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A Tribute to the Memory of Gennadi Henkin

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Born in Moscow on the 26th of October 1942, Gennadi Henkin died in Paris on the 19th of January 2016. He studied in the University of Moscow and, in 1973, he defended his doctoral thesis in Sciences, equivalent to French HDR (Habilitation à diriger des recherches). He had a researcher position at the Institute of mathematical economics in the Science Academy of Moscow, from 1967 to 1991.

I discovered G. Henkin's mathematical work a long time before I met him. It was in the seventies that I read for the first time G. Henkin's mathematical works. Then I was an assistant professor at the University of Nice. At first I have to describe the work atmosphere in mathematics and in France at the beginning of the seventies: of course it seems nowadays unbelievable but nevertheless it was very efficient despite the low level of material conditions. At that time, there were neither individual computers nor web. Mathematical papers were typed by secretaries using typewriters and sent to colleagues by post. Phone and television were still quite luxuries and many people had no phone at home. It was very expensive to phone abroad and we did it only for exceptional events, calling from the university under the control of the administration. The mathematical research teams (especially out of Paris in provincial cities) were generally very small in comparison to those in the present time. In Analytic Geometry and in several complex variables, following H. Cartan's famous theory, sheaves theory, homological methods and geometric constructions like analytic covers, blowing up, were extremely successful and were the most usual and popular methods in that field of research, particularly in France. Lars Hörmander's L^2 methods coming from the Theory of Partial Differential Equations [6] were not well known and appeared as a little bit strange and artificial method in the domain of Complex Analysis. The best we could hope using L^2-methods in bounded pseudoconvex domains in $\mathbb{C}^{\mathbf{n}}$ was to construct global non-trivial holomorphic functions in $L^2(\Omega)$. Mathematicians were very far from thinking they could develop a fine theory of holomorphic functions in several variables as the theory of Hardy spaces $H^p(D)$ in one variable [5], even if some mathematicians such as Charles Ehrenpreis, André Martineau [12] and François

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Norguet [1] had already obtained successful results with integral representations in several complex variables and would like to extend this theory. Therefore, when I learnt in the seventies (probably in 1971) that a Russian mathematician Gennadi Henkin (not well known at that time) had obtained global L^∞ estimates for the $\bar\partial$-equation in a strictly pseudoconvex bounded domain in $\mathbb{C}^{\mathbf{n}}$ [2] (a difficult problem even in the case of the Ball), it appears as an amazing and fascinating event, like a thunderclap out of the clear sky. Moreover, at that time, it was very difficult to interchange results with Russian mathematicians. The post mail needed a lot of time. The first version of the Russian papers were written in Russian language with Cyrillic characters and sometimes an English translation was not easily available. Nevertheless after defending my thesis in 1972 (thèse de doctorat d'Etat equivalent to an habilitation at that time) which used essentially L. Hörmander's L^2 methods, I decided to deeply analyse G. Henkin's papers and to work with them.

At that time, Gennadi Henkin powerfully extended the Cauchy–Leray formula of integral representation of holomorphic functions in a strictly convex domain of C^n to arbitrary differential forms in a strictly pseudoconvex domain. Then he mainly obtained effective, explicit and very precise integral representations for the solutions of the $\overline{\partial}$-equation: $\overline{\partial} u = f$ in these strictly pseudoconvex domains and also for the solutions of the tangential Cauchy operator $\overline{\partial_b}$. He showed that it was possible to solve $\overline{\partial}$-equation on a strictly pseudoconvex domain of C^n with infinite uniform estimations on the domain using an explicit integral quite simple kernel, directly related to the geometry of the domain. The kernel was completely similar to that of the formula of Cauchy–Leray, in the strictly convex case. Lieb and Grauert [10] had obtained the same result using an E. Ramirez de Arellano's formula. But G. Henkin's construction was extremely transparent and it had a major impact on the mathematicians dealing with complex analysis in several variables. For it opened the way to studying a lot of problems which seemed to be quite unreachable before G. Henkin's decisive and founding work. At first it provides L^p and L^∞ estimates up to the boundary for the solutions of the $\overline{\partial}$ operator. It particularly opened the way to studying the algebra of bounded holomorphic functions and Hardy's spaces in several complex variables on pseudoconvex domains in C^n.

