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CORRECTION

Correction to: Asymptotic analysis of passive mitigation of dynamic instability using a nonlinear energy sink network

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Remark In the sequel, the equation numbering follows that of the original paper.

In the original version of the manuscript [1], Result 3.3 which states the conditions to ensure that the system is in a harmless situation fails to describe some possible situations, especially when very different NESs are considered. Therefore, Result 3.3 must be corrected, just like Result 3.4 which results from it and gives a quantitative characterization of the mitigation limit. These corrections do not affect the results presented in Sect. 4 and the numerical validation performed in Sect. 5.

To know the steady-state regime from a given set of parameters, the super-slow behavior of the system must be checked in every successive subspace I_k^a (see Eq. 30) along the trajectory of the system and not only in the last one (I_{2N}^a) as it is done in the original version of the paper. The behavior of the system in a given subspace I_k^a is determined by the relative position between the arrival point $\mathbf{c} = [c_1, \ldots, c_N]$ (i.e., the point on the critical manifold S in I_k^a on which the trajectory arrives after a jump at the slow timescale, it is described more precisely below) and the fixed points (stable and unstable). Note that if I_k^a does not contain fixed point, the direction of the super-slow flow on the critical manifold S is determined by the sign of the function $f(r_1, \ldots, r_N)|_{\mathbf{r}=\mathbf{c}}$ (see Eq. 39).

To follow the trajectory of the system along S and know which subspaces I_k^a are crossed, the transitions at slow timescale (i.e., the jumps) between each I_k^a must be determined. At any point of the phase space, the slow dynamics is described by the *slow subsystem* (13). This is the case when the trajectory starts from the initial conditions. Otherwise, for transitional jumps between two subspaces I_k^a, the trajectory leaves the critical manifold when it passes though a jump point $\mathbf{b} = [b_1, \ldots, b_N]$ (a local extrema of S). If all considered NESs are different, a jump point corresponds to the passage through a fold (defined by Eq. 25) in only one direction. This mean that there exists a direction, here assumed to be the ith direction, such that at $\mathbf{r} = \mathbf{b}$, $dH_i(r_i)/dr_i = 0$ but $dH_n(r_n)/dr_n \neq 0$ (for $n = 1, \ldots, N$ and $n \neq i$). Therefore, the jump point $\mathbf{b}$ has the following form

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$$\mathbf{b} = \left[b_1, \ldots, b_{i-1}, r_i^{M/m}, b_{i+1}, \ldots, b_N\right], \quad (83)$$

where $r_i^{M/m}$ can be r_i^M of r_i^m (see Eq. 25). For the same reason, when the trajectory leaves the jump point to undergo the jump, only g_i in (13b) becomes nonzero, the other g_n (for $n = 1, \ldots, N$ and $n \neq i$) remain equal to zero (because of 14b). Consequently, the transitional jumps are described by the following equation

$$\dot{\phi} = 0 \quad (84a)$$

$$\dot{\xi}_n = 0 \quad \text{for} \quad n = 1, \ldots, N \quad \text{and} \quad n \neq i \quad (84b)$$

$$\dot{\xi}_i = g_i\left(\phi, \xi_1, \ldots, \xi_N, 0\right). \quad (84c)$$

From the previous equation, one can deduce that during an transitional jump from a jump point $\mathbf{b}$ to the corresponding arrival point $\mathbf{c}$, all the coordinates r_n (for $n = 1, \ldots, N$ and $n \neq i$) remains constant. Only the ith coordinate changes from $r_i^{M/m}$ to $r_i^{u/d}$ where $r_i^{u/d}$ can be r_i^u or r_i^d defined by Eq. (29). Therefore, the arrival point can be determined from the jump point as follows

$$\mathbf{c} = \left[b_1, \ldots, b_{i-1}, r_i^{u/d}, b_{i+1}, \ldots, b_N\right]. \quad (85)$$

The jump $r_i^M \rightarrow r_i^u$ (resp. $r_i^m \rightarrow r_i^d$) corresponds to an increase (resp. decrease) of the coordinate r_i.

An easy calculus shows that M identical NESs with parameters a, μ and α are equivalent to one NES with parameters Ma, $M\mu$ and $M\alpha$. This is consistent with the Eq. (53). Therefore, in the NES network, all groups of identical NESs can be replaced by one equivalent NES and the previous reasoning can be used to describe the trajectory of the system.

After these preliminary comments, the corrected versions of Results 3.3 and 3.4 are now given.

Result 3.3 *Definition* 3.1 *states that we consider a set of initial conditions* $\mathbf{r_0} = [r_1(0), \ldots, r_N(0)]$ *as a small perturbation of the trivial solution, i.e.,* $\mathbf{r_0} \in I_1^a$.

