stress ratio |
| $T_N$ | Deviation range in direction of fatigue life |

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✉ Andreas Deinböck
andreas.deinboeck@tu-clausthal.de

[1] Institute for Plant Engineering and Fatigue Analysis, Clausthal University of Technology, Leibnizstraße 32, 38678 Clausthal-Zellerfeld, Germany

[2] Institute of Joining and Welding, Technische Universität Braunschweig, Langer Kamp 8, 38106 Braunschweig, Germany

## 1 Introduction

When calculating the fatigue strength of welded joints, usually one of three (stress based) calculation concepts is used: the nominal stress concept, the structural stress concept, or the notch stress concept. Hobbacher [1] and Radaj et al. [2] give overviews of these different concepts.

The permissible stresses specified by regulations and guidelines [3, 4] are predominantly based on results from fatigue tests on relatively small specimens (with weld lengths of 100 mm or less), e.g., [5].

Two aspects of this approach can be criticized. On the one hand, real components can feature welding seams with lengths of several meters or even hundreds of meters. The designed element thus differs from the abovementioned specimens by several orders of magnitude, and this difference is usually not taken into account. On the other hand, when deriving strength parameters from the abovementioned weld specimens, an approximately homogeneous stress distribution along the weld is generally assumed. Weld seams in real components, on the other hand, are often not subjected to homogeneous stresses due to different loading types and stiffness changes along the seam (in response to ribs, tubes, angles, etc.) and instead show local stress peaks. Thus, the length of the area that is actually highly stressed—and therefore may be critical—may differ significantly from the overall seam length of the component.

To describe the dependency of strength on the weld seam length, accounting for the statistical size effect based on the weakest-link model, as proposed by Weibull [6], is almost inevitable, even if other size influences and support factors cannot be ignored.

Within the concepts of nominal and structural stress, *S*-*N* curves are defined by geometry-dependent fatigue classes (FAT classes), and some component-specific peculiarities are therefore already taken into account. This is not the case for the notch stress concept, in which just a single fatigue class is used. For this reason, the focus in this contribution is on the notch stress concept, even if the discussed principles can be transferred to other concepts as well.

The statistical size effect is widely accepted for nonwelded components and is now included in standards for fatigue assessments, e.g., the FKM guideline [4]. As early as 1998, Haibach and Seeger pointed out that the weakest-link model, which explains the statistical size effect, had not yet been applied to welds at that time, although it was particularly suitable for such welds in a one-dimensional form, [7]. To date, this situation has not changed fundamentally, even though it has been successfully applied to welds in individual cases, as discussed in Section 2.

Therefore, the present paper outlines a procedure to take the statistical size effect of weld seams into account. The procedure can be used for fatigue strength assessments with the notch stress concept. As shown below, in addition to a more precise statement of the fatigue strength in individual cases, this can also improve the overall accuracy of the concept.

To achieve this goal, two strategies are pursued, which are discussed below.

First, fatigue tests are carried out on welded specimens, which are designed in such a way that only the highly stressed weld seam length varies and other influences are largely excluded. Based on these tests, the existence of a statistical weld seam size effect is proven.

In a second step, a large data set of fatigue tests from the literature is gathered and evaluated in order to determine the effects on the notch stress concept when the statistical size effect is taken into account.

## 2 Background

### 2.1 Notch stress concept

The concept of effective notch stress (hereinafter abbreviated to notch stress concept) enables the evaluation of the fatigue strength of complex components. It has obvious advantages for components that do not allow the definition of a nominal cross section and for which nominal stresses therefore cannot be used in calculations.

For the assessment of effective notch stresses, the weld seam in question is modeled for a linear-elastic finite element simulation. In this simulation, both the weld toe and the weld root are replaced by an effective notch root radius $r$. According to various studies, e.g., [1, 2, 8], $r$ equals 1 mm for plate thicknesses $t \geq 5$ mm. The maximum stress concentration factor ($K_t$) determined at the rounded weld foot or root is used to calculate the notch stress, which can be either maximum principal stress (concerning the amount) or a von Mises equivalent stress. In this paper, only the maximum principal stress (concerning the amount) stress is investigated.

The notch stress range determined in this manner is then compared with an *S*-*N* curve, which depends on the used material (steel or aluminum) and the selected notch root radius $r$. For the evaluation of steel components with thicknesses $t \geq$ 5 mm in combination with the principal stresses of the fatigue class $FAT_{225}$ (meaning, a permissible stress range of $\Delta\sigma =$ 225 MPa at $2 \cdot 10^6$ cycles in case of constant amplitude loading), a slope parameter of $m = 3$ is recommended by [1]. This *S*-*N* curve is valid up to $1 \cdot 10^7$ cycles. From there on, it continues with a lower inclination of $m = 22$. The *S*-*N* curve obtained by these parameters is assigned a probability of failure of $P_f = 2.3\%$.

### 2.2 Statistical size effect for nonwelded components

The statistical size effect takes into account the fact that the size of the highly stressed region around the location of a component's maximum stress has an effect on the local fatigue strength. This can be explained on the basis of Weibull's weakest-link model [9]. It is assumed that the failure of a component is based on its largest flaw whose probability of occurrence is greater in a large highly stressed region than in a small one.

The experimental verification in the field of fatigue strength was carried out by Heckel et al. [10–13].

Two possibilities have been established to measure the size of a highly stressed region: The stress integral over surface/volume [12, 14–16] or the highly stressed surface/volume according to Kuguel [17] or Sonsino [18]. The conversion between two components is done in both cases with a power function (Eq. (1)). Hereby the relationship of fatigue strength between two unnotched specimens of different sizes with their highly stressed surface areas A, or volume V, can be described using the Weibull parameter $k_{st}$. In recent studies, multiaxial applications have been investigated using both the surface and volume approaches [19].

