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Research on Mathematics Education in China in the Last Decade: A Review of Journal Articles

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Abstract This article reviews literatures on mathematics education in China in the last decade. It focuses on papers that were published after 2000, and Chinese Social Science Citation Index (CSSCI). Some influential journals in the field of mathematics education, such as the Journal of Mathematics Education and the Mathematics Instruction were also reviewed. The author shares in this article with international scholars the most recent research in the field of Chinese mathematics education. Mathematics education in China mainly focuses on the essence of mathematics classroom instruction, student learning of mathematics, mathematics teacher education, and its curriculum reforms.

Keywords mathematics, mathematics education in China, review

Introduction

Since the 1990s, the world began to pay more attention to how Chinese students learn mathematics, through International mathematics testing and competitions, such as the IAEP, TIMSS, and PISA, which have shown an amazing and outstanding performance of Chinese students in mathematics whereas, it is widely believed that Chinese students are passive in learning mathematics, and their experiences are limited to memorizing, imitating, drilling, and testing (Zheng, 2001; Fan, Huang, Chai, & Li, 2005; Liang, 2005). This phenomenon is referred to as the “Paradox Chinese learners.” The presentations and discussions at the Forum of All Chinese Mathematics Educators at the Ninth International Congress on Mathematical Education (ICME-9), which was held at Tokyo in 2000, once again, resulted in a new wave of interest in “how Chinese learn
mathematics” among international scholars. Chinese scholars, of course, became a natural part of this advocacy. The English edition of *How Chinese Learn Mathematics* (Fan, Wong, Cai, & Li, 2004) which was published in 2004, The authors adopted international perspectives and research methodologies in writing this article, which makes the achievement of Chinese mathematics education a part of the mainstream international mathematics education. The fact that international scholars are paying closer attention to Chinese mathematics education, it motivates Chinese scholars to paint a clearer picture of the most recent progress and development on Chinese mathematics education. This article will review, primarily, papers that have been published after the year 2000, and are included by the Chinese Social Science Citation Index (CSSCI). Some journals are not from the CSSCI, but are influential in the field of mathematics education, such as the *Journal of Mathematics Education* and the *Mathematics Instruction*. The author analyzes these studies, and shares this article with international scholars, the recent research in the field of Chinese mathematics education. The mathematics education discussed in this article mainly focuses on basic education, that is, from the first to the 12th grade.

**The Essence of Chinese Mathematics Education**

The evaluation and interpretation of the “Paradox of Chinese Learners” has not only become a hot topic in both international and comparative education and psychological studies (Biggs 1994; Biggs & Watkins, 1996; Leung, 1995, 2001; Morris, Adamson, Au, Chan, et al., 1996; Watkins & Biggs, 2001), but also has directly promoted the examination on the essence of Chinese mathematics education. Chinese scholars have primarily adopted the cultural perspective when interpreting and elucidating the nature of Chinese mathematics education. Zhang (2005) pointed out that Chinese mathematics teaching could be summarized as teaching the basics in two areas: memorizing basic knowledge and mastering basic skills. The characteristics of this principle are that memorization leads to understanding, speed (of calculation) translates into efficiency, rigor in reasoning forms rationality, and variation supplements repetition (Zhang, 2006). This principle was gradually formed under the influence of the Chinese culture in thousands of years. The Chinese culture of rice cultivation, Confucianism, and the system of civil examination are all reflected on the Chinese mathematics education (Zhang, 2006). Without a solid foundation and a basic knowledge with good skills, creativity and individual development are not possible. The idea of “two basics in mathematics teaching” seeks the balance between foundation (knowledge and skills) and development, and explores ways that help students’ further development on the foundation of
the “two basics” (Zheng, 2006; Zhang, 2008). The meaning of the “two basics” is expanding. For example, Shi (2007) pointed out, in revising the curriculum standards for mathematics in fulltime compulsory education, that as attention is paid to basic knowledge and basic skills, it is also necessary to accumulate “basic mathematic experiences” and develop “ways of mathematical thinking.” This is an important progress in mathematics education research, and it has given further explanation and expansion to the meaning and essence of Chinese mathematics education.

The combination of teacher’s authority and student-centeredness in mathematics classroom also embodies the essence of Chinese mathematics education. Huang (2005) pointed out that student learning behaviors share unique aspects in China as well as in other Confucian Heritage Cultures (CHC), such as the orientation of societal achievement (as opposed to the orientation of personal achievement in the West), the emphasis on working hard, the tendency of attributing success to personal effort, persistence, and the belief of practice making perfect. This philosophy implies that diligence compensates low intelligence. Individuals in the CHC are not necessarily born intelligent, but hardworking. In a CHC environment, classroom teaching provides students with a large amount of drilling. The atmosphere of a typical class is orderly and lively at the same time. An ideal learning environment is one that is quite, where students direct their energy to learning activities and follow directions, and where teachers and students can continue discussions after class. This kind of atmosphere embodies the teaching approach that is “teacher-led and student-centered.” This approach, which is unique to Chinese classroom teaching, challenges the Paradox of Chinese Learners. Some Western scholars believe that the learning of Chinese students is too mechanical, and the reason for which is that they perceive memorization and understanding as dichotomy, and perceive repetition as mechanical learning.

