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CORRECTION

Corrections to: Differentiable McCormick relaxations

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This note lists corrections to various errors in a recent article [1] by Khan, Watson, and Barton. Though these errors appear in the text of [1], they were not present in the C++ implementation used in Section 7 of [1]; hence, the examples in that section were not affected by these errors.

– In the bottom row of Table 1 of [1], concerning relaxations of $\frac{1}{\xi^{2k-1}}$ for $k \in \mathbb{N}$, the entry in the leftmost column should be "$\mathbb{R}_-$" instead of "$\mathbb{R}_+$". (The $B := \mathbb{R}_+$ case is addressed by the earlier row concerning $\frac{1}{\xi^k}$.)

– Proposition 6 of [1] concerns the procedure for obtaining $\mathscr{C}^2$ relaxations of expressions involving odd powers. In this proposition, in the construction of $\overline{\phi}^{\mathrm{C}}$, the "max" function should instead be "min"; the corrected construction is:

$$\overline{\phi}^{\mathrm{C}} : \boldsymbol{x} \to \mathbb{R} : \xi \mapsto \begin{cases} \xi^{2k+1}, & \text{if } \overline{x} \leq 0, \\ \overline{x}^{2k+1}\left(\frac{\xi - \underline{x}}{\overline{x} - \underline{x}}\right) + (\min\{0, \xi\})^{2k+1}, & \text{if } \underline{x} < 0 < \overline{x}, \\ \underline{x}^{2k+1} + (\overline{x}^{2k+1} - \underline{x}^{2k+1})\left(\frac{\xi - \underline{x}}{\overline{x} - \underline{x}}\right), & \text{if } 0 \leq \underline{x}. \end{cases}$$

– In Definition 3 of [1], in the constructions of x^* and y^*, the σ_μ terms should be subtracted rather than added. This affects the Whitney-$\mathscr{C}^1$ relaxations of products described in

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Theorem 6. The corrected constructions are:

$$x^* : (y, \boldsymbol{\zeta}, \boldsymbol{\eta}) \mapsto \underline{\zeta} + (\overline{\zeta} - \underline{\zeta}) \left(\frac{\overline{\eta} - y}{\overline{\eta} - \underline{\eta}} - \sigma_\mu \left(\frac{\underline{\eta} + \overline{\eta}}{(\mu+1)(\overline{\eta} - \underline{\eta})} \right) \right),$$

$$y^* : (x, \boldsymbol{\zeta}, \boldsymbol{\eta}) \mapsto \underline{\eta} + (\overline{\eta} - \underline{\eta}) \left(\frac{\overline{\zeta} - x}{\overline{\zeta} - \underline{\zeta}} - \sigma_\mu \left(\frac{\underline{\zeta} + \overline{\zeta}}{(\mu+1)(\overline{\zeta} - \underline{\zeta})} \right) \right).$$

The proof of Theorem 6 in [1] is valid after this correction.

– Proposition 15 of [1] provides partial derivatives for the relaxations of products described in Theorem 6. In Proposition 15, in the provided expressions for partial derivatives of $\underline{\psi}_{\times,\mathrm{A}}$, the exponents should be $\mu - 1$ rather than $\mu + 1$. The corrected partial derivatives are:

$$\frac{\partial \underline{\psi}_{\times,\mathrm{A}}}{\partial x}(x, y, \boldsymbol{\zeta}, \boldsymbol{\eta}) = \frac{1}{2} \left(\underline{\eta} + \overline{\eta} + (\mu + 1)(\overline{\eta} - \underline{\eta}) \left(\frac{y - \underline{\eta}}{\overline{\eta} - \underline{\eta}} - \frac{\overline{\zeta} - x}{\overline{\zeta} - \underline{\zeta}} \right) \left| \frac{y - \underline{\eta}}{\overline{\eta} - \underline{\eta}} - \frac{\overline{\zeta} - x}{\overline{\zeta} - \underline{\zeta}} \right|^{\mu - 1} \right),$$

$$\frac{\partial \underline{\psi}_{\times,\mathrm{A}}}{\partial y}(x, y, \boldsymbol{\zeta}, \boldsymbol{\eta}) = \frac{1}{2} \left(\underline{\zeta} + \overline{\zeta} + (\mu + 1)(\overline{\zeta} - \underline{\zeta}) \left(\frac{y - \underline{\eta}}{\overline{\eta} - \underline{\eta}} - \frac{\overline{\zeta} - x}{\overline{\zeta} - \underline{\zeta}} \right) \left| \frac{y - \underline{\eta}}{\overline{\eta} - \underline{\eta}} - \frac{\overline{\zeta} - x}{\overline{\zeta} - \underline{\zeta}} \right|^{\mu - 1} \right).$$

References

1. Khan, K.A., Watson, H.A.J., Barton, P.I.: Differentiable McCormick relaxations. J. Glob. Optim. **67**, 687–729 (2017)