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{A_2}{A_1}\right)^{-\frac{1}{k_{st}}} \text{ or } \frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{-\frac{1}{k_{st}}} \quad (1)$$

In this paper, the principle of the highly stressed region according to Sonsino is used. It describes the region as the surface area that is subjected to a stress greater than a certain fraction of the maximum stress, e.g., 90% [18]. For clarification, the fractional part is added as an index, e.g., $A_{90\%}$, for a highly stressed surface area in which all surface parts with the maximum principal stress (concerning the amount) higher than or equal to 90% of the maximum stress are added up.

At present, the statistical size effect is actually considered in fatigue life assessments. The FKM guideline [3], for example, uses the statistical support factor $n_{st}$ to consider the highly stressed surface area $A_{\sigma,\,st}$ to adjust the endurance limit $\sigma_E$ (in combination with other factors), as described in Eq. (2) and thereby the whole *S*-*N* curve.

$$\frac{\sigma E,\text{component}}{\sigma E,\text{material}} = n_{st} = \left(\frac{500\ \text{mm}^2}{A_{\sigma,st}}\right)^{-\frac{1}{k_{st}}} \quad (2)$$

Here, $k_{st}$ is a material group-specific value for the Weibull exponent.

## 2.3 Statistical size effect of welds

At present, multiple investigations have been carried out on the statistical size effect of weld seams. Kreuzer [20] conducted tests on steel sheets with different weld seam lengths. The weld seam length and the number of welds per specimen are varied. It is possible to detect a decrease in fatigue strength as a function of the weld seam length.

Sonsino [21] explains the different fatigue strengths of welded pipes with the highest stressed volume. He modeled the weld toes with mean values of measured weld toe radii and detected a decrease in fatigue strength with an increase in the highly stressed volume.

With a similar approach, an exemplary improvement of the fatigue life estimation of welded sintered components in the notch stress concept has been achieved by Waterkotte et al. [22].

Störzel et al. [23] use the line integral $I_L$ of the stress along the weld seam to take into account a highly stressed weld seam length for thin sheets ($t \leq 5$ mm). In other investigations on thin sheets by Bruder et al. [24] and Baumgartner et al. [25], it is found that both the highly stressed volume and the highly stressed length can be used for the consideration of the statistical size effect, and the same Weibull's exponent is applicable in both studies. Fricke et al. [26] applied the latter approach to extremely small highly stressed areas in order to use the notch stress approach. Here, the statistical size effect can explain that the failure critical location determined within finite element simulations is not identical to the failure location of the component in fatigue tests. Kaffenberger [27] presents a concept that combines Neuber's micro-support effect and the statistical size effect as a surface integral.

In all these investigations on statistical size effects in welds, various methods are used to evaluate the characteristic highly stressed region: highly stressed volumes, surfaces, and weld seam lengths, each using stress integral approaches or 90% of the maximum stress as the threshold value. These investigations have resulted in different values for the Weibull exponents of welded components, as listed in Table 1.

## 2.4 Present status

An examination of the literature reveals that a statistical size effect can be found not only in nonwelded components but also in welded ones. To consider the statistical size effect within a fatigue assessment and to determine a characteristic value for a highly stressed region in a component, a local approach (e.g., the notch stress approach) is required.

**Table 1** Compilation of the Weibull exponents for welds from the literature

| Source | Exponent $k_{st}$ |
|---|---|
| Waterkotte et al. [22], Baumgartner et al. [25] | 8.3 |
| Kaffenberger et al. [27] | 9.0 |
| Bruder et al. [24] | 11.5 |
| Störzel et al. [23] | 16.6 |
| Sonsino [21] | 22.2 |

By using a uniform reference radius within the notch stress approach, the stress fields perpendicular to the weld seam within the critical regions of weld toes and roots are assigned similar characteristics, making them comparable. In this case, it is irrelevant whether a volume, a surface, or a length is used to evaluate the statistical size effect for welds. The exponent of the power function must be very similar in all cases. This is supported by Bruder et al. [24] and Baumgartner et al. [25].

Therefore, it must be adequate to use the highly stressed weld seam length instead of the highly stressed volume or the highly stressed surface as suggested in [23]. This also reflects the line-like nature of the weld seam, and similar to the highly stressed volume or the highly stressed surface, the method requires postprocessing within finite element simulations, which is why the authors favor the highly stressed weld seam length $L_{90\%}$.

Furthermore, it is assumed that the power function, as in Eq. (1), is applicable for the conversion between two different large highly stressed regions.

It should be possible to consider the statistical size effect, similar to Eq. (2), by defining the statistical size factor $n_{st}$ according to Eq. (3), which takes the highly stressed weld seam length into account.

$$n_{st} = \left(\frac{L_{ref}}{L_{90\%}}\right)^{-\frac{1}{k_{st}}} \tag{3}$$

Here, $L_{90\%}$ is the highly stressed weld seam length determined within the finite element simulation of the component in question, $L_{ref}$ is the position of the function at $n_{st} = 1$, and $k_{st}$ is the Weibull exponent that defines the slope of the function that appears as a straight line in a double logarithmic scale, as shown in Fig. 1.

The range of the Weibull exponents in the literature (cf. Section 2.3) and the absence of a suitable position parameter $L_{ref}$ for Eq. (3) in a standardized calculation concept, such as the notch stress concept according to [1] have so far prevented the broad consideration of the statistical size effect of welds.

## 3 Fatigue tests with specimens of differently large highly stressed weld seam lengths

To obtain verified values for the determination of the required Weibull exponent, fatigue tests have been carried out for this study. This ensures that all specimens and test parameters are known. In this section, the design, execution, and evaluation of the fatigue tests on specimens with different highly stressed weld seam lengths are shown and discussed.

