

Special issue on: robust combinatorial optimization

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Since the early days of operational research, mathematical optimization models have been used for the efficient planning of future processes. Inherently, uncertainty in the input parameters leads to less accurate solutions for the problem at hand. Therefore, it is not a surprise that the dealing with uncertainties has been studied as early as the 1950s by introducing probabilities for possible realizations of the data (cf. Dantzig 1955). From this early paper, the area of stochastic programming has emerged. In many applications, however, probabilities for certain realizations are hard to obtain or simply unavailable. Moreover, stochastic programming tends to lead to very large-scale optimization problems, usually much harder to solve exactly than the original problem at hand. In this context, the area of robust optimization was born. Instead of probabilities, an (infinite) set of possible realizations of the input parameters (known as the *uncertainty set*) is introduced and robust feasible solutions are considered, i.e., solutions that are feasible regardless of the realization of the input parameters. Among those solutions, a solution with best worst-case objective value has to be chosen. Soyster (1973) was the first to study such *robust optimization* problems.

In the meanwhile, robust optimization has become a central framework to handle the uncertainty that arises in the parameters of optimization problems. For convex, in particular linear, optimization problems, it has been shown that their robust counterparts can be solved efficiently for a large variety of uncertainty sets (e.g., ellipsoid uncertainty set) (Ben-Tal and Ghaoui 2009). Also, extensions in which the decisions can be adjusted (*two-stage* or *adjustable* robust optimization), depending on the realization of the uncertain parameters have been successfully studied. For optimization problems involving discrete decisions, the situation is more complex. Efficient exact or approximate solution algorithms for such problems must exploit the combinatorial

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structure of the problems at hand. The aim of this special issue is therefore to present state-of-the-art work addressing combinatorial optimization under uncertainty.

We received 17 submissions for this special issue. In total, eight of these submissions have been accepted to be published in this and the next issue of EURO Journal on Computational Optimization:

- This issue starts with an invited contribution of Buchheim and Kurtz (2018) on combinatorial optimization with cost uncertainty. The survey presents several NP-hardness and polynomial results (including new ones) related to the type of uncertainty set: discrete, polyhedral, ellipsoidal, interval, and budget. For the NP-hard problems, the survey describes the main solution algorithms, based on IP formulations or oracle-based solution algorithms. The survey also covers two-stage adjustable robust optimization, providing again complexity results and solution algorithms.
- Ficker et al. (2018) combine balanced combinatorial optimization with robust optimization. In a balanced optimization problem, the difference between the largest and smallest cost values induced by the solution have to be minimized. Considering an uncertainty set consisting of d possible cost vectors yields the robust balanced optimization problem. The authors introduce the corresponding framework and show, e.g., that the problems considered can be solved in polynomial time if d is fixed, whereas in general a polynomial 2-approximation is derived. For the robust balanced assignment problem, computations with the 2-approximation and an integer programming formulation are presented.
- Gabrel et al. (2018) study a portfolio optimization problem under uncertainty on the stock returns. The authors introduce a new robustness criterion, called p -robustness, which maximizes the return in a proportion p of the finite, but large, set of scenarios, guaranteeing a minimum return over all scenarios. This problem can be modeled as integer linear program and computations reveal the performance of ILP software and a tailor-made heuristic.
- Hamaz et al. (2018) study the basic cyclic scheduling problem where the processing times are uncertain. As the uncertainty appears in the right hand side of the associated optimization model, a two-stage approach is required. The cycle time of the schedule have to be decided upon in the first stage, such that the tasks can be processed, regardless of the outcome of the uncertain processing times. Based on combinatorial properties of the problem, three algorithms are presented for this problem and tested in a computational study.
- Well-known uncertainty sets are the so-called budget uncertainty and ellipsoidal uncertainty. Kurtz (2018) combines these two uncertainty sets in the context of combinatorial optimization with uncertain objective. He shows that the robust problem is NP-hard for general budgeted-ellipsoidal uncertainty, but can be solved in polynomial time for several combinatorial problems if the uncertainties are uncorrelated (and the budget is fixed).
- Büsing et al. (2018) consider a problem in the context of stacking at, e.g., container terminals. Focus of the paper is on construction rules for an uncertainty set on the basis of a probability distribution on the uncertain parameters. The impact of

the construction rules on the robustness of the solution is analyzed in a detailed computational study.

- Kargar et al. (2018) study mechanism designs that satisfy Dominant Strategy Incentive Compatibility and Ex-post Individual Rationality, assuming that the types of the buyer and the seller are discrete. In the first part of the paper, several properties of these mechanisms are investigated and compared with those arising with continuous types. Then, the authors relax the unique common prior assumption, thereby introducing ambiguity into the problem framework and leading to a robust optimization problem. The performance of the latter is assessed numerically with different ambiguity sets.
- In Multipolar robust optimization (Ben-Ameur et al. 2017), Ben-Ameur et al. consider adjustable robust optimization problems. Instead of allowing a full adaptation of the second-stage variables to the realization of the uncertain parameters, the authors propose to restrict the values of these variables to convex combinations of second-stage solutions for the so-called *poles*. These poles are selected in such a way that the uncertainty set is covered by the poles in the sense that it is contained in the convex hull of the poles. Complexity and convergence results are presented as well as computations showing the potential of the approach.

References

- Ben-Ameur W, Ouorou A, Wang G, Zotkiewicz M (2017) Multipolar robust optimization. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-017-0092-4>
- Ben-Tal A, El Ghaoui L, Nemirovski A (2009) *Robust optimization*. Princeton University Press, Princeton
- Buchheim C, Kurtz J (2018) Robust combinatorial optimization under convex and discrete cost uncertainty. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-018-0103-0>
- Büsing C, Knust S, Le XT (2018) Trade-off between robustness and cost for a storage loading problem: rule-based scenario generation. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-018-0094-x>
- Dantzig GB (1955) Linear programming under uncertainty. *Manag Sci* 50:197–206
- Ficker A, Spieksma F, Woeginger G (2018) Robust balanced optimization. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-018-0093-y>
- Gabrel V, Murat C, Thiele A (2018) Portfolio optimization with pw-robustness. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-018-0096-8>
- Hamaz I, Houssin L, Cafieri S (2018) A robust basic cyclic scheduling problem. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-018-0100-3>
- Kargar K, Bayrak HI, Pinar M (2018) Robust bilateral trade with discrete types. *EURO J Comput Optim (under revision)*
- Kurtz J (2018) Robust combinatorial optimization under budgeted-ellipsoidal uncertainty. *EURO J Comput Optim.* <https://doi.org/10.1007/s13675-018-0097-7>
- Soyster AL (1973) Convex programming with set-inclusive constraints and applications to inexact linear programming. *Oper Res* 21:1154–1157