Chinese mathematics education stresses that repetition with variation is beneficial for learning. This point of view has been well explained in the philosophy of “teaching with variation.” The study by Gu et al. (2005) demonstrates that the Chinese “teaching with variation” includes both “conceptual variation” and “procedural variation.” “Conceptual variation” emphasizes understanding a concept from multiple perspectives. It encourages utilizing different teaching materials and examples to help students understand the nature of a concept or changing nonessential aspects of problems to make the essential aspect of a concept clearer. “Procedural variation” is a gradual progression of teaching activities. It helps with the understanding of concepts, prepares for problem solving, and constructs a system of mathematical experiences. At surface level, several aspects of Chinese mathematical teaching can easily be perceived by Western researchers as teacher-centered and
spoon-feeding, such as large class sizes, teacher-controlled class activities, and the teacher’s highly efficient and easily understood lectures. However, even with a large class size, if a teacher adopts the strategy of “teaching with variation,” students are still able to participate in learning activities. These arguments and Huang’s viewpoints are mutual supportive.

The Chinese mathematics education pursues the balance between building a good foundation for students and their further development, and the balance between students’ active learning and teacher’s effective instruction. The teacher’s controlling in mathematics classrooms does not necessarily put students in a passive position, and repetition does not necessarily equal to mechanical drilling.

When Chinese scholars elucidate the nature of Chinese mathematics education, they also study, from a more microscopic perspective, how the philosophy of Chinese mathematics education is embodied in classroom instruction. The majority of research at the micro level focuses on classroom instruction.

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**Research on Teaching Mathematics in the Classroom**

Chinese scholars adopt a variety of research methods in studying mathematics instruction, such as experiments, analysis of class video, survey, and design. Researchers have studied the features, typical lessons, and essential elements of mathematics instruction in k-12 schools, and have pointed out the problems that exist as well. Many international scholars have become interested in mathematics instruction in the unique context of Chinese culture, thus, “mathematics classroom instruction in China” has become a popular topic in the field of comparative studies.

**Research by Chinese Scholars**

Research by Chinese scholars suggests that Chinese mathematics classroom teaching strives for student further development while providing the “two basics” (Zhang, 2006). Shao and Gu (2006) systematically analyzes both the external and internal features of teaching the “two basics.” Their research shows that teaching the “two basic” has a relatively stable structure. A typical class progresses as the following: lectures on knowledge and skills, examples and applications of knowledge and skills, and practice and drill on knowledge and skills. To put it another way, this approach allows students to understand the concepts, which is followed by the teacher’s examples showing how the concepts work, and lastly, students master the concepts through practice and exercise. There are five elements in a standard lesson: going over prior knowledge that is relevant to the
new topic, introducing the new topic, analyzing and lecturing on the new topic, applying the new topic to situations, summary and homework assignments. Every element has its own purpose and basic requirements. Teaching the “two basics” is a teacher-controlled and highly efficient model. It pays special attention to memorizing basic knowledge and applying basic skills. Every lesson has specific instructional tasks and objectives, including knowledge and skill topics that the teacher needs to cover, the teacher’s objectives for the lesson, the basic training that students need (problems that they need to solve), students’ objectives for the lesson, to what extent are these objectives achieved (for example, the rate of problems correctly solved). The teacher organizes the lesson and controls class activities around these objectives.

Other researchers proposed specific teaching models, based on the philosophy of the “two basics,” targeting particular student groups in the local area with specific instructional objectives. For example, the “situation-problem” approach was proposed for students who lack interest and motivation in learning in the Southwest (Lu & Wang, 2001; Xia & Wang, 2003; Yao & Lu, 2005; Lu & Wang, 2006). The core element of this approach is to train students to ask questions and solve problems. The specific procedures in the classroom follow this order: (1) review and prepare for the new topic, and raise questions; (2) present vivid and easily understood situations and encourage students to ask questions; (3) guide students to focus on class topic and ask relevant questions; guide student questions in line with instructional objectives and avoid off-topic questions; (4) assign to students “situational problems,” and facilitate student questions as they work on mathematics; (5) facilitate a discussion when students raise questions, instead of answering their questions immediately; and (6) guide students in solving problems as problems arise and avoid forcing students into a pre-designed “situation–problem,” thus avoiding student passivity in the new model (Lu & Wang, 2001, pp. 12–13). Researchers have expounded this teaching strategy at the theoretical level and constructed a “situation–problem” model in teaching mathematics (Lu & Wang, 2006; Xu & Lu, 2008).

The “situation–problem” teaching model underlines four interconnected and dynamic essential elements: setting the mathematical situation is the precondition, raising mathematical questions is the core, solving mathematical problems is the objective, and applying mathematical knowledge is the end. The entire process can be seen as a cycle of situation, problem, solving problem, applying mathematics, and situation again. This is an open and dynamic system that continuously extend itself and where its elements are organically connected (Xu & Lu, 2008, p. 87).

As Chinese scholars study the mathematics classroom teaching models, they also examine, through both experiments and analysis, critical components in mathematics instruction. Some researchers focus their analysis on mathematical
classroom dialogues (Li & Ni, 2007). Their research reveals that, regarding the source of authority, expert teachers tend to leave it to the teacher-student community to assess the reasonability of questions and answers, while non-expert teachers tend to evaluate on their own student responses; regarding the type of questions, expert teachers tend to ask more interpretive, analytical and comparative questions than non-expert teachers, while non-expert teachers tend to ask more questions that require simple recall of information.

Regarding “explorative learning,” some researchers carried out long-term experiments in mathematics classrooms (Wu, 2004). This study suggests that the ultimate goal of “explorative learning” in mathematics classrooms is to cultivate the sense of exploration, which includes the sense of guessing, using examples, categorizing, and determining the scope of exploration. Based on the above goal, the researchers avoided the conventional laws of deduction, memorization, and application, and reorganized mathematical content according to the structure of knowledge and the process of human cognition, to meet the needs for explorative learning.