### 3.1 Specimen design

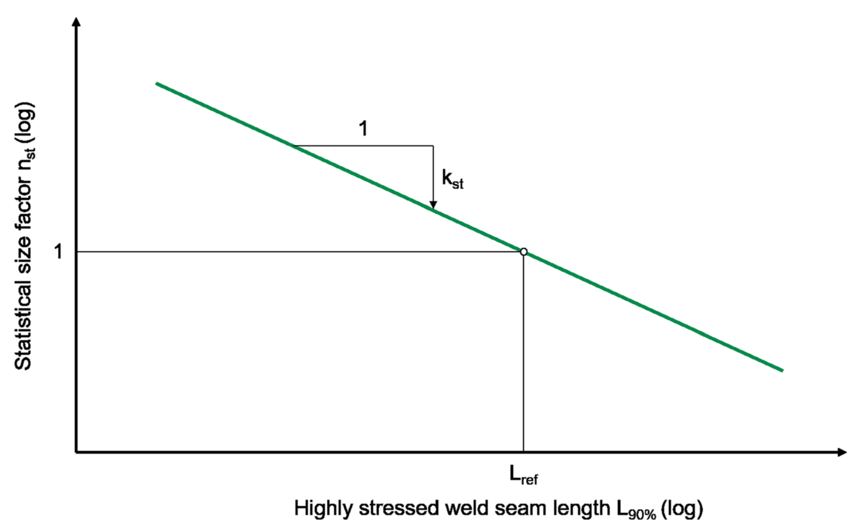
Fatigue tests have been carried out on welded specimens that were designed in such a way that only the highly stressed weld seam length varies in order to exclude other influences. Based on these tests, the statistical size effect of weld seams is verified.

The aim of this section is to explain how to design different fatigue specimens that can be manufactured from the same stack of welded raw material. The highly stressed length varies among the different specimen geometries, but other influences (base material, weld geometry, stress gradients, etc.) are kept constant.

To achieve this, only the area where the load is introduced to the weld seam is changed; the weld geometry itself (full or partial penetration cross joints) and the loading type (tension/compression) are kept the same. This leads to the different specimen geometries shown in Fig. 2 as an example of the variations in the full-penetration cross joints.

All manufactured specimens have a plate thickness of $t$ = 10 mm and vary in length of weld seam between 60 and 500 mm.

Fig. 1 Statistical size factor according to Eq. (3)



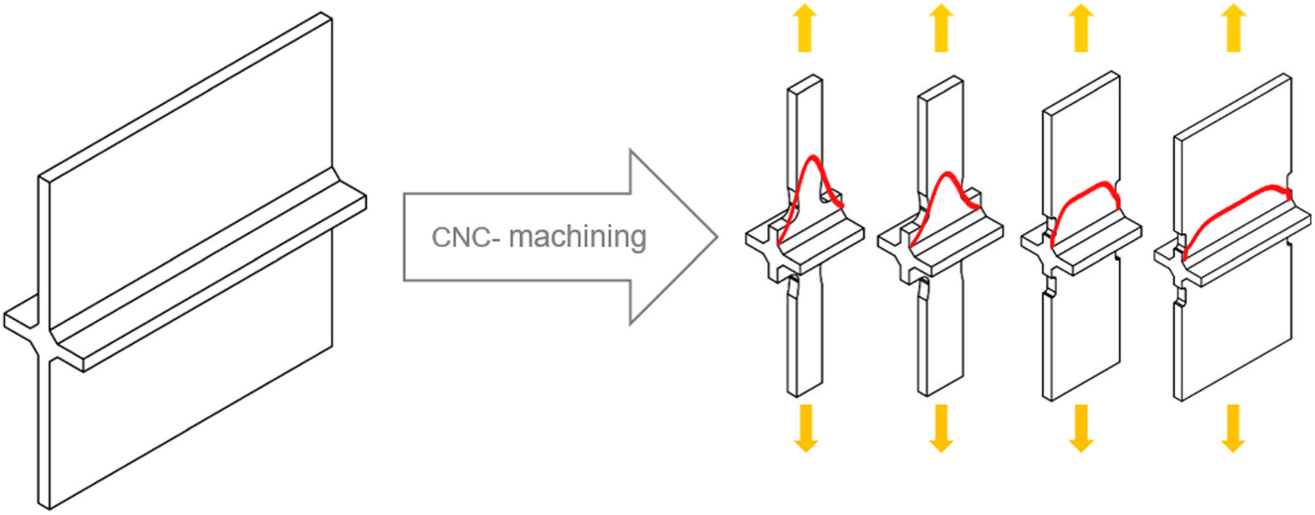



**Fig. 2** Raw material with uniformly welded joints (left side) from which different specimens can be machined that show varying stress distributions along the weld seam and therefore varying highly stressed lengths (schematic, right side)

According to the IIW recommendation, it is required that a plane-strain state is present at the critical point to be able to apply the notch stress concept [3]. Therefore, the strain state at the critical point of all specimens has been examined within the finite element models. All geometries fulfill this condition to a sufficient degree.

As seen in Fig. 2, the designed geometries result in different stress distributions along the weld seams, as shown schematically by the red curves. Applying the 90% criteria, different highly stresses weld seam lengths $L_{90\%}$ are achieved.

The stress distributions in Fig. 2 are determined by evaluating the stresses within the finite element simulation for each geometry (Fig. 3):

1. The specimen is modeled within the finite element program according to Hobbacher [1], Rennert et al. [4], and Merkblatt DVS 0905 [8]. For symmetry reasons, partial models may be used, as shown in Fig. 3b.
2. For the examined partial penetration cross joint with fillet welds, the focus is on the root notch (Fig. 3a); for the same geometry but with full penetration, the focus is on would be the weld toe. However, depending on the specimen geometry, the stress maximum can also shift from the root to the weld toe both in the FE simulation and in the experiments. Therefore, a general statement of the failure location, depending on the weld penetration, is not possible.
3. Stresses (greatest principal stress in terms of amount) are evaluated on a path along the failure critical seam area. The path is set through the occurring point of maximum notch stress. This is shown as an example in Fig. 3e for a partial penetration cross joint geometry with the stress maximum in the weld root.
4. $L_{90\%}$ is determined with this stress distribution, as shown schematically in Fig. 4, by normalizing the stress along the evaluation path to its stress maximum and applying the 90% criterion.