Let us now give some hints on Henkin's construction. For $(\xi, \eta) \in C^n \times C^n$, we set $< \xi, \eta >= \sum_{j=1}^{j=n} \xi_j \eta_j$.

On the set $E = \{(\xi, \eta) \in C^n \times C^n \mid < \xi, \eta > \neq 0\}$, we consider the Cauchy–Leray differential form defined by:

$$\mu =< \xi, \eta >^{-n} \omega'(\xi) \wedge \omega(\eta), \tag{1}$$

where: $\omega'\xi) := \sum_{j=1}^{n} (-1)^{j-1} \xi_j \, d\xi_1 \wedge \cdots \wedge d\hat{\xi}_j \wedge \cdots d\xi_n$
and $\omega(\eta) := d\eta_1 \wedge \cdots \wedge d\eta_j \wedge \cdots \wedge d\eta_n$. It is easy to see that μ is closed and is the pullback of a closed differential form defined on the open subset $\tilde{E} = \{([\xi], \eta) \in P^{n-1} \times C^n \mid < \xi, \eta > \neq 0\}$ of $P^{n-1} \times C^n$ where $[\xi]$ is the line of P^{n-1} defined by $\xi \in C^n \setminus \{0\}$.

We now consider a map s of class C^2:

$$s : (\overline{\Omega} \times \Omega) \setminus \Delta \to C^n, \ (\zeta, z) \to s(\zeta, z), \tag{2}$$

where Δ is the diagonal of $\mathrm{C^n}$, such that

$$< s(\zeta - z), \zeta - z > \neq 0. \tag{3}$$

As s can also be considered as a section of a trivial fiber bundle with fiber $\mathrm{C^n}$, s is usually called a section. Then the map $\hat{s}$ defined by:

$$\hat{s} : (\overline{\Omega} \times \Omega) \setminus \Delta \to \mathrm{C^n} \times \mathrm{C^n},\ (\zeta, z) \to (s(\zeta, z), \zeta - z)), \tag{4}$$

takes its values in E.

The differential form $\hat{s}^{\star}\mu$ which is the pullback of μ by $\hat{s}$ is called the Cauchy–Leray form associated with s. It only depends on the image of s in $\mathrm{P^{n-1}}$. In the following integral formulas, we have to consider components of $\hat{s}^{\star}\mu$ of appropriate bidegrees in ζ and in z but for the simplicity we do not write explicitly these bidegrees. Indeed that immediately results from restrictions coming from the complex dimension. Then (under very mild supplementary assumptions on s, we omit for simplicity) for an holomorphic function f on Ω, of class $\mathrm{C^1}$ on $\bar{\Omega}$, we have the following Cauchy–Leray integral representation of f which essentially results from Stokes formula:

$$f(z) = c_n \int_{\partial\Omega} f(\zeta)\ (\hat{s}^{\star}\mu), \tag{5}$$

where $z \in \Omega$ and c_n is a constant only depending on n (in fact we only have to consider in (5) the component of $(\hat{s}^{\star}\mu)$ of bidegree $(n, n-1)$ in ζ and degree 0 in z).