For instance, after studying the organization of operational laws in mathematics textbooks of elementary schools, they pointed out that the current arrangement is to learn mathematical operations first, and then learn about operational laws, and the nature of deduction is not included.

These features lead both teachers and students to focus on individual operations, but fail to see the entire picture. In order for textbooks to serve the purpose of training students’ sense of exploration, the researchers reorganized these textbooks. They put the mathematics operations in two volumes: addition and deduction in volume I and multiplication and division in volume II. They put the instruction of the laws of addition and subtraction after teaching the operations of addition and subtraction, and the instruction of the laws of multiplication and division after teaching the operations of multiplication and division. The new order helps to develop students’ awareness of speculating and providing evidence for arguments. For instance, the learning of laws of subtraction provides students’ the possibilities of parallel thinking when they learn division operation, and after experiencing the connection between addition and multiplication, it is easier for students to catch the connection between subtraction and division. This approach helps students to understand and master the laws of the four operations and the internal connection between them.

Studies by Chinese scholars analyzes the basic features of mathematics classroom instruction and its effectiveness, which include, for example, the following: (1) an emphasis on reviewing prior knowledge and introducing the new topic, and basing the new topic on prior knowledge; (2) gradual progression step by step, and an emphasis on the structure of the lesson; (3) an emphasis on analysis of typical cases and examples; (4) an emphasis on reviews and
reinforcement; (5) an emphasis on practice and drill; and (6) an emphasis on feedback. Meanwhile, researchers also admit that, in large class instruction, teacher lecturing takes up the majority of class time, which may not be the most effective approach. The real challenge for Chinese mathematics classroom teaching is to maintain all of the beneficial features listed above and promote students’ participation at the same time (Liang, 2005).

**Studies by Scholars in Other Countries**

As Chinese scholars examine the features and models of mathematics instruction through observation, survey, interview, and analyses of class videos, scholars in other countries also try to study the distinguished features of Chinese mathematics instruction through various means.

When comparing mathematics classroom instruction between American and Chinese teachers, Ma (1999), a Chinese scholar who studied in the U.S., noted that Chinese mathematics teachers lay more emphasis on students’ understanding of concepts and theories from different angles whereas their American counterparts pay more attention to specific operations. American teachers usually only know the operations, but not the advantages and disadvantages of the operations, while Chinese teachers stress not only skillful operations, but also easier and more effective operations. It is not possible to achieve this goal by simply relying on mechanical training; the key is to encourage multiple solutions, and help students understand the advantages and disadvantages of each solution. “This is an important feature of Chinese mathematics instruction” (Zheng, 2001).

Dr. Lynn Paine, professor at Michigan State University, has been observing and studying Chinese mathematics instruction for years. Paine (2002) describes the Chinese mathematics instruction as a Virtuoso Model. She was impressed by the step-by-step instruction method (from easy ones to difficult ones). She believes that Chinese teachers’ thorough understanding of mathematics content comes primarily from the unique in-service teacher training model that emphasizes collaborative exploration.

Francis Lopez-Real and others studied three continuous mathematics lessons taught by a mathematics teacher in Shanghai. They interviewed the teacher, analyzed her lessons, and provided specific comments on the teaching. They found that the teacher tried to balance between teacher-centered instruction and student-centered exploration. They concluded that the teacher emphasized, to the same degree, both a mathematics foundation and student exploration. However, they also discovered that the teacher had a good control of the entire class. The exploration was limited to a small scale, and is not open or to a large scale (Lopez-Real, Mok, Leung, & Marton, 2005, p. 315). Researchers analyzed her
lessons using the five-component model: exploration, instruction (prior knowledge/review), summary, practice, and homework assignment. They found that her teaching was primarily composed of three components: prior knowledge/review, exploration, and guided practice. During the observed lessons, 32% of class time was spent on prior knowledge and review, 19% on exploration, and 45% on guided practice. The above comprised 96% of total class time (Lopez-Real et al., 2005, p. 316).

Huang and Wang (2007) applied the Learners’ Perspective Study (LPS) research model to their comparative study on mathematics classroom instruction in Shanghai, Hong Kong, and Macau. Their main method was observing and analyzing the video of teaching of binary linear equation. By coding instructional behavior, they analyzed the introduction of new concepts, the introduction of algorithms, or the complexity of mathematics problems. They concluded that mathematics instruction in Shanghai was most efficient, and time was allotted for student exploration. The teacher in Shanghai tended to introduce mathematics concepts through contextualized problems (which also included some review), and stress summing up laws that govern problem-solving, and reviewing previous knowledge when introducing new algorithms.

The above studies reflect some fundamental characteristics of Chinese mathematics instruction, for example, approaching concepts and principles from different angles, step by step progression of instruction, emphasizing multiple solutions to one problem, facilitate student exploration, applying mathematics concepts into practice with teachers’ guidance, and so on.

**Research on Mathematics Teacher Education**

Obviously, research on mathematics instruction cannot be separated from the professional development of mathematics teachers. The training of mathematics teachers has been a hot topic for Chinese scholars in the last decade. Besides summarizing and reflecting on experiences in training mathematics teachers, Chinese scholars primarily focus their research on mathematics teachers’ knowledge and skills, professional development of in-service mathematics teachers, and the curriculum and instruction for pre-service teachers. Most studies employed methods such as experimental design, action research, case studies, questionnaire and in-depth interview.