The values for $L_{90\%}$ that are achieved with the shown fatigue specimens range from 31 and 503 mm.

The tests focus on cross joints with fillet welds, with both full and partial penetration, and butt welds, all of which are made from plates with a thickness of $t = 10$ mm. Two steel grades, S355NL and S690QL, are tested with similar specimen geometries, as shown in Fig. 2.

The specimens are taken from the same batches of robot-welded blanks that were postweld heat treated below the lower critical temperature to achieve low residual stresses. The weld seam quality is evaluated according to ISO 5817 [28], and the results on average achieve quality grade B. For modeling and evaluation of the finite element analysis, the average weld seam geometry and the axial and angular misalignment from scans of each individual specimen are used. For the modeling of the root gap of the cross joints, the actual penetration depth is determined after the fatigue tests.

Throughout the literature, often nominal stresses $S$ and stress concentration factors $K_t$ are used to describe the connection between loads and notch stresses $\sigma$ for one single component (Eq. (4)). Here, loads are described by nominal stresses, which requires a reasonable definition of a nominal cross section.

$$K_t = \frac{\sigma}{S} \tag{4}$$

The definition of such a nominal cross section, however, may be difficult for complex components or load situations.

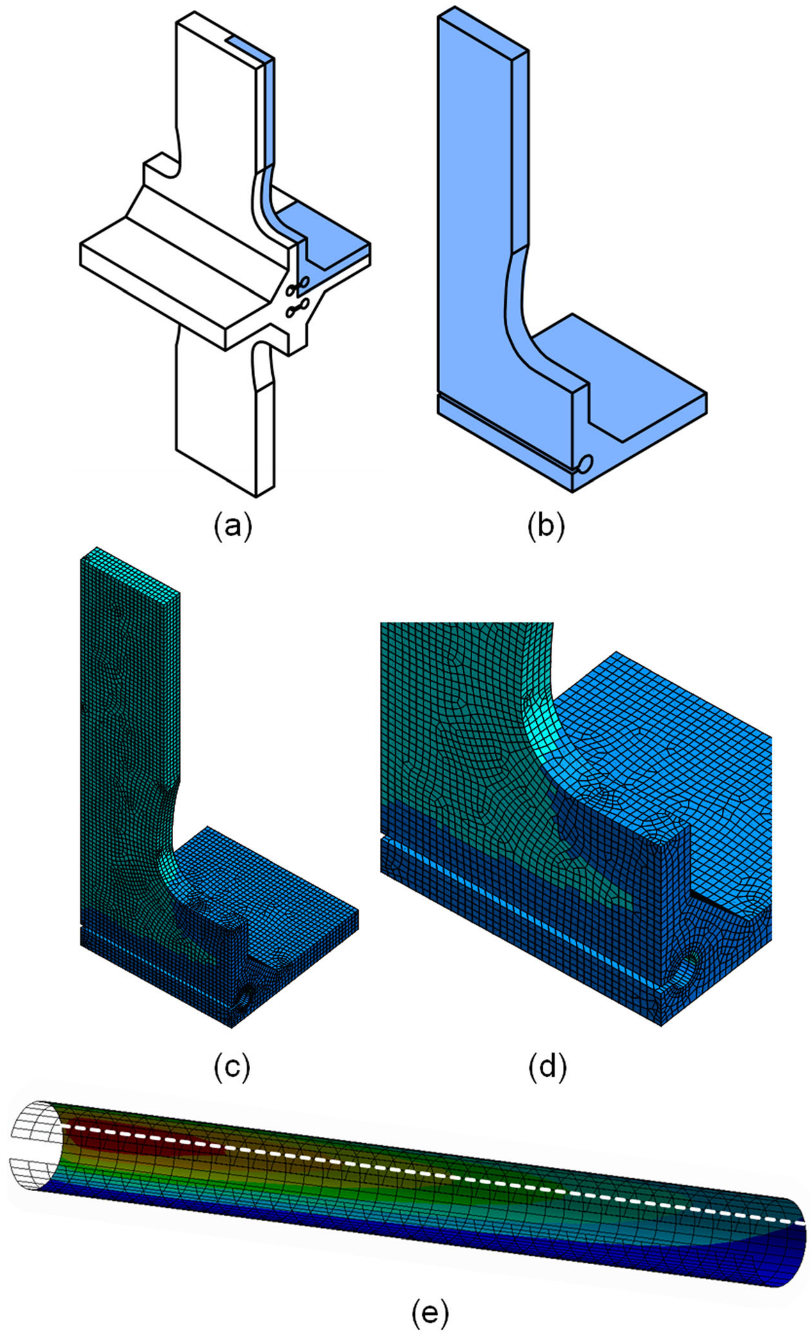



**Fig. 3** Schematic procedure of the simulation of a partial penetration cross joint to determine $L_{90\%}$. **a** Illustration of the full model. **b** Illustration of the eighth model. **c** Finite element model. **d** enlargement of the area of interest. **e** Stress evaluation of the root notch with evaluation path

The specimen geometries may be a good example for such hard to define nominal cross sections since the stress distribution within the base plate in front of the weld seam differs significantly from the one within the seam itself. For cases like these, it may be more unambiguous to use loads $F$ and proportionality factors $c$ to describe the dependency between loads and notch stresses Eq. (5).

$$c = \frac{\sigma}{F} \tag{5}$$

### 3.2 Test execution and evaluation

The constant amplitude fatigue tests are carried out according to German standard DIN 50100 [29], with a constant stress ratio of $R = 0.1$ ((Eq. (6)).

$$R = \frac{F_{\min}}{F_{\max}} \tag{6}$$

The specimens are tested to fracture or a load cycle number of $5{\cdot}10^6$, and the number of cycles performed within the tests ranges