After a transient response from $\mathbf{r_0}$ *the trajectory reaches the critical manifold S. If the slow-flow has a stable fixed point* $\mathbf{r_s^*} = \left[r_{s,1}^*, \ldots, r_{s,N}^*\right]$ *(trivial or nontrivial) in* I_1^a*, i.e.,*

$$\mathbf{r_s^*} \in I_1^a, \quad (86)$$

then the system is in a ***harmless situation*** *(trajectories reach inevitably* $\mathbf{r_s^*}$*) and undergoes complete suppression for the trivial fixed point or mitigation through periodic response (PR) for the nontrivial fixed point.*

If condition (86) is not respected, the trajectory reaches the jump point of I_1^a*. Note that* I_1^a *has only one jump point. The coordinates of the corresponding arrival point* $\mathbf{c}$*, contained in the next crossed* I_k^a*, are determined from those of the jump point through Eq. (85).*

The following reasoning is valid at each arrival in a new I_k^a*. In each* I_k^a *crossed by the critical manifold S there are two exit points corresponding to the passage by a fold in two different directions (here the ith and the jth directions). Therefore, these two exit points are jump points denoted* $\mathbf{b^{(i)}}$ *and* $\mathbf{b^{(j)}}$

$$\mathbf{b^{(i)}} = \left[b_1^{(i)}, \ldots, b_{i-1}^{(i)}, r_i^{M/m}, b_{i+1}^{(i)}, \ldots, b_N^{(i)}\right], \quad (87)$$

and

$$\mathbf{b^{(j)}} = \left[b_1^{(j)}, \ldots, b_{j-1}^{(j)}, r_j^{M/m}, b_{j+1}^{(j)}, \ldots, b_N^{(j)}\right]. \quad (88)$$

Three situations are possible:

- <u>*Situation 1.*</u> *There are no fixed points in* I_k^a*, then, depending of the sign of the function* $f\left(r_1, \ldots, r_N\right)|_{\mathbf{r=c}}$*,* $\mathbf{b^{(i)}}$ *or* $\mathbf{b^{(j)}}$ *is reached and the trajectory leaves* I_k^a*.*
- <u>*Situation 2.*</u> *The arrival point is between an unstable fixed point and* $\mathbf{b^{(i)}}$ *(resp.* $\mathbf{b^{(j)}}$*), then* $\mathbf{b^{(i)}}$ *(resp.* $\mathbf{b^{(j)}}$*) is reached and the trajectory leaves* I_k^a*.*
- <u>*Situation 3.*</u> *A stable fixed point is the first neighboring point of the arrival point* $\mathbf{c}$*, and then the stable fixed point is reached.*

Afterward, if the trajectory leaves I_k^a*, the new arrival point and the next* I_a^k *are determined and so on.*

The procedure stops when one of the following conditions are met resulting in harmless or harmful situations:

– [HlS]*: The system is in a* ***harmless situation*** *if*

- [HlS(a)]*: situation 1 or 2 holds, and along the trajectory a* I_k^a *is met again. In this case, the system is in a harmless situation corresponding to a mitigation through strongly modulated response (SMR).*
- [HlS(b)]*: situation 3 holds in* I_k^a *with* $k \neq 2N$*. In this case, the reached fixed point has a small*

amplitude and the system is in a harmless situation corresponding to a mitigation through periodic response.

- [HfS]*: The system is in a **harmful situation** if situation 3 holds in $I_k^a = I_{2N}^a$. Indeed, in this case, the reached fixed point has a large amplitude close to that of the case without NES.*

Note the procedure will stop due to the finite number of subintervals I_k^a.

Result 3.4 *From a given set of parameters, Result 3.3 gives a theoretical prediction of the resulting steady-state regime and therefore allows to know if the system is a harmless or harmful situation. Consequently, the mitigation limit, denoted ρ_{ml}, can be predicted theoretically as the first value of ρ for which situation 3 holds in $I_k^a = I_{2N}^a$ (i.e., scenario HfS).*

As usual in the example shown in Figs. 7 and 8 of the original paper, the mitigation limit corresponds to the value of the parameter ρ corresponding to the transition from harmless to harmful situation. In this example, three NESs are considered ($N = 3$) and the last harmless situation before the transition (with respect to ρ) to harmful situation corresponds to a SMR in which the trajectory reaches $I_{2N=6}^a$. The subinterval I_6^a contains unstable fixed points and the larger one is denoted $\mathbf{r_u^*} = \left[r_{u,1}^*, r_{u,2}^*, r_{u,3}^*\right]$. Therefore, the harmless situation corresponds to the scenario HlS(a) described above. Moreover, the arrival point $\mathbf{c}$ in $= I_6^a$ is due to a jump in direction r_3 from a jump point $\mathbf{b} = \left[b_1, b_2, r_3^M\right]$. Consequently $\mathbf{c} = \left[b_1, b_2, r_3^u\right]$ and the mitigation limit is the solution of $r_{u,3}^*(\rho) = r_3^u$. Indeed, at this special value, the response of the system switches from HlS(a) to HfS in which $\mathbf{r_u^*}$ is reached. On Fig. 7, the mitigation limit corresponds therefore to the intersection between the branch of the r_3 coordinate of the larger unstable fixed point and r_3^u as it is defined in the original version of the paper.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest concerning the publication of this manuscript.

Reference

1. Bergeot, B., Bellizzi, S.: Asymptotic analysis of passive mitigation of dynamic instability using a nonlinear energy sink network. Nonlinear Dyn. **94**(2), 1501–1522 (2018). https://doi.org/10.1007/s11071-018-4438-0