**Research on Mathematics Teachers’ Knowledge and Skills**

Mathematics teachers’ knowledge comprises three components: mathematics content, pedagogy, and educational psychology. They are interconnected and dynamically developed, and all have an impact on mathematics instruction, thus,
Chinese scholars have studied all the components. Li, Yu, Tang, and Huang (2007) studied mathematics teachers’ knowledge in elementary and middle schools through survey and interview. The questions in their survey are in three areas: mathematics content knowledge, pedagogical knowledge, and psychological knowledge. Each of the three areas includes some sub-areas. For instance, mathematics content knowledge includes its theory, appreciation of the nature of mathematics, understanding of approach of solving its problem, and knowledge of mathematics history. Their survey showed that mathematics teachers in both elementary schools and high schools are confident about their knowledge. They have a good understanding of the knowledge of mathematics methodology, but only limited understanding of its ontology.

Through survey and interview, they also studied the impact of mathematical knowledge structure on mathematics instruction (Li et al., 2007). Study showed that teachers perceive mathematics knowledge and education knowledge are more important to its instruction. There is not a significant difference between the two. Psychology knowledge comes next. After quantified their data, researchers concluded that 76.6% of teachers believe that knowledge relate to mathematics concepts, its principles, and theories have great impact on its instruction. This group of knowledge ranked first in terms of importance on mathematics instruction. About 68.1% teachers believe that knowledge relate to its methodology has impact on its instruction; 45.2% teachers believe that the understanding of mathematics ontology is important to its instruction. These conclusions provide valuable reference for the training of in-service teachers. Does the rich educational value of mathematics history worth its teachers to reevaluate on it? Is it necessary for mathematics educators to integrate its history into their teaching?

At the same time that some researchers survey mathematics teachers’ knowledge structure, others studied on mathematics teachers’ knowledge and capacity in specific areas (Huang & Li, 2006; Zhou & Bao, 2009; Zhang et al., 2007). Zhou and Bao (2009), take mathematical proof as an example, surveyed on high school mathematics teachers’ understanding of mathematics, their mathematical knowledge and perception of its instruction. They first investigated, through survey, if teachers appreciate the difference between mathematical proof and proof in everyday life. Their conclusion was that Chinese high school mathematics teachers have a clear understand of mathematical proof. Next, they provided several mathematical problems to test teachers’ proof capacity. They concluded that Chinese high schools mathematics teachers have a solid grasp on mathematical proof. Last, they examined teachers’ understanding of mathematics instruction, including distinguishing different mathematical proof levels among students, diagnose and analyze difficulties and barriers that students encounter during their learning of mathematical proof, and main strategies of teaching
mathematical proof. Data showed that Chinese teachers appreciate the major difficulties the students face when learning mathematical proof and corresponding solutions although, at the same time, they are not concerned by it different levels among students. These conclusions echo Li and Yu’s research well.

The study by Zhang, Shen, and Lin (2007) was primarily concerned with elementary school mathematics teachers’ capability of processing information in its classrooms. By analyzing classroom video and measuring the time needed before reacting to situations, they examined mathematics teachers’ ability and speed in processing information. The results indicated that, while stressing on content instruction, more experienced teachers also paid attention to managing classroom activities and classroom atmosphere. Factors that had a significant impact on teachers’ ability to process information and professional development include teachers’ mathematical knowledge, experience in teaching, and experience in general classroom instruction (Zhang, et al., 2007, p. 76).

These research findings provide valuable reference for pre-service and in-service teacher training. With in-service teacher training, it is especially important to emphasize not only improving teachers’ mathematical knowledge and its instruction, but also developing teachers’ ability in organizing classroom activities and adjusting classroom atmosphere. This will help teachers’ development in all dimensions: mathematics content, pedagogical knowledge, and psychological knowledge, thus continuously improving teachers’ content knowledge and instructional capacity.

Has in-service teacher training been utilizing the research findings mentioned above? We will find the answer in the research studies that follow.

**Research on the Forms of Professional Development of In-Service Mathematics Teacher**

At policy level, the continuing education of in-service teachers is highly valued in China. There are various forms of training for in-service teachers, including, for instance, short training courses, topical workshop, classroom demonstrations, and seminars. No matter what form is taken; teachers must eventually apply into practice what they learn in mathematics, pedagogy, and psychology. The reality is that most teachers who participated in such trainings found it difficult to integrate what they learned into their classroom instruction, which seems to be a puzzle. Some researchers turned such puzzles into their research topic, and have been fruitful in shedding lights on the professional development of in-service mathematics teachers.

For example, there is a study with the title *A Faculty Development Model for*
In-Service Teachers That Connects Instructional Improvement with the Learning of Theories (Wang, Zhou, & Gu, 2006). This model requires teachers to begin with problems encountered in everyday classroom instruction, and seek solutions while learning educational theories. For example, a topic for classroom instructional improvement could be “how to effectively conduct a mathematics lesson,” and the related theoretical study could be “how to facilitate active mathematical thinking.” This makes teachers try to improve their teaching under the guidance of theories. The study indicates that, in the professional development of in-service teachers, it is important that teachers give advice to one another, and those who are well-versed in theory guide those who are not. It is also important for teachers to exchange ideas through case studies, and to emphasize putting theory into practice after learning something new (Wang, et al., 2006, p. 111).

The study by Gu, Yang, and Wang described an effective professional development model for in-service mathematics teachers, that is, professional development through instructional case studies (Gu & Wang, 2003; Gu & Yang, 2007; Yang, 2008). “Instructional Case” is the keyword in this study, which refers to real life instructional cases. It is the description and re-presentation of an instructional problem and the decision-making process, that is, “tell the story of a teaching situation” (Yang, 2008, p. 72).

