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A note on risk aversion and herd behavior in financial markets

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Abstract We show that differences in market participants risk aversion can generate herd behavior in stock markets where assets are traded sequentially. This in turn prevents learning of market's fundamentals. These results are obtained without introducing multidimensional uncertainty or transaction cost.

Keywords Herd behavior · Risk aversion

JEL Classification G1 · G14 · C11 · D82

1. Introduction

The literature on rational herding pioneering by Bikhchandani, Hirshleifer and Welch [1992] and Banerjee [1992] among others, proves that sequential interaction of rational investors can generate imitative behavior (herding) that prevents learning of the economy's fundamentals. However, in the herding models transaction prices are exogenous and constant, therefore their predictions cannot be directly extended to stock markets. To what extent the endogeneity of trading prices in financial markets can prevent herding phenomena and guarantee full information aggregation?

Avery and Zemsky [1998] (AZ henceforth) and Lee [1998] study the occurrence of herding in stock markets when trading is sequential and prices are endogenous. AZ show that in order to generate herding behavior in a Glosten and Milgrom [1985] style model, it is necessary to introduce multidimensional uncertainty such as, for

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example, event uncertainty or uncertainty on the proportion of informed agents in the economy. Nevertheless, as in Glosten and Milgrom [1985], in the long run all these phenomena vanish and all private information is eventually incorporated into prices. In Lee [1998] an exogenous transaction cost may prevent traders from revealing their private information leading to information aggregation failure.

Within a simple sequential trade model, this note shows that herding, contrarian behavior[1] and information aggregation failure can occur even in the absence of both multidimensional uncertainty and transaction cost. Differently from AZ, in our model traders and market makers interpret past histories in the same way. Nevertheless, when market makers and traders differ in their risk aversion, the same information affects market makers' quotes and traders' valuations differently. This is sufficient to generate herding, contrarian behaviors and long run informational inefficiency. Section 2 presents the model and our results. Section 3 concludes. Proofs are in appendix.

2. The model and our results

We consider a sequential trade model similar to Glosten and Milgrom [1985]: a risky asset is exchanged for money among market makers and traders. At each trading period, a randomly selected trader has a unique opportunity to buy or sell one unit of the asset at the most attractive ask (A_t) or bid price (B_t) respectively. Prices are competitively posted by market makers. We denote with $\mathbf{v} = \mathbf{V} + \varepsilon$ the liquidation value of the asset, where ε has a normal distribution $N(0, \sigma)$ with $\sigma > 0$, $\mathbf{V} \in \{\underline{V}, \overline{V}\}$ with $\underline{V} < \overline{V}$ and $\Pr(V = \overline{V}) = \pi_0$. $\mathbf{V}$ and ε are independently distributed. Each trader receives a private signal $\mathbf{s} \in \{l, h\}$ with $\Pr(\mathbf{s} = l|\mathbf{V} = \underline{V}) = \Pr(\mathbf{s} = h|\mathbf{V} = \overline{V}) = p \in (\frac{1}{2}, 1)$. Signals are conditionally i.i.d. across traders and independent from ε. Note that we have $\underline{V} < E[\mathbf{v}|s = l] < E[\mathbf{v}] < E[\mathbf{v}|s = h] < \overline{V}$.

Let H_t be the history of trade (past quantities and prices) up to date $t - 1$. All agents observe H_t and update their beliefs according to Bayes' rule. We denote $\pi_t = \Pr[\mathbf{V} = \overline{V}|H_t]$ the public belief at time t and $\pi_t^s = \Pr[\mathbf{V} = \overline{V}|H_t, s]$, $s \in \{h, l\}$, an informed traders' belief at time t. A trader's action $\mathcal{A} \in \{buy, sell, no\,trade\}$ is said to be *not informative* at date t if it does not affect the public belief that is $\Pr[\mathbf{V} = \overline{V}|H_t, \mathcal{A}] = \pi_t$. Note that the learning process in the economy regards only the realization of $\mathbf{V}$ and not ε, still $E[\mathbf{v}|H_t] = E[\mathbf{V}|H_t]$.

A risk averse agent of our economy has utility function $u(\mathbf{v}x + m) = -\gamma e^{-\gamma(\mathbf{V}x+m)}$, where x and m are respectively the amount of risky asset (*inventory* henceforth) and money in his portfolio. Thus, in our setting, risk averse agents can differ in their inventory x, their cash m and their risk aversion γ. We assume that the set Φ of all possible inventories held by the agents in the economy is finite. The distribution of the inventories is exogenous, orthogonal to $\mathbf{v}$ and constant across time. We denote with $f(X)$ the probability that a trader has inventory $x \in X \subset \Phi$. Last, we assume that the set of possible coefficients of risk aversion admits a strictly positive lower bound $\underline{\gamma} > 0$.

[1] See next section for a precise definition of these behaviors.