Particularly, taking for s, $s = s_b := \overline{\zeta - z}$ so that $< s_b, \zeta - z >= |\zeta - z|^2 > 0$ for $\zeta \neq z$ (s_b is often called the Bochner–Martinelli section), we obtain for an arbitrary open-bounded domain of class C^2, the Bochner–Cauchy–Martinelli integral representation formula:

$$f(z) = c_n \int_{\partial\Omega} f(\zeta) \left[\sum_{j=1}^{n} (-1)^{j-1} \frac{\overline{\zeta_j - z_j}}{|\zeta - z|^{2n}} \bigwedge_{k \neq j} \mathrm{d}\bar{\zeta}_k \wedge \omega(\zeta) \right]. \tag{6}$$

In the case of a strictly convex bounded domain Ω defined as $\Omega := \{\zeta \in \mathrm{C^n} \mid \rho(\zeta) < 0\}$ where ρ is a strictly convex function defined in a neighbourhood of $\bar{\Omega}$ with $\mathrm{d}\rho(\zeta) \neq 0$ for $\zeta \in \partial\Omega$, we can construct a section s such that for $(\zeta, z) \in \partial\Omega \times \Omega$, $s_j(\zeta, z) := \frac{\partial\rho}{\partial\zeta_j}(\zeta)$ (or $s(\zeta, z) = \bigtriangledown\rho(\zeta)$ or $\partial\rho(\zeta) = \sum_{j=1}^{n} s_j(\zeta, z)\mathrm{d}\zeta_j$) so that s is holomorphic in z for $\zeta \in \partial\Omega$ (and even constant in z). We obtain the Cauchy–Leray integral representation for holomorphic functions on a strictly convex open set which can be written as follows:

$$f(z) = c_n \int_{\partial\Omega} f(\zeta) \frac{\partial\rho \wedge (\partial\bar{\partial}\rho)^{n-1}}{< \partial\rho(\zeta), \zeta - z >^n}. \tag{7}$$

For a $\bar{\partial}$-closed differential form f of bidegree (p, q), we have the following Cauchy–Martinelli integral representation of f on every open-bounded domain Ω of class C^2:

$$c(p,q,n)f(z) = \bar{\partial}_z \left[\int_{\Omega} f(\zeta) \wedge (\hat{s_b}^{\star}\mu) \right] + \int_{\partial\Omega} f(\zeta) \wedge (\hat{s_b}^{\star}\mu), \tag{8}$$

where $z \in \Omega$ and $c(p,q,n)$ is a constant only depending on p, q, n. We consider the component of $(\hat{s_b}^{\star}\mu)$ of bidegree $(p, q-1)$ in z and of bidegree $(n-p, n-q)$ in ζ in the first integral (resp. of bidegree $((p,q)$ in z and bidegree $(n-p, n-q-1)$ in ζ in the second one).

Of course the term we obtain integrating on the boundary $\partial\Omega$ is a stumbling block to solve the $\bar{\partial}$ equation. To obtain an explicit solution of the $\bar{\partial}$ equation on a strictly pseudoconvex domain Ω, the main G. Henkin's idea was to use an explicit homotopy between the Bochner section s_b and a section s_h which is holomorphic in z when $\zeta \in \partial\Omega$. In the case of a strictly convex domain, he considers the preceding section $s_h(\zeta, z) := \bigtriangledown\rho(\zeta)$ and he defines for $t \in I = [0,1]$, $\zeta \in \partial\Omega$ and $z \in \Omega$ the homotopy function g:

$$\begin{aligned} g(t,\zeta,z) &:= t\frac{s_h}{<s_h, \zeta - z>} + (1-t)\frac{s_b}{<s_b, \zeta - z>} \\ \hat{g}(t,\zeta,z) &:= (g(t,\zeta,z), \zeta - z). \end{aligned} \tag{9}$$

Let us observe it results from the strict convexity of the function ρ that $\mathrm{Re}(< s_h, \zeta - z >) = \mathrm{Re}(< \bigtriangledown\rho(\zeta), \zeta - z >) > 0$ for $(\zeta, z) \in \partial\Omega \times \Omega$ so that g is well defined and takes its values in E. As (modulo a numerical factor) $g = s_b$ for $t = 0$ and $g = s_h$ for $t = 1$, a new application of Stokes Formula on the manifold $I \times \partial\Omega \times \Omega$ gives:

$$\begin{aligned} c(p,q,n)f(z) = \bar{\partial}_z &\left[\int_{\Omega} f(\zeta) \wedge (\hat{s_b}^{\star}\mu) + \int_{I\times\partial\Omega} f(\zeta) \wedge (\hat{g}^{\star}\mu) \right] \\ &+ \int_{\partial\Omega} f(\zeta) \wedge (\hat{s_h}^{\star}\mu). \end{aligned} \tag{10}$$