This project focused on providing guidance to practicing teachers by analyzing their instructional cases. Topics that had received guidance from this project included the following: “from objects to equations: the mathematicalization process in teaching divisions with remainders in elementary schools,” “designing scaffolds and guiding explorations: teaching the Pythagorean Theorem in secondary schools,” “deducting new knowledge from prior concepts: teaching regular polygon in a mathematics activity class in secondary schools,” and so on. Apparently instructional cases are real life instructional situations where the content is the medium and the focus is on a narrow topic. Instructional case studies refer to the process of conducting a teaching session that can be used as an instructional case.

The researchers administered a survey at the beginning of their research project to learn about teachers’ needs for training, and the results showed that teachers needed the professional guidance of instructional case analysis and they needed to reflect on the entire process, from case analysis to follow-ups of their action after the training. Based on the data they collected and subsequent analysis, they developed the following “action education model for faculty development that is based on instructional case analysis” (Gu & Wang, 2003, p. 11).

The “action education” in the above diagram is a way of education that targets at learning practical wisdom and applying theories into practice. It was originated from the needs for professional development of teachers, and adopted from
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action research some reasonable components and revised them.

Research on the forms of professional development of in-service mathematics teachers in China is still in its infancy, and some studies in this area are nothing more than summary of experience and reflection on practice.

**Research on the Curriculum and Instruction for Pre-Service Mathematics Teachers**

The study on the training of pre-service mathematics teachers is a component of its teacher education research, and is also a topic that has received much attention these days (Li, 2004). Not much has been done on the topic in China; however, the limited existent studies can still give us some inspiration.

Regarding instruction, Gu, Zhou and others conducted experimental studies on instruction that encourages exploration and research, and on the role of the professor in instruction (Gu, 2006; Zhou, 2005). To answer the call from the Department of Education to “promote instruction that encourages research and to cultivate college students’ creativity,” Gu (2006) in elaborating on the features of research-oriented instruction and the roles that instructors play in this kind of instruction. Based on practices in teaching abstract algebra and mathematical culture in the undergraduate curriculum, he analyzed the role of the instructor. He suggested, for example, in research-oriented instruction the instructor become the “architect,” one who organizes, inspires, guides, encourages, and facilitates (Gu, 2006, p. 4).

Zhou’s (2005) study approaches this topic from a more micro level and focuses on how an instructor can teach an effective lesson at the undergraduate level. He expounds “what a good lesson is,” and suggests ways of self assessment. He believes that a good lesson has the following characteristics. One, the instructor is able to connect the dots of knowledge and make them a line, and along the line completes all the entire process. Two, the instructor is able to skillfully utilize the knowledge being taught at the moment to scaffold learning that will happen later, so that students can experience the connection within the scope of the content, and connect lines of knowledge and make them a plane. Three, the instructor is able to view the course being taught from a higher perspective, not only from the perspective of a narrow discipline, but also from the perspectives of natural sciences and humanities.

Regarding curriculum studies for pre-service mathematics teachers, although appropriate research design and methods were not employed, such summary of experience is worth sharing. In addition, this is an area that receives much attention among researchers in the world. Zhang and Zhu’s (2006) article explains the concept of curriculum reform for pre-service teacher training, that is, “to have adequate theoretical foundation, to focus on practice, to have innovative
content, and to improve the total quality of the teacher candidate.” This concept stresses that mathematics teacher education should serve the reform and development of education at K-12 level, and the curriculum for its teacher education should include foundation courses and modular professional courses. To be more specific, current model of four years of teacher training is divided into two segments: the first two years are for building the foundation when students complete their general education and the mathematical foundation as teacher candidates; and the last two years are for the professional training of mathematics teachers, when students learn modern pedagogical theories, the art of teaching, and practice their teaching skills. This is a typical model for mathematics teacher education in college, although it is not the only model. Mathematics education programs at each college should have their own curriculum with their unique features.

**Research on How Students Learn Mathematics**

How students learn mathematics is the sub-area that is most productive and bears most fruit within mathematics education research. Studies in this sub-area may be put into various categories, such as student gender, grade level, cognitive level, area of mathematics, research method, and so on. Chinese researchers have produced much scholarship in this sub-area and I will present my review of them by grade level: elementary school, middle school and high school.

**Research on How Elementary Students Learn Mathematics (1st to 6th grade)**

While Chinese scholars emphasize the study of elementary school students’ understanding of mathematics concepts and their skills (Liu & Chen, 2007; Liang, 2006; Wu & Xie, 2005), they are also concerned with the study of affective learning, learning anxiety, and students who have difficulties with it (Luo, 2008; Chen, 2007).

For example, Liu and Chen (2007) tested the ability of picture viewing of 405 students from the 2nd grade to the 6th grade by administering a “pictorial test.” The test, which is intended to measure students’ ability to recognize images, included questions on three dimensions and in three aspects. The three dimensions were recognizing image, reading image, and drawing images, and the three aspects were from life to life, from life to mathematics, and from mathematics to mathematics. The authors stated that, the “image recognition” questions required that students recognize the different perspectives of the spaces or geometrical objects being presented when they are looked at from different directions or angles.

The “image drawing” questions required that student draw perspectives of
spaces or objects from predetermined directions and angles. The “image reading” questions required that students construct the entire space or geometrical object based on perspectives from certain direction or angle. Regarding the three aspects, questions in the “from life to life” aspect required that students analyze the relationship between objects in life from the “real-life” perspective, based on where they are in life, and realize that different points of view are correspondent with different perspectives. Questions in the “from life to math” aspect required that students analyze objects in life that have certain patterns, from the “mathematical” perspective, therefore are more abstract. Questions in the “from to mathematics” aspect required that students analyze, through abstract thinking and imagination, the relationship and characteristics of geometrical shapes, based on the geometrical shapes given, and complete the transition from two dimensional figures to three dimensional ones.

