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RESEARCH ARTICLE - CIVIL ENGINEERING

Development of Improved Frequency Expressions for Composite Horizontally Curved Bridges with High-Performance Steel Girders

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Abstract

Composite curved I-girder bridges are often used in modern highway systems, but the open sections of I-girders mean that these structures suffer from low torsional resistance. The curvature also results in quite complex behaviors due to the coupled bending and torsional responses of curved I-girder bridges. High-performance steel, which adds strength, enhances durability and improves weldability, addresses both the economic and structural problems associated with curved bridges. However, as yet, there are no simplified design methods in the form of practical equations with which to optimize the design parameters of curved bridges and their dynamic behavior remains controversial. This study evaluated the effects of various design parameters on the free vibration responses of curved HPS I-girder bridges. A sensitivity analysis of 278 prototype simple-span and continuous bridges was conducted using CSIbridge software to create a set of simple, practical expressions for the fundamental frequencies of these structures.

Keywords Curved bridges · Frequency · Finite element method · High-performance steel

1 Introduction

Horizontally curved high-performance steel (HPS) I-girder bridges can offer an economical solution for modern highways where roadway alignments need a smooth and curved transition, reducing costs and controlling pollution due to car emissions. In curved bridges, the centerlines of the girder webs in the sections rising up from the abutments are not collinear with the cords between the abutments. These eccentricities result in high torsional moments, causing high out of plane deformations and rotations in the bridge cross sections [1–3]. High-performance steel (HPS) is often utilized to address these problems as it provides more economic and durable bridges, a significant advantage that enables engineers to design longer and shallower bridges. To enhance the torsional resistance of open-section bridges, cross-frames and diaphragms are applied to interconnect girders in both curved and straight bridge superstructures [4–8]. Many studies have sought to define the exact behavior of curved or bent beams. In one of these earlier studies, Culver [9] developed several analytical models with which to investigate the interactions between curved girders and cross-frames. The effect of erection sequencing on the induced stress and deflections has also been studied in an attempt to enhance the design guidelines and improve the stability of the bridges [10–12].

To reduce the complexity of curved bridge design and improve the capacity of design codes to predict the responses of curved bridges, several researchers have tried to model bridge superstructures as an assembly of simple systems, for example by applying the grillage technique, orthotropic plate methods or other relatively simple forms [13–15]. However, the accuracy of these methods remains problematic and the results produced may not be acceptable due to the simplifications involved. Although finite element modeling (FEM) has been found to be a reliable method for evaluating the performance of curved bridges, the time and resources required mean that it is seldom feasible in a typical bridge design office, especially during the preliminary design stages [16,17].

Several analytical and numerical analyses have been performed to evaluate the dynamic behavior of composite I-girder bridges. Christiano and Culver [18] developed a the-

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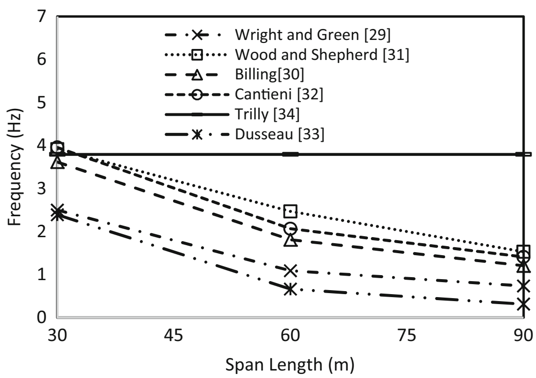

Fig. 1 Fundamental frequencies obtained using various equations

oretical method using the differential equations governing the free vibration of a simple-span curved girder, deriving two sets of equations to determine the fundamental frequencies of such bridges based on Lagrange multiplier concepts. Maneetes and Linzell [19] studied the effects of cross-frame and lateral bracing on the dynamic responses of a single-span curved I-girder bridge using both experimental and numerical solutions, while Yoon et al. [20] presented a finite element formulation based on a free vibration analysis of composite bridges, taking into account the stiffness as well as the mass matrices of the curved beam elements. The findings of extensive studies have indicated that grillage methods are not in fact a reliable way to evaluate the dynamic behavior and free vibration of bridges [3]. Barth and Wu [21] performed a simulation and an experimental study to capture the vibration characteristics (bending frequencies and mode shapes) of straight HPS girder bridges.