But the basic fact is as $q \geq 1$ and as s_h is holomorphic in z, the component of $(\hat{s_h}^{\star}\mu)$ of bidegree (p,q) in z is 0 for $\zeta \in \partial\Omega$. The third integral in (10) vanishes. Hence, we obtain the explicit solution of the $\bar{\partial}$-equation:

$$c(p,q,n)f(z) = \bar{\partial}_z \left[\int_{\Omega} f(\zeta) \wedge (\hat{s_b}^{\star}\mu) + \int_{I\times\partial\Omega} f(\zeta) \wedge (\hat{g}^{\star}\mu) \right]. \tag{11}$$

As the kernels defined by $(\hat{s_b}^{\star}\mu)$ and $(\hat{g}^{\star}\mu)$ in (10) are explicit and as the kernel $(\hat{g}^{\star}\mu)$ is closely connected with the strict convexity of the boundary, we can get precise estimations with them and prove that if f is bounded up to the boundary then the solution of the $\bar{\partial}$ is too bounded up to the boundary. In the general case of a bounded, strictly pseudoconvex, open set $\Omega := \{\zeta \in \mathbb{C}^{\mathbf{n}} \mid \rho(\zeta) < 0)\}$ where ρ is a strictly plurisubharmonic function defined in a neighbourhood of $\bar{\Omega}$ verifying $d\rho(\zeta) \neq 0$ for $\zeta \in \partial\Omega$, G. Henkin constructs a global section $\hat{s}_h$ on $(\overline{\Omega} \times \Omega) \setminus \Delta$ with values in E which is holomorphic in z for $\zeta \in \partial\Omega$ by glueing all together the section $\hat{s}_b$ and local sections $\hat{s}_h$ alongside the boundary $\partial\Omega$ obtained from the local Taylor development at the order 2 of a defining function ρ of Ω and correcting it by an adapted global resolution of the $\bar{\partial}$-equation on a neighbourhood of $\bar{\Omega} \times \bar{\Omega}$.

After this first pioneer G. Henkin's work, one of the most striking results in this field of research has been the characterisation of the zeros of Nevanlinna class functions with Blaschke condition, which we have independently obtained Henkin [3] and I [14,15]. Reading G. Henkin's writing, I was pleased to see we had operated with perfectly parallel processes and shown the obvious efficiency in this type of problems of the concept of closed positive current due to Pierre Lelong. Let us now more explain this link with Pierre Lelong's work.
The Nevanlinna class $N(\Omega)$ (resp. the space $H^p(\Omega)$, $0 < p < \infty$) is the set of holomorphic functions f on Ω such that

$$\begin{aligned}&\limsup_{\epsilon\to 0,\epsilon>0}\int_{\partial\Omega_\epsilon}\log^+|f|\,\mathrm{d}S_\epsilon<\infty,\\ &\text{resp.}\\ &\limsup_{\epsilon\to 0,\epsilon>0}\int_{\partial\Omega_\epsilon}|f|^p\mathrm{d}S_\epsilon<\infty,\end{aligned} \tag{12}$$

where for $\epsilon > 0$, $\Omega_\epsilon := \{\zeta \in \mathbb{C}^{\mathbf{n}} \mid \rho(\zeta) < \epsilon\}$ and $\mathrm{d}S_\epsilon$ is the Euclidian area of $\partial\Omega_\epsilon$. We trivially have

$$H^\infty(\Omega) \subset H^p(\Omega) \subset N(\Omega), \tag{13}$$

for all p where $H^\infty(\Omega)$ is the space of holomorphic bounded functions in Ω. A complex hypersurface X (with given multiplicities and singularities) of Ω verifies the Blaschke condition if (by definition):

