## **EDITORIAL**



## **Foreword**

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The international conference "Complex Analysis and Related Topics 2018" (CART 2018) took place from April 23 to April 27, 2018, at the Euler International Mathematical Institute in Saint Petersburg. It was the second edition of the conference of the same name which took place in Saint Petersburg in 2014.

The aim of the conference was to bring together leading experts from different branches of Complex Analysis and its applications. The scope of the conference encompassed the whole spectrum of problems in contemporary Complex Analysis as well as topics in Functional and Harmonic Analysis, Operator Theory, Geometry, Topology and Dynamical Systems inspired by or related to various Complex Analysis problems. Among the main themes of the conference one could mention the following:

- Zero sets and interpolation in spaces of analytic functions;
- Operators in spaces of analytic functions (such as shifts, Toeplitz and Hankel, truncated Toeplitz, etc.) and their spectral properties;
- Approximation theory (including Padé) in the complex plane; and orthogonal polynomials for planar measures;
- Functions of several complex variables and applications to complex geometry (including the problems of holomorphic immersion);
- Applications to harmonic analysis and spectral theory of differential operators.

The conference attracted a number of prominent specialists in Analysis from many countries (Finland, France, USA, Israel, Slovenia, Switzerland, Belgium, Poland, Sweden, Spain, Italy, Norway) as well as from several important Russian mathematical centers (Moscow, Saint Petersburg, Kazan, Ufa, Vladivostok). The program included 20 plenary talks and 29 session talks. The list of plenary speakers was as follows:

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia



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A. Aleman, A. Aptekarev, K. Astala, Z. Błocki, A. Bufetov, E. Chirka, F. Forstnerič, J. Globevnik, H. Hedenmalm, G. Kozma, A. Kuijlaars, F. Kutzschebauch, A. Logunov, P. Mattila, N. Nikolski, S. Petermichl, A. Poltoratski, E. Saksman, S. Smirnov, M. Sodin.

The present volume consists of invited papers by CART 2018 participants mostly based on their talks at the conference. Let us give a short survey of the contributions to the volume (in alphabetical order).

The paper by E. Abakumov and E. Doubtsov describes the univalent functions generating bounded Volterra operator between two growth spaces with radial weights. Direct and inverse problems for vector logarithmic potentials with external fields are studied by A. Aptekarev, M. Lapik, and V. Lysov. N. Arcozzi, P. Mozolyako, and K.-M. Perfekt investigate multi-parameter potential theory on the weighted d-tree (Cartesian product of several copies of uniform dyadic tree), which is connected to discrete models of weighted Dirichlet spaces on the polydisk, and establish some basic properties of the respective potentials, capacities and equilibrium measures. The paper of A. Bagapsh extends a classical result about weighted averages of harmonic functions to solutions of second-order strongly elliptic systems of PDE with constant coefficients in a disk.

The survey paper by S. Charpentier, K. Fouchet, O. Szehr, and R. Zarouf presents several recent results concerning estimates of the inverses of matrices. In particular, it discusses the constructive approach to the Schäffer conjecture suggested recently by the last two authors who showed that the critical asymptotics can be achieved by explicit Toeplitz matrices.

The note by D. Dmitrishin, K. Dyakonov, and A. Stokolos discusses an interesting open question of finding the exact "Koebe constant" for the class of univalent polynomials of a given degree in the disk. E. Doubtsov and F. Kutzschebauch study a well-known factorization problem; namely, they seek to find out whether any matrix in  $SL_n(R)$ , where R is a commutative unital ring, is a product of elementary matrices with entries in R. They consider two approaches, one based on the notion of the Bass stable rank and the other on construction of a null-homotopy. K. Dyakonov describes the extreme points in the class of functions of the form  $|f|^2$  for f in the unit ball of the kernel of some Toeplitz operator on the Hardy space  $H^p$ .

K. Fedorovskiy and P. Paramonov study properties of the harmonic reflection operator in simple Carathéodory domains in  $\mathbb{R}^N$ . They obtain several new sharp necessary and sufficient conditions for  $Lip^m$ -continuity of the reflection operator. The paper by K. Isaev, K. Trounov, and R. Yulmukhametov deals with the classical problem of representation by series of exponentials in normed spaces of holomorphic functions. B. Khabibullin and F. Khabibullin study zero sets for the class of analytic functions f on the unit disk that satisfy the estimate  $|f| \leq \exp(M)$  where M is some subharmonic function. The authors relate the properties of zero sets with the growth of the Riesz measure associated to M.

I. Kayumov and K.-J. Wirths survey various methods of estimating certain functionals involving the coefficients of Bloch functions. The work of M. Mazalov is concerned with the properties of bianalytic capacities which appear in problems of uniform approximation by bianalytic functions on compact sets in the complex plane.

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The author discusses properties of these capacities and their relationship to Calderon commutators.

The paper by N. Nikolski and A. Volberg studies in detail the following interesting phenomenon (observed for the first time by V. V. Kozlov in 1948 in the context of orthogonal bases): given a frame  $(u_n)$  in a real  $L^2$  space with a non-discrete measure, it is not possible that the negative parts  $(u_n^-)$  satisfy  $\sum_n (u_n^-)^2 < \infty$  on a set of positive measure. The authors show the sharpness of their conditions and also extend this result to unconditional bases in reflexive Banach spaces.

Criteria for approximability of functions by solutions of homogeneous second order elliptic equations (with constant complex coefficients) in the norms of the Whitney  $C^1$ -spaces on compact sets in  $\mathbb{R}^2$  are obtained in the paper of P. Paramonov and X. Tolsa; these criteria are given in terms of the respective  $C^1$ -capacities. S. Petermichl, S. Pott and M. C. Reguera prove a bilinear Carleson embedding theorem with matrix weight and scalar measure, generalizing the well-known weighted bilinear Carleson embedding theorem. Their proof is based on a new type of maximal function whose norm in the matrix weighted setting does not grow with dimension.

A survey paper by H. Queffélec and R. Zarouf discusses various forms of the Bernstein inequality for polynomials. The paper by V. Todorčević gives an overview of some results on the class of functions with subharmonic behaviour and their invariance properties under conformal and quasiconformal mappings.

The contributions to the volume cover a wide spectrum of themes and reflect many different aspects of modern Analysis. At the same time almost all of them are based on complex-analytic methods. We hope that this special volume of Analysis and Mathematical Physics will be of interest to specialists in Complex Analysis and its numerous applications.

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