Figure 1 shows the fundamental frequencies values defined based on the equations proposed by various researchers for steel girder bridges. According to the graph, Trilly's equation [22] generates the most conservative values for the fundamental frequency, although all follow the same trend with increasing span length. Some past reports on vibration analysis are quite controversial, as they suggest very high values for the fundamental frequency. Wu [23] summarized these reports, concluding that fundamental frequencies are highly dependent on the geometry of bridges and that beam theory and the simplified equations proposed so far are not capable of predicting the correct values. Nassif et al. [24] presented an investigation of vibration control (e.g., acceleration and velocity) in HPS bridges using an experimentally validated three-dimensional dynamic computer model to evaluate both the deflection criterion and depth-to-span (D:L) limitation produced by the current design codes. They concluded that neither set of limitations could provide effective vibration control for steel girder bridges under normal truck traffic conditions and these limits therefore represent a poor method for controlling bridge vibration or ensuring human comfort for those traveling over them [23,25]. Several studies have shown that the current AASHTO serviceability criteria may be insufficient for controlling bridge vibration and frequency-based limits are more rational than span-based limits [25,26].

This paper presents the results of a simulation that applies CSIbridge V20 [27] to design and analyze the fundamental frequencies of curved HPS I-girder bridges. A parametric study examines the variables that could influence the free vibration characteristics of typical composite HPS I-girder bridges. The results from a comparison study indicated that existing frequency prediction equations are not sufficiently accurate for curved composite I-girder bridges. Hence, nonlinear regression methods are used to derive two sets of more reliable and practical expressions with which to estimate the fundamental frequencies of curved HPS I-girder bridges that capture the effects of all the important parameters.

2 Finite Element Modeling and Verification

In this section, the general attributes of the proposed finite element modeling (FEM) using CSIbridge [27] are described. The results obtained from two field tests and a laboratory study are used to verify the effectiveness of the proposed modeling technique for curved HPS bridges.

2.1 Bridge Sections

A four-node shell element with six degrees of freedom at each node that combines membranes and plate-bending behavior was used to model the bridge's concrete slab and steel girders. Each element has its own local coordinate system for defining material properties and loads, and for interpreting the output. Stresses and internal forces and moments in the element local coordinate system were evaluated at the 2-by-2 Gauss integration points and extrapolated to the joints of the element. The bridge modeler in CSIbridge [27] was used to model the prototype bridge. This option takes into account the full composite action between the reinforced concrete slab and the HPS I-girders at the serviceability limit state, as recommended by AASHTO LRFD [6]. Cross section properties were defined for all frame-elements and diaphragm properties imported (X or V types) and manually defined (built-up plate sections). The parapets were modeled as a single frame element. Note that it is critical to place the connecting nodes of the parapets at the centroid of the barrier section. Additionally, the parapets were attached to the deck using a series of rigid links in order to contribute to the stiffness of superstructure. Figure 2 shows the parapet model assumptions applied.

The discretization of the models was performed using the auto-meshing option, defining a good expectation ratio for all elements. The boundary conditions for simple-span

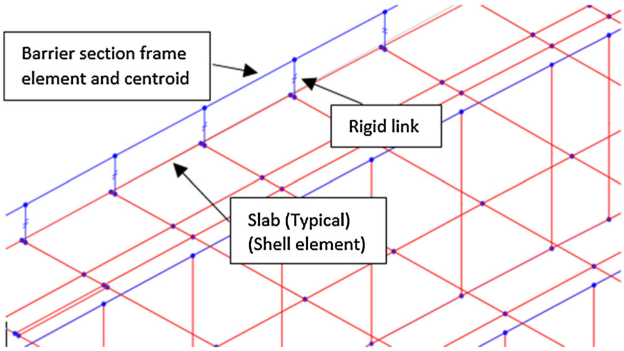

Fig. 2 Parapet modeling assumptions

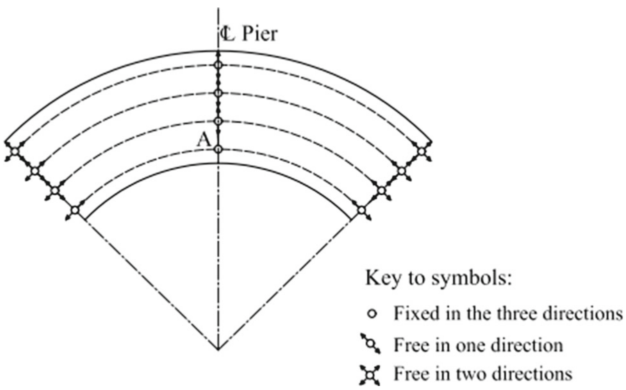

Fig. 3 Selected boundary conditions

and continuous curved bridges were defined based on the proposed tangential bearing arrangement recommended by Samaan et al. [28], shown in Fig. 3. In the elastic range of structural behavior, any slab cracking present exerts only a negligible effect on the bridge responses [29]; hence, the slab was assumed to be uncracked and steel reinforcements were therefore not modeled.