The results indicate that there was no significant difference in the ability of image viewing in students from the 2nd to the 4th grade who had studied or encountered the relevant content; students from lower grades are as capable as, or more capable than, students from higher grades in understanding images. There was no significant difference between boys and girls, either. The study also found that there were differences in the cognitive development in the three dimensions among students at different ages. The 6th graders had poor performance in recognizing and drawing images than students in other grades, but had good performance in reading images than students in other grades. These results have implications for adjusting the geometry curriculum in the future for elementary schools.

Wu and Xie’s (2005) study expounds the true meaning of mental arithmetic, which is especially stressed in elementary mathematics instruction in China, and discloses an unfortunate deviation, which is the tendency to view mental arithmetic as simpler calculation on paper, to distinguish the two only by the time needed for calculation, and to improve mental arithmetic only through mechanical memorization. Teachers fail to see, from the psychological perspective, the value of mental arithmetic for developing students’ mathematical capabilities.

If we do not have research on the psychological aspect of how students learn mathematics, we would lack benchmark in assessing how mathematics instruction helps with student development, and students would find mathematics inaccessible and develop anxiety for mathematics. Luo’s (2008) survey found that 20.8% of elementary school students who were uninterested, indifferent or lack enthusiasm to mathematics exhibited low mathematics anxiety. Other students (54.7%) exhibited moderate mathematics anxiety: they had moderate unease and sense of urgency, were able to keep concentration, and could think actively and logically. Still others (24.5%) experienced intense anxiety when they
encountered mathematics or some sections of mathematics: they were overly tense and fearful, and could not think logically. Such things are happening to elementary school students, which prompts us to take the responsibility to study how to ease their mathematics anxiety.

**Research on How Students Learn Mathematics in Junior High School (7th to 9th Grade)**

There is also a rich body of research regarding mathematics learning among junior high school students, including research on how students learn mathematics concepts (geometry or probability and statistics), students’ mathematics ability (the ability to solve mathematics problems and self-control), and students’ general ideas of mathematics. It is regrettable that Chinese scholars did not conduct much standardized research on algebra learning in recent years. This article will present the research methods and achievements of Chinese scholars in recent years by giving a few examples.

Since China started the curriculum reform in 2000, probability and statistics have become part of middle school mathematics content. For this reason, research in this area came along. In order to find out what content of probability and statistics can be relatively grasped with ease or with difficulty, Xu and Chen (2007) investigated the learning of probability and statistics among junior high school students. Their research showed that, in statistics, students were able to understand that “mean, median, and mode are representatives of numbers....” They also understood well random sampling, and are able to accurately distinguish discrete data and continuous data. They, of course, encountered difficulties, too, when learning statistics. For example, they had trouble to understand the multiple forms of presentation of frequencies. They also needed improvement in obtaining and processing information from charts and diagrams. Students also had difficulty in understanding the concept of probability from a statistical perspective, for example, most students did not fully understand the concept of probability. These relatively objective conclusions may serve as guidance for teaching of probability and statistics in middle school, which was launched not long ago.

The teaching of geometry has always been stressed in mathematics education, and for that reason, research in this area is most fruitful. Zhang and Lian (2007) studied the various kinds of strategies that junior high school students use to solve word problems in geometry. Huang and Li (2007) studied, using qualitative methods, students’ understanding of geometric proofs. Zhang and Lian (2007) analyzed midterm exam papers of eighth graders in three categories: those with high, intermediate, and low proficiencies. They found that students at different proficiency levels adopted different strategies when solving word problems. High
proficiency students utilized “bird-eye view” strategy, which means that they were able to describe effectively the problem in mathematics language, and understand problems at a deeper level. They could skillfully integrate prior knowledge with the conditions and the problem, create intermediate conditions, and gradually get closer to solving the problem. They had masterly control of all elements in problem solving (for example, conditions given, the problem, prior knowledge, and so on). Students of intermediate proficiency used strategies that were experience-based. They were able to connect the given conditions with the problem, analyze the new problem mechanically based on prior problem solving experiences, and they lacked a holistic understanding of the problem. They relied too much on previous experience and could not adequately adapt to new problems; they tended to solve problems in an established routine.

Low proficiency students adopted the trial and error strategy, which means they simply derive some conclusions from known conditions without understanding the direction of the problem. These conclusions may have nothing to do with the solution. They were unable to apply previous knowledge to their problem solving, thus made the entire process one of trial and error.

The mathematics ability of junior high school students’ and their epistemic beliefs about mathematics have become a hot topic in Chinese mathematics education research in recently years. Research on the ability to solve mathematics problems (Xu & Yang, 2007), the ability of self monitoring (Yum 2004), and beliefs about mathematics (Zhou, Tang, & Huang, 2008) enriched the perspectives of Chinese mathematics education research. In order to study students’ beliefs about mathematics education, Zhou, et al., (2008) first interviewed students and mathematics teachers, and obtained preliminary data. Based on this information, they surveyed students with questionnaires that had open-ended questions. After processing their data using SPSS 11.5 and AMOS 6.0, they concluded that there are five interconnected, yet independent, factors in junior high school students’ beliefs about mathematics: the structure of mathematics knowledge, the stability of mathematics knowledge, the speed in learning mathematics, the ability in learning mathematics, and the methods of learning mathematics. The stronger the beliefs, the more effective they are in boosting student learning.

Research on How High School Students Learn Mathematics (10th to 12th Grade)

There is relatively less research done on high school students’ mathematics learning, and among the limited studies completed, most of them focused on the learning of mathematics concept from a micro perspective, for example, the understanding of infinity among high school students (Wang & Zhou, 2006), the understanding of sampling and function (Wang, 2005). In recent years more
attention has been given to the study of mathematical abilities, such as ability of mathematical reflection and the ability to build mathematical models among high school students (Zhang, Zhao & Yang, 2008; Xu & Lu, 2008).