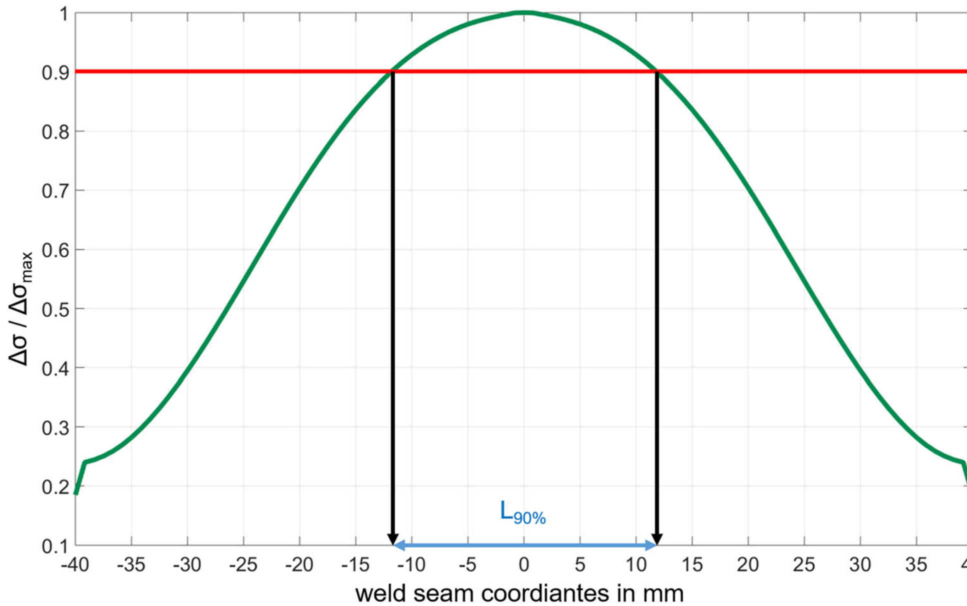



**Fig. 4** Determination of $L_{90\%}$ by using the 90% criteria

from approximately $5{\cdot}10^4$ to $5{\cdot}10^6$ cycles. The tested forces are then converted into notch stresses according to Eq. (4). All tests shown in Fig. 5 have been mean stress corrected according to Fricke [3] to a stress ratio of $R = 0.5$ with a mean stress sensitivity $M_\sigma = 0.3$. Furthermore, the *S-N* curves for $P_f = 50\%$ (green) and $P_f = 2.3\%$ (black) are shown in Fig. 5. A fixed slope of $m = 3$ has been used for both, based on Hobbacher [1].

## 4 Evaluation of the influence of the highly stressed weld seam length

To show the dependency between the experimental fatigue strength and the highly stressed length, both values are compared with each other. Therefore, one reference strength from each *S-N* curve needs to be chosen. For this purpose, the fatigue strength at the mean number of cycles for all test results ($N = 7{\cdot}10^5$) is chosen, as shown in Fig. 6a. Figure 6 shows all 19 test series used, separated by color, with the corresponding individual test results and the *S-N* curve.

The fatigue strength of each *S-N* curve is tabulated in Table 2. In addition, the highly stressed weld seam length $L_{90\%}$ and the fatigue strength for every single *S-N* curve are also listed in Table 2. The data pairs of $L_{90\%}$ and $\Delta\sigma$ ($N = 7{\cdot}10^5$) are plotted in Fig. 6a. For a better optical comparison, the same *S-N* curves are already shown in Fig. 6b, after application of the statistical support factor. By comparing Fig. 6 a and b, the reduction of the scatter of the test series can already be seen.