2.2 Verification of Finite Element Models

2.2.1 Colquitz River Bridge

The results of dynamic field tests on the Colquitz river bridge in British Columbia, Canada [30] were selected to verify the proposed FEM technique. The bridge has five spans ($2 \times 14\,\text{m} + 2 \times 17\,\text{m} + 18\,\text{m}$), each composed of six steel girders $W33 \times 141$, and a bridge width and slab thickness of 11.9 and 0.10 m, respectively. The diaphragms and cross-braces members are made of U-shaped steel ($MC18 \times 42.7$) and spaced 3.5 and 4.5 m apart along the end and intermediate spans, respectively, of the bridge. For the ambient test, six force-balanced accelerometers were used to collect and process acceleration data during the day and under varying traffic conditions. Pullback tests were carried out by loading the bridge bents with a special assemblage using a force of 90 kN, and then quickly releasing the applied load to induce free vibrations in both the longitudinal and transverse directions. Nearby abandoned railway bridge piers provided adequate anchor points for the pullback testing. The software capabilities of hybrid bridge evaluation system (HBES) were used to analyze records on site shortly after the data were collected. The resulting torsional and vertical modes exhibited some coupling, which is typical in this type of analysis when periods are close in value [30]. In order to comprehensively verify the results of the FEM, the entire bridge was simulated in the present study using the CSIbridge package software according to the prescribed modeling technique. The first three vertical frequencies obtained by the bridge model (with parapet effects) were 6.12, 7.48 and 8.75 Hz, differing from the field test frequencies of 5.95, 7.14 and 8.35 Hz by about 3.50, 4.54 and 4.72%, respectively. Without modeling the parapet, the first three vertical frequencies obtained by the FEM method were 6.03, 7.25 and 8.50 Hz, which indicates that including the parapet shifts the results by 1.32, 1.52 and 1.76%, respectively. These results indicate that the FEM technique adequately predicts the vibration responses of composite girder bridges. Since the fundamental frequency (the first natural frequency) is the most pronounced, the parapets may not noticeably influence the vertical frequency of these bridges.

2.2.2 Laboratory Test of a Quarter-Scale Model Bridge

Additional verification was performed through experimental tests of a quarter-scale model of a 4.5 m long simple-span straight I-beam bridge with five scaled girders at a spacing of 0.45 m and width of 1.80 m [31]. These I-beams had the following dimensions: flange thickness of 4.80 mm, flange width of 58 mm, web thickness of 3.5 mm and total height of 203 mm. Channels with dimensions of $25 \times 9.5 \times 0.30\,\text{mm}$ and a spacing of 15.8 cm were used as shear connectors. Here, solid diaphragms were applied at the abutments and spaced at one-third intervals along each span, consisting of $76 \times 50 \times 52\,\text{mm}$ angles welded to the beam webs about 13 mm below the top of the beam. A comparison of the FEM and experimental results for the mid-span deflections of all girders under four 20 kN concentrated loads at the mid-span and symmetric about the longitudinal axis of bridge is shown in Fig. 4.

Although the experimental and FEM results compare fairly well, the FEM results are slightly higher than those obtained in the laboratory, varying by up to 1.5, 4.6 and 3.4% for the external, intermediate and central girders, respectively. This discrepancy is likely because the experimental loads were not localized at an exact position but rather distributed over the entire steel supporting plate. The same phenomenon has also been reported by Wegmuller [32] in a comparison between FEM and experimentally obtained

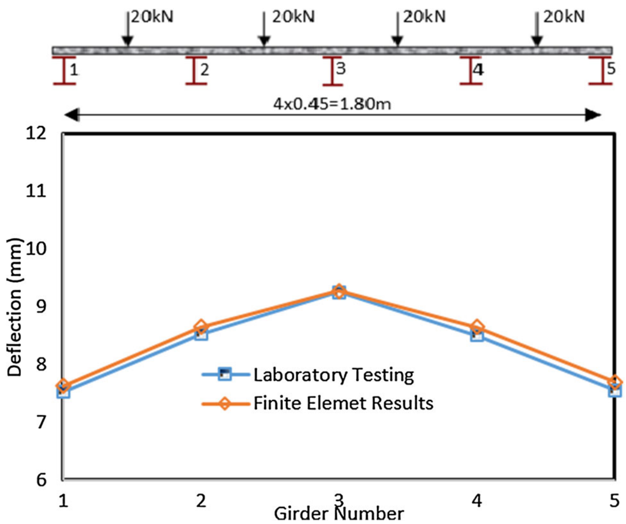

Fig. 4 Deflection distribution in the bridge girders at the mid-span cross section