Zhang and his colleagues (2008) reported their project on the study of mathematical reflection among high school students. Their project was completed in 7 steps. On one hand, they examined the nature of mathematical reflection and the factors that hinder students’ mathematical reflection, through literature review, observation, and surveys. On the other hand, they proposed a model for training students’ mathematical reflection, based on literature research and practical experience, as well as questionnaire, observation, interview, and homework analysis. The model reflects a complete process for training students’ mathematical reflection, which includes four parts: stimulating questions, modeling, practicing, and assessment, where the core is reflection.

Xu (2008) conducted experimental studies, from a comparative perspective, on students from the ninth to the eleventh grade. The results showed that students’ ability of building mathematics model was limited, but the ability did increase as they moved into higher grades. On average, students were only able to simplify real life situations and arrive at a real life model, but unable to translate the real life model into a mathematics model. The experiment also revealed that students’ basic mathematics ability decreased, which is somewhat worrisome.

**Research on Reforms of Mathematics Curriculum**

The Ministry of Education issued the *Mathematic Curriculum Standards for Full-time Compulsory Education* (trial version) (2001) and the *Mathematics Curriculum Standards for High Schools* (trial version) (2004). These mathematics standards were first experimented in some regions, and then gradually spread to other regions. These events announced the official beginning of the current reform in mathematics curriculum in basic education in China, and along with the reform, came research in this area. Research in mathematics curriculum reform has touched on a variety of topics, such as the basic principles of mathematics curriculum reform, the teaching and learning of mathematics and teacher training in the guidance of the new curriculum standards, the debates on the reform itself, and so on.

**The Philosophy of Mathematics Curriculum Reforms in China**

Chinese scholars systematically analyzed the beliefs that the mathematics curriculum reforms are based on, namely, the beliefs on mathematics and mathematics education.
Results show that the current mathematics curriculum reform reflects beliefs like “the nature of mathematics education is about mathematics itself” (Liu, 2006), “what nurtures mathematics education research is mathematics” (Shen, 2005), and the fundamental goal of mathematics education is “to promote student overall development from a mathematical perspective” (Ma, 2005).

More specifically, the philosophy of the current mathematics curriculum reform stresses that mathematics should service the public; mathematics is a supplement of natural language and is a tool that people need for communication, and mathematics should become an indispensable part of every citizen’s cultural preparation. In addition, some researchers also emphasize that, on one hand, mathematics is rational—it is rigorous, systematic, and deductive science. On the other hand, mathematics is perceptual—it is inductive science based on experience (Wang, 2002; Ding, 2002; Huang, 2001). Zhang (2000) points out in his study that the current mathematics curriculum reform challenged the idea that “only the system of pure deduction and proof is authentic mathematics,” and stated that mathematics is more than a series of dry, tedious, and esoteric symbols and formulas.

Based on these philosophies on mathematics, the current mathematics curriculum reform has made it clear that mathematics education should be student-oriented, and the ultimate goal is the holistic development of the student. It requires teachers break up from the education model of “knowledge-centered” and “subject-centered”, but be concerned with the social function of mathematics education and the goal of developing individuality, and keep a balance between “serving the class as a whole” and “recognizing individual differences” (Lou, Chang & Fan, 2008). Some scholars point out in their studies that mathematics education should be connected to everyday life, use activities, and are individualized. Teachers should take mathematics back to its origin, which is life, while allowing life to enter the world of mathematics. However, teachers should be cautious and should not make the connection with life a goal in itself; if they do, it would hinder students’ development in mathematical thinking and give students false impressions of mathematics (Li, 2004; Zhang, 2003).

Debates among scholars regarding the current mathematics curriculum reform are intense. Critics of the reform point out that it is a “top down” approach, it abandoned the superior features of traditional Chinese education, it undermines the function of mathematics in training students’ logical thinking, urban life is over-represented, its content is difficult to handle, and it has caused confusion in mathematics instruction (Zhang, 2003; Yang, 2002).

Scholars examined the problems encountered from the perspectives of cognitive dissonance and multiple intelligences. For example, Luo (2007) analyzed the problems mentioned above from the perspective of cognitive
dissonance and concluded that effective measures for reducing dissonance included strengthening theoretical training, improving instructional environments, and establishing scientific assessment systems.

**Mathematics Education Under the Guidance of Mathematics Curriculum Reform**

One of the features of Chinese mathematics education is to value the “two basics” in mathematics instruction. Scholars (Senior Seminar in Mathematics Education, 2003) assured the value of instruction methods that pay particular attention to the “two basics,” such as “meaningful memorization,” “concise lecturing and more practice,” “variation in instruction” and “practice making perfect.” Meanwhile, they further proposed the “new two basics” in mathematics instruction, and emphasized the balance between the “two basics” and creativity. They stressed that instructional design should reflect the philosophy of the new mathematics curriculum, and keep creativity and practical skills at the center. They insisted that creativity without a solid foundation is no different than a fantasy, and lying the foundation without a creative spirit is silly drill. The best method for teaching creativity is “to create again” in mathematics instruction (Gu, 2003). The idea that creativity can only be acquired in the process of being creative delivers a similar message with Freudenthal’s theory that mathematics instruction should follow the principle of mathematicalization and creativity. Regarding mathematics content, the goal is not to teach everything, but to teach the connection between the segments, and eventually a system of knowledge would take shape. Regarding mathematics skills, the goal is not to teach every skill, but to teach adaptation; the target should not be simple repetition of skills, but the application of skills (Tang, 2005).

