



# Intensification, Diversification and Learning in metaheuristic optimization

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Published online: 14 March 2019

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Intensification, Diversification and Learning (IDL), separately or together, are fundamental components of methods for metaheuristic optimization. Although each may be studied independently of the others, in the more effective metaheuristic algorithms they are often interdependent, as legs of what may be called an IDL triangle, each leg intimately relying on the presence of the other two, so that a focus on any subset of the three is implicitly understood to operate in relationship to the missing component(s).

While numerous proposals have been made for methods in all three of these classifications, there remains a significant unexplored area, both conceptually and empirically, for integrating various elements of the three-sided IDL relationship. In some cases, general formulations await to be made complete by filling in specific details, and in other cases details await to be tested to determine their impact. New ideas have not always been thoroughly compared with previous ones, to identify their relative contribution. And within the space of possibility spanned by the legs of the IDL triangle, it is reasonable to speculate that there remains a vast region of formulations not yet conceived.

The power of modern metaheuristic algorithms to generate better solutions to complex optimization problems than could be found in the past motivates a closer look at the link between algorithmic effectiveness and the combined effects of intensification, diversification and learning. The goal of this special issue is to provide a forum advancing the state-of-the-art of metaheuristic design based on the IDL interrelationship. A brief summary of the contributions in this special issue of the *Journal of Heuristics* for addressing these concerns is as follows.

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## **Diversification-based learning in computing and optimization**

**Fred Glover and Jin-Kao Hao**

The authors present a diversification-based learning (DBL) framework that is shown to go beyond the opposition-based learning (OBL) framework that has recently become the focus of numerous research initiatives. By unifying and extending earlier proposals in metaheuristic search, the paper defines a collection of flexible approaches for creating intensification and diversification strategies and describes potential applications of DBL to various subfields of machine learning and optimization.

## **Intensification, learning and diversification in a hybrid metaheuristic: an efficient unification**

**Vincius R. Mximo and Mari C. V. Nascimento**

A hybridization of Intelligent Guided Adaptive Search (IGAS) with Path-Relinking is developed to solve large-scale instances of the Maximum Covering Location Problem (MCLP). The IGAS approach is a learning-based metaheuristic proposed recently to incorporate adaptive memory into the well-known Greedy Randomized Search Procedure (GRASP) that involves 3 stages consisting of construction, local search and learning. For the learning stage, the authors use an unsupervised learning algorithm called Growing Neural Gas (GNG), and join this with the Path-Relinking algorithm which explores the intermediate solutions between two solutions to find better quality solutions. The combination of GNG and Path-Relinking obtained high quality solutions, outperforming the best results found in the literature for the MCLP problem.

## **Iterated backtrack removal search for finding k-vertex-critical subgraphs**

**Wen Sun, Jin-Kao Hao and Alexandre Caminada**

The authors propose an iterated backtrack-based removal (IBR) heuristic to find critical subgraphs in a graph  $G$ . Their method extends the popular intensification-oriented removal strategy and introduces two new diversification-oriented search components a backtracking mechanism to reconsider some removed vertices and a perturbation strategy to escape the trap of local optimality. A computational study of 80 benchmark graphs shows the effectiveness of the proposed method in terms of solution quality and run-time efficiency compared to state-of-the-art algorithms in the literature.

## **A combined approach for analysing heuristic algorithms**

**Jeroen Corstjens, Nguyen Dang, Benot Depaire, An Caris and Patrick De Causmaecker**

Insight into the behavior of a heuristic algorithm is sought by investigating how its rules separately and in combination correlate with performance, in relation to the problem instance solved. Both functional analysis of variance and multilevel regression analysis are studied including a combined methodology that can provide more insights than when both approaches are used separately. The case studies analyse a large neighbourhood search algorithm applied to the vehicle routing problem with time windows and an iterated local search algorithm for the unrelated parallel machine scheduling problem with sequence-dependent setup times.

## **Clustering-driven evolutionary algorithms: an application of path relinking to the QUBO problem**

**Michele Samorani, Yang Wang, Yang Wang, Zhipeng Lu and Fred Glover**

The paper addresses the challenge of selecting parent solutions in evolutionary population-based algorithms to yield better offspring and provide improved solutions to populate successive generations. The proposed method, which derives from scatter search and path relinking, improves the efficiency of the evolutionary process by a strategy of clustering the solutions and focusing on solution combinations where the parents alternatively belong to the same cluster or to different clusters. The efficacy of this approach is demonstrated in application to the widely studied Quadratic Unconstrained Binary Optimization (QUBO), by focusing on path relinking algorithm joined by tabu search, to obtain outcomes that improve the best results in the literature for hard QUBO instances.