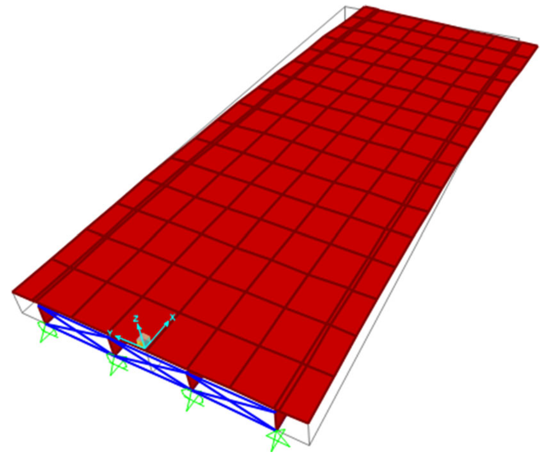

Fig. 5 FE model of a 30 m span length bridge with $k = 0.1$

results. This suggests that the proposed FEM technique accurately predicts the response of I-girder bridges to static loading.

2.3 Geometric and Material Properties of Bridges

High-performance steel (HPS) bridges need less steel due to their higher yield stress, but consequently the live-load deflections of these bridges are more likely to exceed AASHTO's deflection limits [5]. As design optimization must take into account the AASHTO limits on performance and economy, an important first step is to select a set of key parameters and establish a matrix that covers a wide range of steel bridge types. Based on this database, bridges can then be designed and optimized for various combinations of these parameters to capture a least weight approach using CSIbridge [27]. The initial designs are performed by neglecting the AASHTO criteria for deflection in order to design girders that meet other AASHTO LRFD strength and serviceability criteria for curved bridges. Any girders that then fail the AASHTO live-load deflection criteria must be redesigned. Table 1 shows the geometric properties and material parameters for three prototype bridges. Figure 5 shows a typical 30 m span length bridge modeled in CSIbridge [27].

Four different span lengths (L), 30, 45, 60, and 90 m, were considered in this investigation, with span-to-radius (k) values for the slenderness ratio increasing from 0 to 1.20 at 0.2 intervals. Three types of cross section, with girder spacings of 2.8, 3.5, and 2.5 m, were utilized, along with the American Iron and Steel Institute (AISI) standard parapet design, which has a height of 0.87 m and bottom and top widths of 0.4 and 0.15 m, respectively. The prototype bridges were modeled using three different high-performance steel materials: HPS 50 W (Fy = 345 Mpa), HPS 70 W (Fy = 485 Mpa) and HPS 100 W (Fy = 620 Mpa). The concrete deck was modeled with a modulus of elasticity of 28,000 Mpa, a Poisson's ratio of 0.2 and a density of 24 kN/m^3. The concrete deck thickness was 0.20 m for all the cross sections.

3 Sensitivity Analysis

A sensitivity analysis provides useful information regarding the influence of various parameters on the free vibration responses of bridges. The goal here is to characterize the key parameters affecting a bridge's natural frequency so that they can be taken into account in the development of effective expressions for natural frequencies. The investigation variables are the span length, the span-to-radius curvature ratio (k), the number of girders, the girder spacing and the number and arrangement of the cross-braces. A detailed discussion of each of these parameters is presented in this section, and the key parameters are identified.

Table 1 Characteristics of three prototype curved I-girder bridges

Set	L (m)	HPS types (W)	L/D ratio	S (m)	No. girder	W (m)
1	30, 45, 60, 75, 90	50	20	2.80	5	13.0
2	30, 45, 60, 75, 90	70	25	3.50	4	13.0
3	30, 45, 60, 75, 90	100	30	2.50	4	9.50

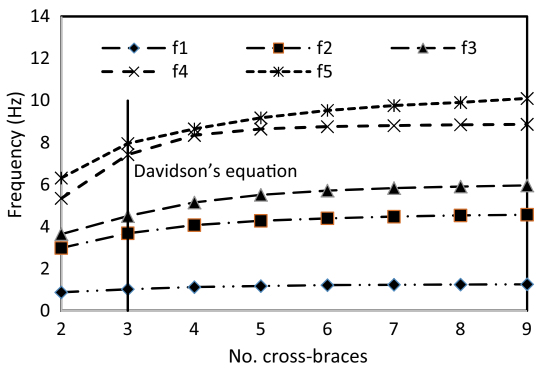

Fig. 6 Effect of the number of cross-bracing lines on a bridge's natural frequencies