Research on mathematics instruction in this wave of reform is more concerned with the implementation of the above mentioned ideas. For example, Xu (2003) suggested in his research that the study of mathematics instruction should focus on “case analysis,” and teachers’ instructional behaviors should be improved through action research and other research methods that are closely tied to practice. “One good instructional case is worth more than a dozen clichés” (Senior Seminar on Mathematics Education, 2003). Instructional cases come from the classroom, and the classroom should be the ideal laboratory for testing instructional theories. Regarding learning from colleagues, the traditional observation of demonstrative lessons should be gradually replace with instructional case analysis; open classes that are the result of repeated practice, similar to what goes on before a competition, should be abolished, and be replaced with intuitive and authentic instructional cases. Wang (2006) examined the characteristics of highly efficient mathematics instruction and the two
dimensions of instructional efficiency: time invested in instruction and the outcome achieved. People generally recognize and affirm the importance of instructional effectiveness. Effective instruction is instructional activities where the teacher is successful in achieving instructional objectives and helping students with their development.

**Mathematics Learning in the Guidance of the Current Mathematics Curriculum Reform**

The mathematics curriculum reform also requires students to adjust their learning, such as, mathematics learning should be practical, meaningful, and challenging, and learning activities should include observation, experimentation, speculation, proving, communication, and so on. The important methods for learning mathematics include hands-on practices, active exploration, cooperation, and communication. Research in this area mainly focuses on cooperative learning (Qi, 2005; Tang, 2005), communication in learning mathematics (Huang, 2007), and exploration and reflection (Xu, 2006; Zhang, 2008).

Qi (2005) pointed out in her research that the relationship between the teacher and the student in traditional classrooms is like the one between the farmer and the individual plants on the farm; it is a “one on one” relationship. In small group cooperative learning, however, every member has a clearly defined partner, and everyone relies on one another. Everyone has clearly defined responsibilities, and has equal chance to be successful as well. Cooperative learning is not only a means for learning the content, but also a means for training the awareness of cooperation and cooperative skills. Tang (2005) contended that the conditions for cooperative learning must be present: individual group members have similar preparations, mutual trust and support, shared objectives, clear division of labor and individual responsibilities, and the capability to communicate and integrate the finished product. In cooperative learning, all members do their parts and have equal opportunity for participation. The skills required for cooperative learning include listening with respect, asking questions, organizing learning activities, inspiring others, and summarizing and generalizing. Zhang proposed a complete process for training student mathematical reflection, which included four parts: inspiring questions, demonstration, practice, and assessment, with reflection running through all parts. Reflection commences when students encounter questions or confusion; where “the cognitive subject senses, evaluates, explores, monitors, and adjusts the process and outcome of his or her own mathematical thinking. Reflection is a high level mental process; it guides, controls, determines, and monitors mathematical cognition” (Zhang, 2008, p. 38).
The Role of the Teacher in the Current Mathematics Curriculum Reform

The current mathematics curriculum reform raised the bar for teachers. The role of the teacher in curriculum reform may be summarized as the leader, organizer, manager, designer, practitioner and researcher in instruction, and the participant, usher, guide, and collaborator in student learning (Zhang, 2001). Teachers need to be action researchers, who explore and reflect on their teaching, and deepen the understanding of their instructional practice and its contexts, in order to make their teaching more appropriate and justifiable.

Some studies emphasized the special role of the teacher and contended that teachers are the key factor determining the outcome of instructional reforms, and any theory that weakens the role of the teacher is deemed to be flawed. No mathematics curriculum reform would be successful if teachers do not participate from the beginning to the end (Senior Seminar on Mathematics Education, 2003). In the brewing stage of a reform, teachers should be independent and active observers; in the planning stage, teachers should be indispensable participants; when the philosophy is publicized, teachers should accept it whole-heartedly; when the curriculum is devised, teachers should enthusiastically make their suggestions; when the curriculum is circulated, teachers should be active learners; in the implementation stage, teachers should be the down-to-earth practitioner; in the evaluation stage, teachers should be the authoritative spokesperson (Luo, Chang & Fan, 2008).

Obviously, in front of such high expectations, teachers are faced with many confusions and challenges. From the very beginning of the reform, mathematics teachers had no options but accepting everything, and they passively adapted to the new circumstance. Teachers do not have a solid foundation in curriculum theories or advanced educational theories that reflect the current time. They carry with them traces of the old system, and became the barrier of the modern reform. Even if they accept the new ideas, they may not be able to apply them. They also lack time and energy (Li & Chen, 2004).

Summary

This literature review of research on mathematics education in China mainly focuses on the essence of mathematics education, mathematics classroom instruction, student learning of mathematics, mathematics teacher education, and mathematics curriculum reforms. Research on mathematics education in China in the last decade has also covered international and comparative study of mathematics education, mathematical problem solving, mathematics education
for ethnic minorities, mathematics textbooks, the content of mathematics curriculums, and so on. This article is not able to review these areas due to the limit put on the length of the article.

A rich variety of topics has been covered in the research on mathematics education in China; however, the research methods employed are not completely accepted by international standards. A large portion of existent studies are based on experiences and perceptions, therefore the viewpoints expressed are subjective, and lack of evidence. For this reason, the outcomes of these studies are not able to enter the international stage. Chinese scholars have realized this problem and have been trying to communicate and collaborate with international colleagues, and to employ research methods that are internationally accepted in studying mathematics education in China. The goal is to eventually establish a system of theories on mathematics education that has Chinese characteristics.

References


implications for geometry curriculum].