## **Diversification methods for zero-one optimization**

**Fred Glover, Gary Kochenberger, Weihong Xie and Jianbin Luo**

New diversification strategies for zero-one optimization are introduced that significantly extend strategies previously introduced in the setting of metaheuristic search, building on ideas proposed in Scatter Search and Path Relinking evolutionary algorithms. The outcome yields easily implemented strategies for partitioning assignments of values to variables, accompanied by augmentation and shifting processes to create greater flexibility and generality. Detailed pseudocode and numerical illustrations show the operation of these methods and the collections of solutions they create.

## **A UAV location and routing problem with spatio-temporal synchronization constraints solved by ant colony optimization**

**Oktay Yilmaz, Ertan Yakici and Mumtaz Karatas**

The paper addresses the challenge of optimizing location and routing an unmanned aerial vehicle fleet together with allocating available capacity to potential locations. The problem includes synchronization constraints with time windows, capacity monitoring and a limit on the number of sorties. A mixed integer programming formulation and an ant colony heuristic approach are proposed, comparing the heuristic to a commercial solver, a greedy heuristic and a simpler version of the suggested heuristic. The heuristic provides the best solutions, while the commercial solver produces only poor solutions in longer time periods. A learning component of the heuristic makes a significant change.

## **Patient classification considering the risk of restenosis after coronary stent placement**

**Halenur Ahn Mahmutoullari, Serhan Duran, Ertan Yakici and Mahmut Ahn**

Aging and some lifestyle habits cause plaque accumulation in the blood vessels resulting in narrowing of the arteries. Tiny wire mesh tubes called stents are used in balloon angioplasty to keep the vessels open. However, the stented vessel has a risk of re-narrowing due to the recovery response of the body and this is called in-stent-restenosis. The paper classifies patients according to their risks of restenosis by developing a

UTADIS (Utilites Additives Discriminates) model and then improving classification performance by a non-dominated sorting based multi-objective evolutionary algorithm (NSGA-II). Finally, 10,435 records of cardiac patients treated at the Cardiology Department of Ondokuz Mays University Hospital are scanned and computational experiments conducted with 420 conforming observations (including 65 lesions having in-stent-restenosis) to demonstrate the efficiency of proposed methodology.

### **An effective multi-wave algorithm for solving the max-mean dispersion problem** **Jiawei Song, Yang Wang, Haibo Wang, Qinghua Wu and Abraham P. Punnen**

The authors propose an effective multi-wave algorithm for the max-mean dispersion problem, based on enhancing neighborhood search algorithms by incorporating the notion of persistent attractiveness in memory based strategies. In each wave, a vertical phase and a horizontal phase are alternately performed to reach a boundary solution, followed by a concluding horizontal phase to search for further solution refinement. Experimental results from the proposed approach compare favorably with those from state-of-the-art algorithms in the literature. Additional analysis discloses the benefits of the key algorithmic ingredients.

### **Adaptive multiple crossover genetic algorithm to solve workforce scheduling and routing problem**

**Haneen Algethami, Anna Martnez-Gavara and Dario Landa-Silva**

The workforce scheduling and routing problem assigns personnel to visits across various geographical locations and involves numerous scheduling and routing constraints with the goal or minimising operational cost. This paper presents a novel adaptive multiple crossover genetic algorithm to tackle the combined setting of scheduling and routing problems. Experimental results show that the combined performances of all the operators were better than using only one operator used in isolation, advancing the understanding of the best method to use crossover operators for this highly-constrained optimisation problem.

### **Intensification, diversification, and learning via relaxation adaptive memory programming: a case study on resource constrained project scheduling**

**R. Christopher L. Riley and Cesar Rego**

The potential benefit of integrating memory structures derived from solution landscapes with complementary characteristics is validated on the classical Resource Constrained Project Scheduling Problem by integrating Lagrangian dual information with a rudimentary tabu search approach via the Relaxation Adaptive Memory Programming (RAMP) framework. While the simple tabu search component alone is not competitive with the best performing tabu search algorithms, when dual information is added to the procedure the resulting integrated RAMP algorithm outperforms all of them, supporting the RAMP premise that primal-dual relationships make it possible to effectively explore the interplay between intensification, diversification, and learning.

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