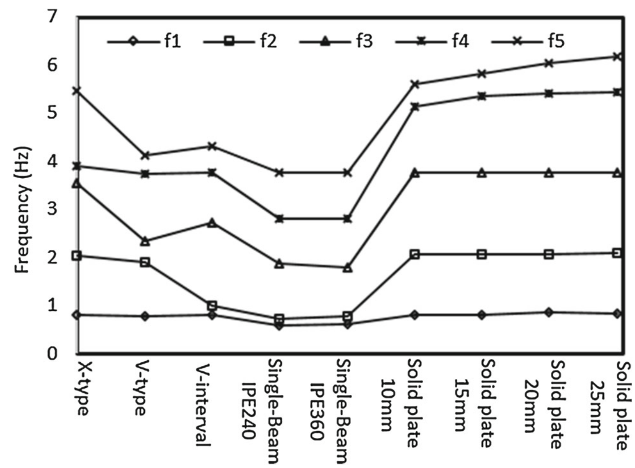

Fig. 7 Effect of cross-brace type on a bridge's natural frequencies

3.1 Effect of the Number of Cross-braces

Curved girder bridges have a high torsional moment due to their curvature. The open nature of these bridges results in low torsional resistance to cracking, so cross-braces are applied to enhance their torsional stiffness. It is thus vital to determine the effect of this variable on the bridge's free vibration responses. The effect of the number of cross-bracing lines on the first natural frequency of a 30 m long bridge with a curvature ratio of 1.20 is shown in Fig. 6. The minimum required number of cross-braces obtained from Davidson's equation [33] is also indicated. Although the number of braces appears to have only an insignificant effect on the first and second frequencies and the vertical mode shapes of the bridge, it does decrease the natural frequency values for the higher modes of vibration. Thus, for all the prototype bridges modeled in this study, cross-braces are placed 6 m apart, with a minimum of five bracing lines for each span.

3.2 Effect of the Cross-brace Type

In order to study the effect of the various types and stiffnesses of bracing systems on the free vibration responses of curved HPS bridges, several different bracing configurations were modeled for a 90 m long bridge with a curvature ratio of 0.60. The following bracing systems were considered: *X*-type, *V*-type, inverted *V*-type, single beam and solid plates; the results are shown in Fig. 7. Once again, although the type of bracing systems had an insignificant impact on the first natural frequency of the bridge, it did have an important impact on the higher frequencies. For instance, the second natural frequency of a bridge with *X*-type bracing was up to 12, 48 and 54% higher than bridges with *V*-, inverted *V*- and single beam bracing systems, respectively. The effects of the stiffness of the cross-bracing and the end diaphragms on the free vibration of the bridge are also shown in Fig. 7.

These results suggest that the stiffness of secondary members has a significant influence on the bridge's fundamental frequencies. For example, enhancing the thickness of the solid steel plates from 10 to 25 mm changes the first and second frequencies by up to 2%. However, although increasing the plate thickness also increases the natural frequencies of the higher modes, it should be avoided due to possible brittle connection failures of the diaphragms under seismic loads. Thus, it is preferable to adjust the thickness of the steel X-bracing rather than the plate thickness to enhance the torsional resistance of both single-span and continuous bridges.

3.3 Effect of the Span Length

Many researchers have described the importance of span length for the dynamic responses of straight bridges. For example, the current North American codes [5,6] offer equations for the dynamic impact factor of bridges as a function of span length. Wright and Walker [34] developed the following theoretical equation based on beam theory to determine the fundamental frequencies. Note that this equation incorporates the span length as an important factor:

$$f_{\mathrm{sb}} = \frac{\pi}{2L^2}\sqrt{\frac{E_b \cdot I_b \cdot g}{w}} \qquad (1)$$

where $E_b \cdot I_b$ and w are the flexural rigidity and the weight per unit length of the composite steel girder, respectively. Figure 8 shows the effect of span length on the natural frequencies of bridges with a slenderness ratio (L/D) of 25 and a curvature ratio of 0.20. Increasing the span length from 30 to 90 m decreases the natural frequencies by up to 65%. From a practical standpoint, the span length is thus an important parameter affecting the dynamic response of a curved bridge.