$$\int_X |\rho(z)|\, d\sigma(z) < +\infty, \tag{14}$$

where $d\sigma$ is the area element of X. A classical result of Lelong [7] states that the integral is locally well defined despite the singularities of X. In the case of one variable when Ω is the unit disc, X is a discrete sequence $\{a_j\}$, $j \in N$ of Ω (each a_j may be repeated with multiplicity) such that Blaschke condition simply means that $\sum_{j=1}^{\infty}(1-|a_j|) < +\infty$. The classical theory of $H^p(\Omega)$ spaces for $n = 1$ states that Blaschke condition characterises the sets of zeros of functions in $N(\Omega)$ and in $H^p(\Omega)$ for all $1 \le p \le \infty$. For a given sequence verifying Blaschke condition, the explicit Blaschke product $B = \Pi_{j=1}^{\infty}\frac{\bar{a}_j}{|a_j|}\frac{a_j-z}{1-\bar{a}_j z}$ provides a bounded holomorphic function in Ω vanishing precisely at each a_j with given multiplicities. For $n > 1$ the characterisation of the sets of zeros of functions in $H^p(\Omega)$ necessarily depends on p (Rudin [13]) and should probably be a very intricate problem. Nevertheless, in 1975, G. Henkin and I have proved that Blaschke condition characterises zeros sets of functions in the Nevanlinna class when Ω is a strictly pseudoconvex bounded domain of class C^2 such that $H^2(\Omega, Z) = 0$. It easily results from an extension to several variables of the Jensen formula (Stoll, Lelong [8] or [9]) that the condition is necessary. Of course the main difficulty is to prove the condition is sufficient, that is, to construct something like a "Blaschke product" in several complex variables.

We followed the successful Pierre Lelong's method [8]. He built in $\mathbb{C}^{\mathbf{n}}$ the equivalent of the canonical Weierstrass product (that is an holomorphic function F in $\mathbb{C}^{\mathbf{n}}$ of minimal growth vanishing on a given zeros set) as a plurisubharmonic potential $\log|F|$