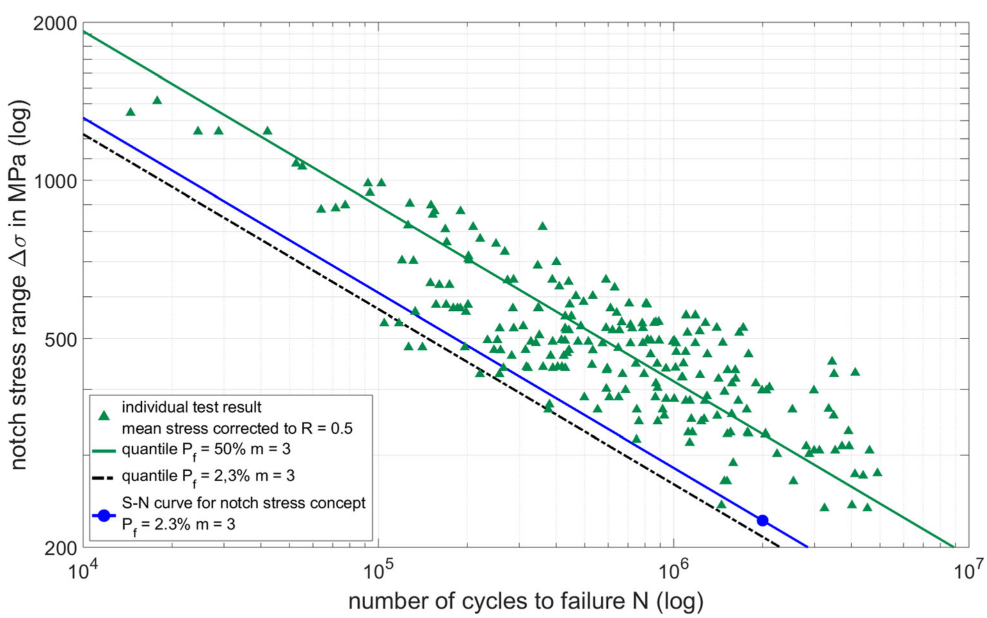



**Fig. 5** Individual test results on a notch *S-N* plot, with mean stress corrected to the stress ratio $R =$ 0.5

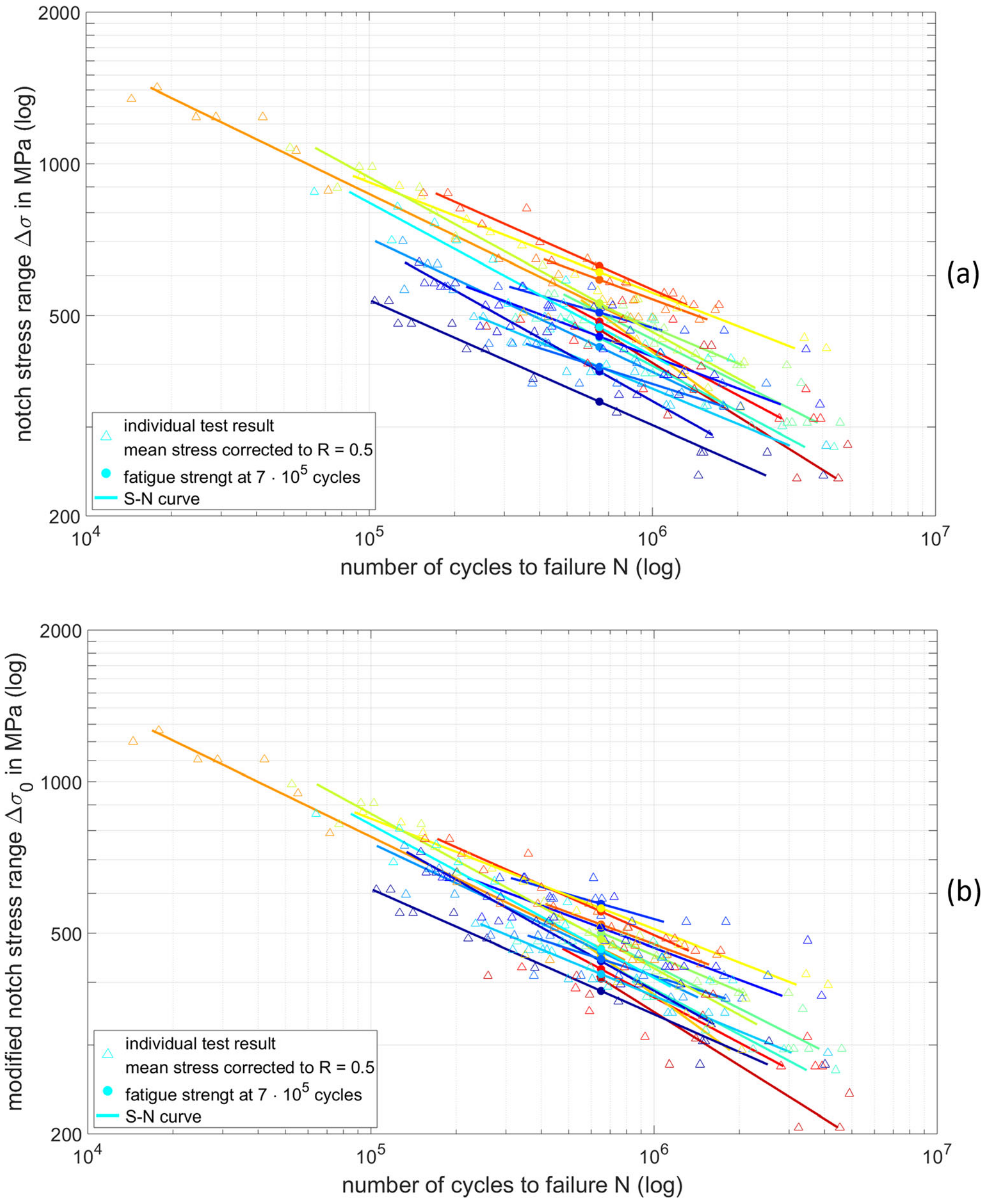



**Fig. 6** Individual test series with S-N curves and the corresponding evaluation points at $7{\cdot}10^5$ cycles. **a** According to the notch stress concept. **b** According to the notch stress concept with the statistical support factor applied

It can be noted:

1. A dependency between $\Delta\sigma_{\mathrm{Median}}$ and $L_{90\%}$ can be seen for all five-test series. With increasing length of the highly stressed weld seam length, the notch stress range decreases.
2. The Weibull exponents determined with these data sets are between 6 and 11.
3. The statistical size effect does not seem to be the only influence present in the data. The batches show a notable distance between each other. Influencing factors can be different base materials and in some cases different joints. Furthermore, the welding differs between the batches in the number of welding layers used. The notch stress concept assumes that there is no influence of these aspects on the local strength, but a similar deviation range can be seen in the data evaluated by Pedersen et al. [30].
4. The average Weibull exponent of $k_{\mathrm{st}} = 9.2$, $x \approx 9$ (magenta) seems to be suitable to describe the tendency of the whole set of data.

## 5 Database with fatigue tests from the literature

To be able to assess the influence of the statistical size effect on the accuracy of the notch stress concept, a large database with test results from the literature is compiled.

**Table 2** Different test series $L_{90\%}$ and converted effective notch stress $\Delta\sigma$ ($N = 7\cdot10^5$)

| Test series | Highly stressed weld seam length $L_{90\%}$ in mm | Effective fatigue strength based on notch stresses $\Delta\sigma$ ($N = 7\cdot10^5$) in MPa |
|---|---|---|
| Batch 1: S355NL cross joint partial penetration | 34.8 | 486 |
| | 43.4 | 523 |
| | 92.8 | 507 |
| | 246.4 | 433 |
| | 493.1 | 387 |
| Batch 2: S690QL cross joint partial penetration | 32.4 | 469 |
| | 43.4 | 505 |
| | 105.6 | 452 |
| | 220.1 | 395 |
| | 502.9 | 337 |
| Batch 3: S355NL cross joint full penetration | 38.0 | 627 |
| | 58.0 | 610 |
| | 451.4 | 507 |
| Batch 4: S690QL cross joint full penetration | 39.0 | 589 |
| | 58.0 | 529 |
| | 453.4 | 454 |
| Batch 5: S355NL butt joint | 86.0 | 524 |
| | 110.0 | 474 |
| | 443.0 | 395 |

Therefore, fatigue test data have been digitalized from the following sources: [31–39].