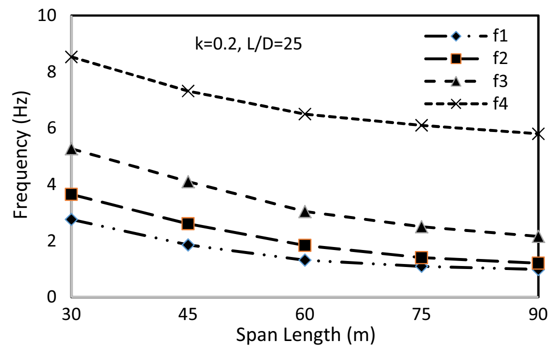

Fig. 8 Effect of span length on the fundamental frequencies of curved bridges ($k = 0.20$, $L/D = 25$)

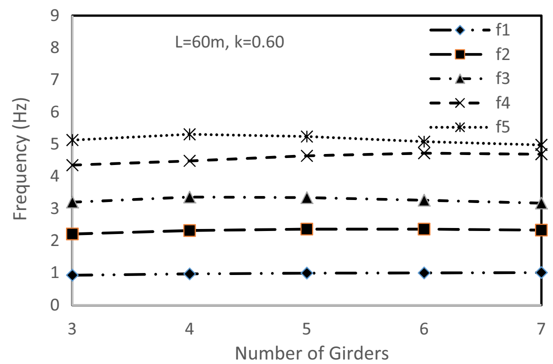

Fig. 9 Effect of the number of girders on a bridge's natural frequencies

3.4 Effect of the Number of Girders

Analyzing the performance of a 60 m long curved bridge with a curvature ratio of 0.6 to investigate the effect of the number of girders on the bridge's natural frequencies revealed that although increasing the number of girders results produced higher torsional stiffness in the structure, the natural frequencies of the bridge's superstructure were enhanced only slightly. The results, presented in Fig. 9, show that the first and second frequencies increase by approximately 3% as the number of girders increases from three to seven, for example.

In practice, this does not influence the free vibration sufficiently to justify its inclusion as an important model parameter. These relatively small changes are due to the fact that the prototype bridges have almost the same flexural stiffness, irrespective of the number of girders. Previous research has also indicated that the number of girders has no impact on the natural frequencies of straight bridges [35].

3.5 Effect of the Span-to-Radius Curvature Ratio

Figure 10 shows the relationship between the first natural frequencies (f1) and curvature ratios ($k = L/R$) of five bridges with lengths of 30, 60 and 90 m and various slenderness ratios (L/D). The graph indicates that the curvature ratio has an important effect on the free vibration of curved superstructures, with an increase in the curvature ratio significantly reducing the natural frequencies. For instance, for the 60 and 90 m bridges, the natural frequencies decrease by up to 56 and 39%, respectively, when the span-to radius ratio increases from 0 to 1.20. The curvature ratio also has a significant impact on the mode shapes.

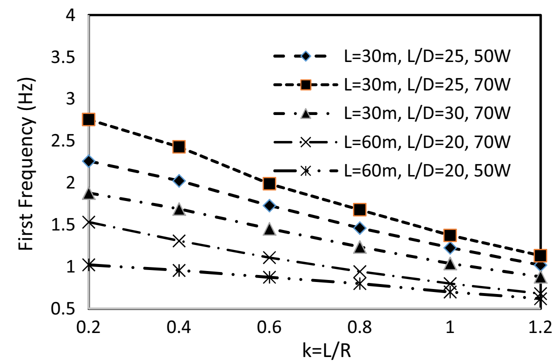

Fig. 10 Effects of the bridge curvature ratio on the fundamental frequency

Figure 11 shows the first three mode shapes for both straight and curved bridges with 30 m long spans. These reveal that any curvature in a bridge exerts a higher torsional impact on mode shapes. For example, the first mode shape for a straight bridge is purely flexural, but in a curved bridge it consists of a combination of flexural and symmetric torsional modes. Thus, the effect of the curvature ratio on both the mode shapes and frequencies will be significant and should thus be included in the parametric study.

3.6 Effect of Slenderness (Span-to-Depth) Ratio

AASHTO (5) specifications limit the value of the slenderness ratio to a maximum of 25 in order to control the maximum live-load deflection through controlling the bridge's bending stiffness. However, this approach became somewhat controversial with the introduction of high-performance steel (HPS), which has approximately 40% higher yield stress than conventional steel and hence allows engineers to design shallower and longer spans. Since HPS bridges are more likely to suffer from large deflections, the slenderness ratio plays a significant role in the dynamic behavior of these systems. The effects of different slenderness ratios on the natural frequencies of a 30 m long bridge are presented in Fig. 12. The natural frequencies decrease almost linearly as the L/D ratio increases, irrespective of the curvature ratio. For example, in a bridge with a curvature ratio of 0.80, the first natural frequency decreases by approximately 28% when the L/D ratio increases from 20 to 30. From a practical standpoint, the impact of the slenderness ratio is considerable and should

