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CORRECTION

Correction to: Conservative and Semiconservative Random Walks: Recurrence and Transience

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The aim of this note is to correct the errors in the formulation and proof of Lemma 4.1 in [1] and some claims that are based on that lemma. The correct formulation of the aforementioned lemma should be as follows.

Lemma 4.1 *Let the birth-and-death rates of a birth-and-death process be* λ_n *and* μ_n *all belonging to* $(0, \infty)$. *Then, the birth-and-death process is transient if there exist* $c > 1$ *and a value* n_0 *such that for all* $n > n_0$

$$\frac{\lambda_n}{\mu_n} \geq 1 + \frac{1}{n} + \frac{c}{n \ln n}, \tag{1}$$

and is recurrent if there exists a value n_0 *such that for all* $n > n_0$

$$\frac{\lambda_n}{\mu_n} \leq 1 + \frac{1}{n} + \frac{1}{n \ln n}. \tag{2}$$

Proof Following [2], a birth-and-death process is recurrent if and only if

$$\sum_{n=1}^{\infty} \prod_{k=1}^{n} \frac{\mu_k}{\lambda_k} = \infty.$$

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Write

$$\sum_{n=1}^{\infty}\prod_{k=1}^{n}\frac{\mu_k}{\lambda_k}=\sum_{n=1}^{\infty}\exp\left(\sum_{k=1}^{n}\ln\left(\frac{\mu_k}{\lambda_k}\right)\right). \tag{3}$$

Now, suppose that (1) holds. Then, for sufficiently large n

$$\frac{\mu_n}{\lambda_n}\leq 1-\frac{1}{n}-\frac{c}{n\ln n}+O\left(\frac{1}{n^2}\right),$$

and since the function $x \mapsto \ln x$ is increasing on $(0, \infty)$, then

$$\begin{aligned}\ln\left(\frac{\mu_n}{\lambda_n}\right)&\leq \ln\left(1-\frac{1}{n}-\frac{c}{n\ln n}+O\left(\frac{1}{n^2}\right)\right)\\&=-\frac{1}{n}-\frac{c}{n\ln n}+O\left(\frac{1}{n^2}\right).\end{aligned}$$

Hence, for sufficiently large n

$$\sum_{k=1}^{n}\ln\left(\frac{\mu_k}{\lambda_k}\right)\leq -\ln n - c\ln\ln n + O(1),$$

and thus, by (3), for some constant $C < \infty$,

$$\sum_{n=1}^{\infty}\prod_{k=1}^{n}\frac{\mu_k}{\lambda_k}\leq C\sum_{n=1}^{\infty}\frac{1}{n(\ln n)^c}<\infty,$$

provided that $c > 1$. The transience follows.

On the other hand, suppose that (2) holds. Then, for sufficiently large n

$$\frac{\mu_n}{\lambda_n}\geq 1-\frac{1}{n}-\frac{1}{n\ln n}+O\left(\frac{1}{n^2}\right),$$

and, consequently,

$$\ln\left(\frac{\mu_n}{\lambda_n}\right)\geq \ln\left(1-\frac{1}{n}-\frac{1}{n\ln n}+O\left(\frac{1}{n^2}\right)\right).$$

Similarly to that was provided before, for some constant C',

$$\sum_{n=1}^{\infty}\prod_{k=1}^{n}\frac{\mu_k}{\lambda_k}\geq C'\sum_{n=1}^{\infty}\frac{1}{n\ln n}=\infty.$$

The recurrence follows. □

As $n \to \infty$, asymptotic expansion (4.5) obtained in the proof of Lemma 4.2 in [1] guarantees its correctness. However, the corrected version of Lemma 4.1 requires more delicate arguments in the proofs of Lemma 4.2 and Theorem 4.13 in [1]. Specifically, in the proof of Lemma 4.2 instead of limit relation (4.6) we should study the cases $d = 2$ and $d \geq 3$ separately in terms of the present formulation of Lemma 4.1.

In the formulation of Theorem 4.13 in [1], assumption (4.12) must be replaced by the stronger one:

$$\frac{L_n}{M_n} \leq 1 + \frac{2-d}{n} + \frac{1-\epsilon}{n \ln n},$$

for all large n and a small positive ϵ. In the proof of Theorem 4.13 in [1], we should take into account that for large n

$$\frac{\lambda_n(1,d)}{\mu_n(1,d)} = 1 + \frac{d-1}{n} + O\left(\frac{1}{n^2}\right)$$

is satisfied (see the proof of Lemma 4.2), and hence,

$$\frac{p_n}{1-p_n} \asymp \left[\frac{\lambda_n(1,d)}{\mu_n(1,d)} \cdot \frac{L_n}{M_n}\right] \leq 1 + \frac{1}{n} + \frac{1-\epsilon}{n \ln n} + \frac{C}{n^2},$$

for a fixed constant C and large n. So, according to Lemma 4.1 the process is recurrent.

Note that the statements of Lemma 4.1 are closely related to those of Theorem 3 in [3] that prove recurrence and transience for the model studied there.

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