Overall the database contains 223 *S-N* curves with 2265 individual test results. Below, only the data that meet the following requirements is considered:

- The base material is steel
- The specimen material was gas metal arc welded
- The sheet thickness of the base material is ≥ 10 mm
- Individual test results are documented
- Enough information about the specimen geometry is given to generate a detailed finite element model according to Merkblatt DVS 0905 [8]
- Test series with failure at the end of the weld seam are not considered

With these criteria, 147 test series with 1097 individual tests remained for evaluation.

For each specimen geometry, a detailed finite element model is generated, and the test results are converted from load or nominal stresses to notch stresses.

The scatter band of the individual test results from this database is shown in Fig. 7. The mean stresses of all shown tests have been evaluated according to Fricke [3] to a stress ratio of $R = 0.5$ with a mean stress sensitivity $M_\sigma = 0.3$.

In addition to the individual test results, Fig. 8 also shows the total averaged *S-N* curve for a fixed slope of $m = 3$ ($P_f =$ 50% green) and the *S-N* curve for the notch stress approach ($P_f = 2.3\%$ black) according to Hobbacher [1]. The quantile for a failure probability of 2.3% of the individual test results is shown in black dotted lines. The deviation range of the test results in the direction of the number of cycles is $T_N = 9.65$.

$$T_N = \frac{\text{quantile}\ (P_f = 90\%)}{\text{quantile}\ (P_f = 10\%)} \tag{7}$$

It can be recognized that $FAT_{225}$ can be reproduced well with the given database and its quantile with a failure probability of 2.3%.

In addition to information on the notch stresses, the highly stressed weld seam length $L_{90\%}$ is determined using the finite element model for each individual test series.

To show the influence of the statistical size effect, the notch stress range $\Delta\sigma$ of each individual test result is shifted to $\Delta\sigma_0$ on the basis of its highly stressed length $L_{90\%}$ and the reference length of $L_{ref}$ using Eq. (8) and Eq. (9). This process yields new results for the notch stress strength, designated $\Delta\sigma_0$.

$$n_{st} = \left(\frac{L_{ref}}{L_{90\%}}\right)^{-\frac{1}{k_{st}}} = \left(\frac{135\ \text{mm}}{L_{90\%}}\right)^{-\frac{1}{9}} \tag{8}$$

$$\Delta\sigma_0 = n_{st} \cdot \Delta\sigma \tag{9}$$

The Weibull exponent of $k_{st} = 9$ leads to the shown reduction of the deviation range. However, the vertical shift of the

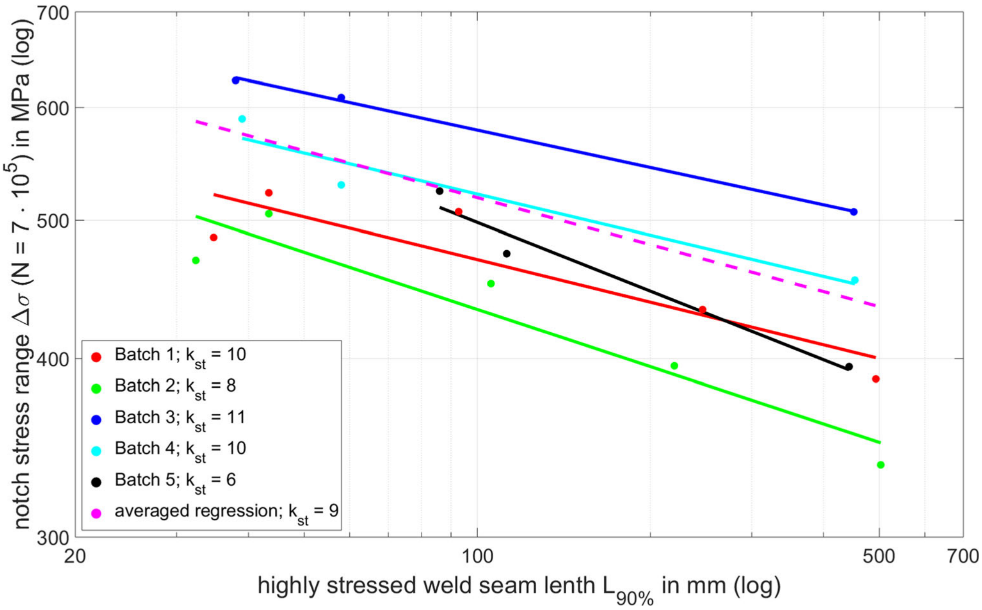



**Fig. 7** Dependence between the notch stress strength at $\Delta\sigma$ ($N = 7{\cdot}10^5$) and $L_{90\%}$

test results is determined by the reference length, which was chosen to $L_{\text{ref}} = 135$ mm. This length enables the *S-N* curve of the quantile of the failure probability of $P_{\text{f}} = 2.3\%$ to match the *S-N* curve for $\text{FAT}_{225}$ that is defined for the notch stress concept in [1]. Therefore, no new strength values need to be introduced if $n_{\text{st}}$ is considered within the concept. One hundred thirty-five millimeter also corresponds approximately to the mean value of the highly stressed weld seam lengths that occur throughout the geometries within the data base.

By plotting $\Delta\sigma_0$ over the number of cycles, the diagram in Fig. 9 is generated, and a reduced deviation range of $T_{\text{N}} = 8.61$ can be calculated. Furthermore, Fig. 9 shows that the valid FAT class can be maintained by applying the statistical size effect according to Eq. (8). A similar reduction in the deviation range can be found in the individual test results shown in Fig. 6. Here, the deviation range is reduced from $T_{\text{N}} = 4.65$ (Fig. 6a) to $T_{\text{N}} = 3.76$ (Fig. 6b), which represents a decrease of factor 1.24 (24%).