solving in $\mathbb{C}^{\mathbf{n}}$ the so-called today's Lelong–Poincaré equation: $\frac{i}{\pi}\partial\bar{\partial}\log|F| = [X]$, where $[X]$ is the current of integration on the hypersurface X. We solved the same equation in Ω. As P. Lelong, we more generally solved the equation $\frac{i}{\pi}\partial\bar{\partial}V = T$ where T is a given closed positive current of bidegree $(1, 1)$ in Ω verifying the Blaschke condition (T is positive if for all differential forms α_j of bidegree $(0, 1)$, $1 \leq j \leq n-1$, the (n, n) current $i^{n-1}T \wedge \alpha_1 \wedge \bar{\alpha}_1 \wedge \alpha_2 \wedge \bar{\alpha}_2 \wedge \cdots \wedge \alpha_{n-1} \wedge \bar{\alpha}_{n-1}$ is a positive measure). Taking for T the current of integration $[X]$ on the hypersurface X, Lelong [8] proved that the solution V verifies $V = \log|F|$ for some holomorphic function F in Ω vanishing on X with the right multiplicities.
P. Lelong's views have also taken a prominent part in the following way. Let ρ be a smooth strictly plurisubharmonic function defining the bounded open set $\Omega = \{z \in \mathbb{C}^{\mathbf{n}}; \rho(z) < 0\}$. The Blaschke condition on T can be written as $\int_\Omega -\rho(i\partial\bar{\partial}\rho)^{n-1} \wedge T < +\infty$. Stokes formula provides the following equality: $\int_\Omega -\rho(i\partial\bar{\partial}\rho)^{n-1} \wedge T = \int_\Omega (i\partial\rho \wedge \bar{\partial}\rho) \wedge (i\partial\bar{\partial}\rho)^{n-2} \wedge T < +\infty$. It means that the complex tangential component of T, $(i\partial\rho \wedge \bar{\partial}\rho) \wedge T$, has its coefficients in $L^1(\Omega)$, hence it satisfies something stronger than the Blaschke condition. This strong condition on the behaviour at the boundary of the complex tangential component of T (found in another more restrictive form by P. Malliavin so that I called it the "Malliavin condition" [11]) was the first compulsory and decisive step to solve the equation $\frac{i}{\pi}\partial\bar{\partial}V = T$ with the expected Nevanlinna estimate $\sup_{\epsilon>0}\int_{\rho(z)=-\epsilon} V^+ \mathrm{d}S_\epsilon < +\infty$. Indeed using Poincaré's classical explicit solution for the de Rahm operator d, we can solve at first the equation $dw = T$ in Ω. We consider the component $w_{(0,1)}$ of w of bidegree $(0, 1)$ which is $\bar{\partial}$ closed. Then, using the preceding bound on $(i\partial\rho \wedge \bar{\partial}\rho) \wedge T$, we can estimate the coefficients of $w_{(0,1)}$ and of $|\rho|^{-\frac{1}{2}}\bar{\partial}\rho \wedge w_{(0,1)}$ and show that there are measures of bounded mass on Ω. Of course that means once again that the complex tangential part $\bar{\partial}\rho \wedge w_{(0,1)}$ of $w_{(0,1)}$ verifies a more restrictive condition than $w_{(0,1)}$. Then we have to solve the $\bar{\partial}$-equation: $\bar{\partial}u = f := w_{(0,1)}$ on Ω with the appropriate estimate on u and to take $V := 2\mathrm{Re}\, u$. That is the most difficult part of the construction. We proved:

Theorem 1 *Let f be a current of bidegree $(0, 1)$ on Ω, $\bar{\partial}$-closed, such that the coefficients of f and of $|\rho|^{-\frac{1}{2}}\bar{\partial}\rho \wedge f$ are bounded measures on Ω, then there exists $u \in L^1(\Omega)$ such that*

$$\bar{\partial}u = f, \quad in\, \Omega, \tag{15}$$

and such that u has a boundary value in $L^1(\partial\Omega)$ in the sense of the Stokes formula:

$$\int_{\partial\Omega} u\phi = \int_\Omega f \wedge \phi + \int_\Omega u\bar{\partial}\phi \tag{16}$$

for all forms ϕ of bidegree $(n, n-1)$ and of class C^1 in $\bar{\Omega}$.

That means roughly speaking u and therefore V are in $L^1(\partial\Omega)$. To prove this theorem, we built kernels similar to those of formula (11) but which are more symmetric in the pair (ζ, z) and which highlight the value of u on the boundary $\partial\Omega$. For instance, in the case of the Euclidean ball of $\mathbb{C}^{\mathbf{n}}$, if we apply this kernel to an holomorphic function in $\bar{\Omega}$, we obtain the Poisson–Szegö reproducing kernel for the functions in the Hardy space $H^2(\Omega)$.

That kind of strong estimate of the complex tangential component of a closed positive current T (or consequently of the (0, 1) form $f = w_{(0,1)}$ associated with T by the Poincaré formula) remains today as an essential argument in the numerous lines of research on hard analysis about zeros of functions in Hardy classes on pseudoconvex domains.
These last results had a major impact especially in France where they have contributed to the development of schools of complex analysis in Bordeaux with Eric Amar and Philippe Charpentier, in Toulouse with Anne Cumenge, in Lille with Anne-Marie Chollet, in Paris with Paul Malliavin and his scholars, with Pierre Dolbeault and Christine Laurent, with Nessim Sibony and Nicolas Varopoulos, with François Norguet and Guy Roos. These results have strongly impacted Swedish school around Bo Berndtsson and the American school too with, for instance, W. Rudin, S. Krantz, M. Range, Y.T. Siu.