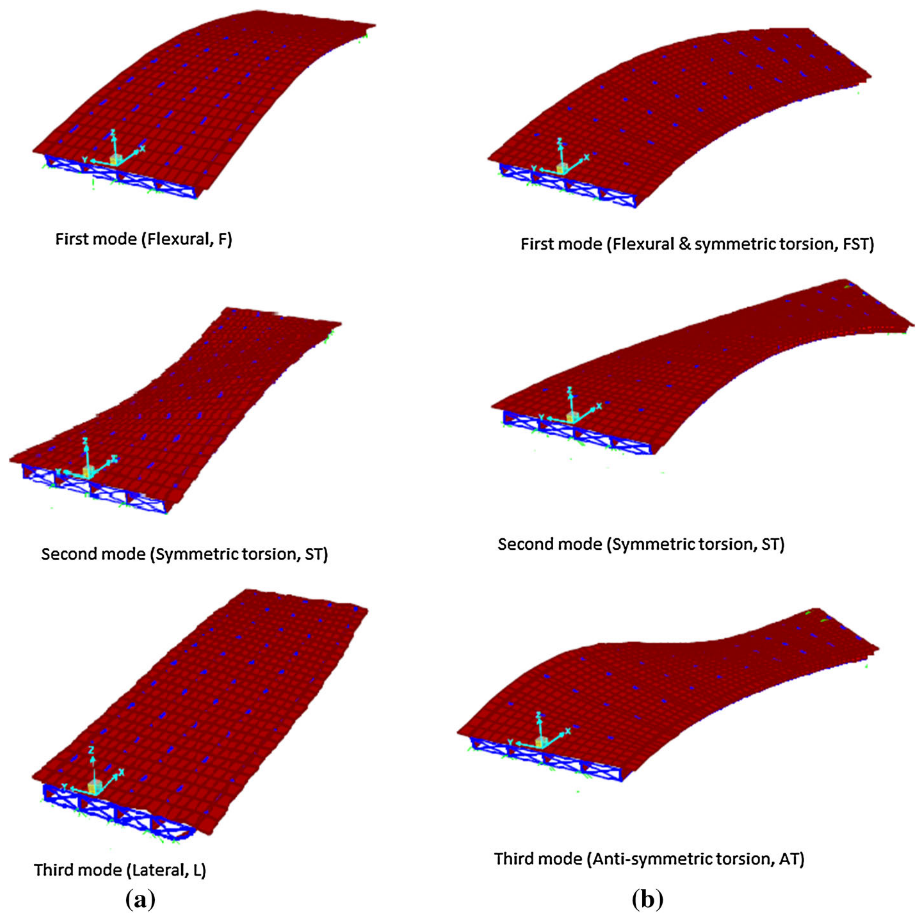

Fig. 11 First three modes of vibration for straight and curved bridges. **a** Straight bridges and **b** curved bridges

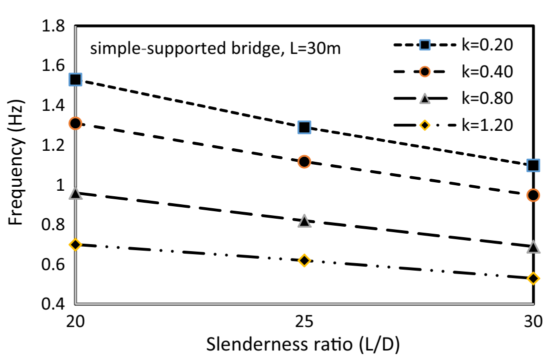

Fig. 12 Effect of slenderness ratio (L/D) on a bridge's natural frequency

thus be taken into account in any parametric study of curved HPS I-girder bridges.