## 6 Conclusion and outlook

By conducting fatigue tests with specimens manufactured from the same welded sheets, it can be shown that there is a detectable decrease in notch stress fatigue strength with increasing highly stressed weld seam lengths which is determined using the 90% criterion.

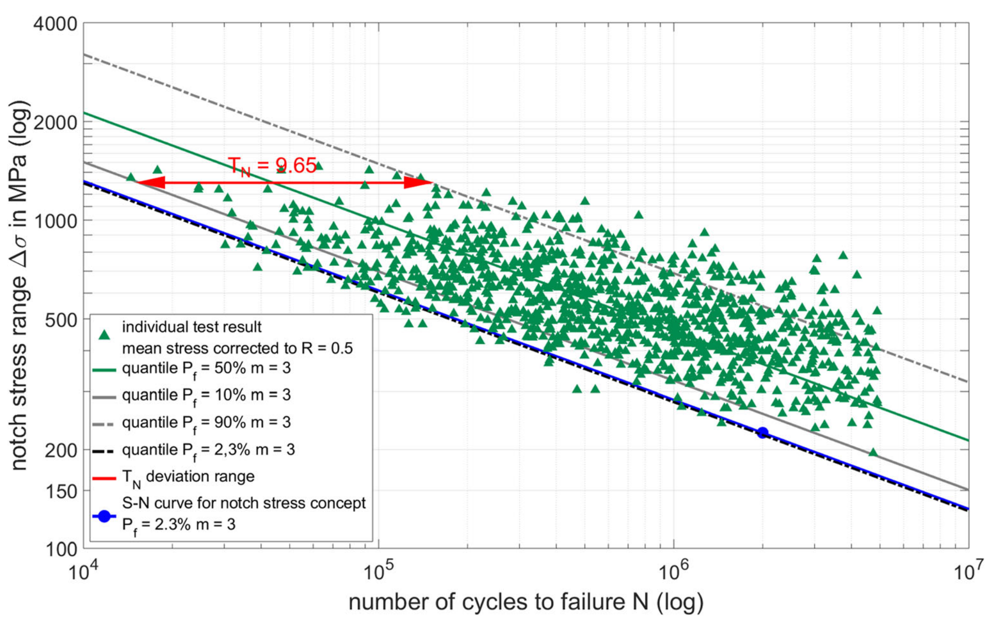



**Fig. 8** Individual test results presented on a notch stress *S-N* plot, with the mean stress corrected to the stress ratio $R = 0.5$

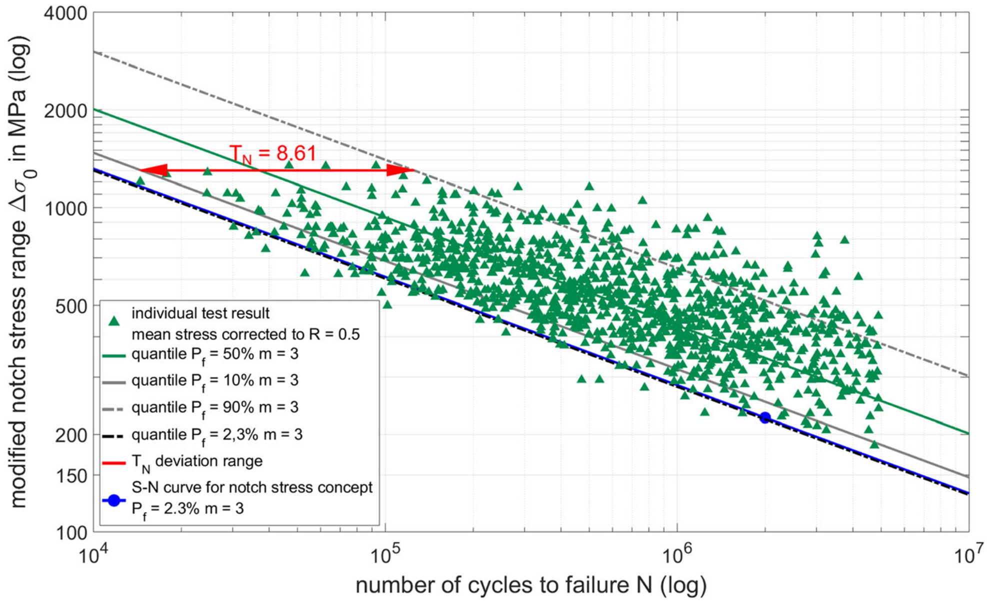



**Fig. 9** Individual test results presented on a notch stress *S-N* plot, with the stress ratio $R = 0.5$ and shifted to a uniform highly stressed weld seam length of $L_{\text{ref}} = 100$ mm

Power laws can be used to describe these dependencies, and a Weibull exponent of 9 has been found. This value lies within the range of values that can be found in the literature (cf. Section 2, Table 1).

The deviation range $T_N$ is reduced by a factor of 1.12 (12%) when the statistical size effect is taken into consideration. This means that the statistical size factor $n_{st}$ according to Eq. (8) can be used to improve

1. The accuracy of calculated notch stress strength values for individual geometries and
2. The overall accuracy of the notch stress concept.

This improvement can be used by applying Eq. (10).

$$\text{FAT}_{\text{modified}} = \text{n}_{\text{st}} \cdot \text{FAT}_{225} = \left(\frac{L_{\text{ref}}}{L_{90\%}}\right)^{-\frac{1}{k_{st}}} \cdot \text{FAT}_{225} \qquad (10)$$

Here, the statistical support factor $n_{st}$ is multiplied by the fatigue strength (fatigue class FAT), and the *S-N* curve is shifted vertically.

The determination of the additional quantity of the highly stressed weld seam length is undoubtedly a considerable additional effort for the user of the notch stress concept. By taking this quantity into account, however, the accuracy of the notch stress concept can be improved, and on average, a strength increase can be made available, which is worth the additional effort.

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