On the other hand, using these new explicit integral kernels solving the $\overline{\partial}$-equation, G. Henkin, J. Leiterer and other mathematicians [4] were able to highlight a complete new approach of the Andreotti–Grauert theory of Stein manifolds and q-convex manifolds and to get more precise results which were out of range before.

G. Henkin has also obtained other famous results in many other connected fields of research as Theoretical Physics, Mathematical Physics, Mathematical Economy, Theory of Information. But we have decided to focus our attention on the results of the strongest importance for his international recognition and his scientific career.

In the same vein, using constructions or integral representations as explicit as possible, he has made fundamental contributions to Integral Geometry, Algebraic Geometry and Mathematical Physics through the study of Abel's transformation, Radon transform and Penrose transform. He has also deeply pushed forward the understanding of the equations of Mathematical Physics, more specifically those of inverse problems.

Between 1970 and 1985, G. Henkin was living and working in Moscow and then it was very difficult for a Russian mathematician to go out and visit foreign countries. It was only in June 1987 that he could participate in a colloquium in Montpellier. He could stop in Paris and I met him for the first time. Earlier, I only knew his articles. I remember with some emotion, his first talk in Paris and his joy and his delight when he discovered a part of Paris and the Seine with my family, on a sightseeing boat. It was the beginning of a long friendship.

Then, what seemed to me quite impossible happened in 1989: the fall of the Berlin wall with all its consequences for the Eastern European countries. G. Henkin could come back several times and quite longer to France. At the end, he could apply for a position in Paris 6 University where he was elected as a full professor in 1991. He had opted to begin a foreign adventure, leaving his country for France. He immediately collaborated with the Complex Analysis Seminar founded by P. Lelong in the sixties and got involved in its administration. In 1981, P. Lelong retired, but P. Dolbeault, J. M. Trépreau and I shared the organisation of the Seminar with G. Henkin. G. Henkin played a leading role there and developed new subjects, concerning particularly Abel's transformation, Integral Geometry and Mathematical Physics. So he has played a vital part in the development of Complex Analysis in several variables related to Mathematical Physics.

For about twenty years he was supervising so many doctoral theses. He did a magnificent job not only thanks to his high level in mathematics but also because he was able to find new research themes accessible to our students and was kind to his students and his colleagues. Many of his former students became full professors, associate professors or assistant professors. I shall mention Tien Dinh who joined us in October 2005, as a full professor after having spent seven years at the University of Orsay. Since that time, he also took his part in the organisation of Complex Analysis Seminar. I will mention Stéphanie Nivoche too, one of his first students who is now full professor in the University of Nice; Pascal Dingoyan who is associate professor at Paris 6 since 1998 and Luc Pirio who, in 2007, was the first of our team to get a position in the National Center of Scientific Research affiliated to Rennes. I have also to remind that his student Bruno Fabre obtained very deep results about Abel transform in relation to P.A Griffiths's work and that he unfortunately died very early in 2010 (he was only 38 years old) before he could achieve major parts of his work.

For many years, G. Henkin was assuming responsibility for pre-doctoral teaching in pure mathematics too, DEA (Diploma in Advanced Studies, now called Master II), which is a key position for defending mathematics in Paris 6 University.

In 1983, he was invited for giving a talk at the World Mathematics Congress in Warsaw.

In 1992, his work on Schumpeter dynamics and non-linear waves theory was rewarded with Kondratiev Price of Russian Academy of Economical and Mathematical Sciences.

In 2011, he received Stefan Bergman Price from the American Mathematical Society for all his scientific works.

He was honoured in two international colloquiums, in Paris, the first in June 2007, and the second one in October 2012.

Gennadi Henkin was above all, a scientist, one of the most eminent representatives of Complex Analysis in the world. He was also a simple, self-effacing, always smiling man. He will continue to be a model of scientific engagement for us and we will always keep in mind the memory of his fruitful works and of all the vocations he has raised.

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