4 Empirical Expressions for Fundamental Frequency

As the comparison of the various previous studies that have investigated this issue presented in Fig. 1 shows, none of the proposed equations adequately predict the fundamental frequency of curved composite girder bridges constructed using high-performance steel. While Wood and Shepherd's equation [36] estimates highly conservative values, the models proposed by Dusseau [37] and Wright and Walker [38] both significantly underestimate the fundamental frequencies of such bridges. To address this problem, the data for 180 simple supported and 85 two- and three-span continuous bridges were collected and analyzed to develop a set of proposed expressions to describe their fundamental frequencies. The proposed expressions take the following form:

$$f_{c \cdot b} = \psi \cdot f_{\mathrm{sb}} \tag{2}$$

where f_{sb} is the fundamental frequency of a straight bridge from Eq. (1). A modification factor, ψ, is applied to take into account the effect of the parameters from the previous section that were found to affect the free vibration of curved HPS I-girder bridges. A regression analysis using a statistical computer package for best fit based on the least squares method for nonlinear data was used to deduce appropriate modification factors as follows:

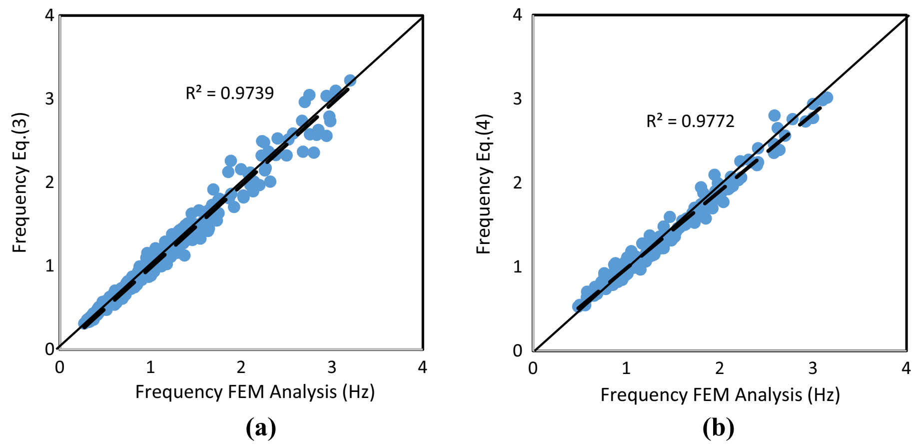

Fig. 13 Comparison of the results obtained by the proposed expression and the FEM analysis. **a** Simple-span and **b** continuous span

Table 2 Comparison between the FEM results and those obtained using the proposed expressions

No. spans	L/D	L (m)	L/R	f_{sb} (Hz)	ψ	Frequency (Hz)	
						$f_{c\cdot b} = \psi \cdot f_{sb}$	FE
1	20	30	1.0	0.5966	2.240	1.337	1.375
1	25	30	0.80	0.6865	1.930	1.328	1.281
1	30	30	1.0	0.5910	1.980	1.172	1.102
2	30	30	1.20	0.8435	2.290	1.936	2.020
2	30	60	0.20	1.2063	1.050	1.275	1.250

- *Simple-span bridges*

$$\psi = 1.160 \times e^{-0.82k} \times e^{0.018S} \times \left(0.80 + 0.008\frac{L}{D}\right) \quad (3)$$

- *Continuous bridges*

$$\psi = 1.137 \times e^{-0.37k} \times e^{0.005\frac{L}{D}} \quad (4)$$

It should be recognized that the effect of the slenderness ratio in the proposed expressions for the modification factor is negligible because the impact of the span length and depth has already been taken into account in Eq. (1). The results of the verification analyses for the proposed expressions are shown in Fig. 13 and Table 2.

Examining the first natural frequency for both simple-span and continuous curved bridges reveals that the difference between the FEM analyses results and those obtained using the expressions proposed here are within 1%. The high coefficient of determination, R^2, obtained in both cases indicates the minimal variation in the results calculated by the proposed expressions and the FEM analysis.

5 Conclusions

This study has presented a detailed numerical analysis of the free vibration characteristics of horizontally curved HPS I-girder bridges. This numerical approach includes an extensive sensitivity study of simple supported and continuous bridges to determine the effect of a number of different variables on the natural frequencies and mode shapes of the bridge structures. Empirical expressions for the fundamental frequencies of such bridges were derived that are suitable for use in design codes and engineering offices. The proposed expressions are a function of the natural frequency obtained from flexural beam theory and are hence applicable to bridges with various slenderness ratios. Based on the results obtained from the sensitivity study, the following conclusion can be derived:

- The slenderness ratio significantly affects the natural frequency of such bridges: the fundamental frequency decreases with increasing span-to-depth ratio.
- The magnitude of the fundamental frequency decreases with increasing span-to-radius curvature ratio for horizontally curved I-girder bridges. The curvature ratio also has a significant effect on the bridge's vibration modes due to the high contribution of the torsional effects induced in curved superstructures.
- Cross-braces between the abutments increase the torsional stiffness of open-section bridges. It is recommended that the maximum spacing for bracing lines be limited to 6 m in order to exert a meaningful effect on bridge responses.

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