We denote with β (resp. α) the agent's buy (resp. sell) *reservation price* that corresponds to the asset's price such that this agent is indifferent between buying (resp. selling) one asset or not trading at all. As shown in appendix, the reservation prices for a risk averse agent whose initial inventory is x and that attaches probability π to the event $\{\mathbf{V} = \overline{V}\}$ are

$$\beta(\pi, x) = \frac{1}{\gamma}\left(- \frac{\gamma^2\sigma^2(2x+1)}{2} + \ln\left(\frac{\pi e^{-\gamma\overline{V}x} + (1-\pi)e^{-\gamma\underline{V}x}}{\pi e^{-\gamma\overline{V}(x+1)} + (1-\pi)e^{-\gamma\underline{V}(x+1)}}\right)\right), \quad (1)$$

$$\alpha(\pi, x) = \frac{1}{\gamma}\left(- \frac{\gamma^2\sigma^2(2x-1)}{2} + \ln\left(\frac{\pi e^{-\gamma\overline{V}(x-1)} + (1-\pi)e^{-\gamma\underline{V}(x-1)}}{\pi e^{-\gamma\overline{V}x} + (1-\pi)e^{-\gamma\underline{V}x}}\right)\right). \quad (2)$$

We adopt exactly the same definition of herding, contrarian behavior and informational cascade as in Avery and Zemsky [1998]:

A trader with private signal s engages in buy (sell) *herding behavior* if: (i) initially he strictly prefers not to buy (resp. not to sell); (ii) after observing a positive history of trades H_t, i.e. $\pi_t > \pi_0$ (resp. negative history, i.e. $\pi_t < \pi_0$), he strictly prefers to buy (resp. sell).

A trader engages in buy (sell) *contrarian behavior* if: (i) initially he strictly prefers not to buy (resp. not to sell); (ii) after observing a negative (resp. positive) history of trades H_t, he strictly prefers to buy (resp. sell).

An *informational cascade* occurs when the actions of all informed traders are not informative.

Note that if an informational cascade starts at a given date t and never ends then, for any subsequent date, the public belief remains stopped at the level π_t and trading prices cannot converge to the fundamental value of the asset. Then the market is informational inefficient.

The two following propositions show that the difference in risk aversion between traders and market makers can originate herd, contrarian behavior and informational cascade. As a general rule, herd or contrarian behavior and informational cascade occur when market makers' quotes and traders valuations for the asset react differently to an history of trade.[2] In our model agents react differently to an history of trade because of the difference in risk aversion between dealers and traders. Take for example, a positive history that increases the public belief π_t. As the public belief π_t approaches 1 a risk neutral agent's valuation for the asset converges to $\overline{V}$. By contrast, a risk averse agent's buy and sell reservation prices will converge toward levels that are in general different from $\overline{V}$. We now turn to the formal statement of our results.

2.1. Risk neutral market makers

We start the analysis with risk neutral market makers and risk averse traders. Bertrand competition among equally uninformed risk neutral market makers leads to bid and ask quotes that are equal to the expected value of the asset given the available information.

[2] Based on this remark, in a model where there is exogenous difference in agent's valuation for assets, Cipriani and Guarino [2003] investigate contagion in financial markets.

Thus, time t bid quote, B_t, is equal to the maximum[3] of the solutions of the equation

$$B_t = E[\mathbf{v}|H_t, \text{ trader sells at } B_t],$$

and the ask quote A_t is equal to the minimum[4] of the solutions of the equation

$$A_t = E[\mathbf{v}|H_t, \text{ trader buys at } A_t].$$

Direct computations show that solutions of these equations always exist and satisfy

$$E[V|H_t, l] < B_t \leq E[v|H_t] \leq A_t < E[V|H, h].$$

Thus, if time-t-trader has signal s and inventory x, then he will buy if $\beta(\pi_t^s, x) \geq A_t$, he will sell if $\alpha(\pi_t^s, x) \leq B_t$ and he will not trade elsewhere. We now state our first proposition.

Proposition 1. *If traders are risk averse and market makers are risk neutral, then as soon as π_t is sufficiently close to* 1 *or to* 0*,*

(i) *a trader whose inventory is bounded away from $-1/2$ and $1/2$ will engage in herd or contrarian behavior depending on his initial attitude to buy or to sell the asset;*
(ii) *if there exist neighborhoods $\mathcal{N}_{-1/2}$ and of $\mathcal{N}_{-1/2}$ of $1/2$ and $-1/2$ respectively such that $f(\mathcal{N}_{1/2}) = f(\mathcal{N}_{-1/2}) = 0$, then an informational cascade occurs.*

Part (i) of Proposition 1 describes individual behavior of risk averse traders. In a setting where market makers are risk neutral and traders are risk averse, we show that, as soon as the public belief π_t is close to 1 (or to 0), a trader whose inventory x is different from $-\frac{1}{2}$ or $\frac{1}{2}$ will choose an action that does not depend on his private signal. Therefore, depending on his initial attitude to buy or to sell the asset, such a trader will engage in herd or contrarian behavior. For instance, a trader who initially would have sold the asset, and whose inventory x satisfies $x < -\frac{1}{2}$, will engage in buy herding behavior after a sufficiently long positive history. The following table summarizes the situations that lead to herding or contrarian behavior as defined in the previous section.

Traders' inventory	Trader's initial attitude	Positive history (π_t close to 1)	Negative history (π_t close to 0)
$x < -\frac{1}{2}$	seller	buy herding	buy contrarian behavior
$x > \frac{1}{2}$	buyer	sell contrarian behavior	sell herding

Part (ii) of Proposition 1 explains the relation between long run behavior of prices and the distribution trader's characteristics. The condition on f in part (ii) means that the inventories of all traders in the economy are bounded away from 1/2 and $-1/2$.

[3] Any other solution would not be an equilibrium as there would exist a larger bid that would provides positive profit to the market makers.

[4] Any other solution would not be an equilibrium as there would exist a smaller ask that would provides positive profit to the market makers.

Thus, from part (i), for a public belief π_t close to 1 or to 0, there exist no trader that would submit an informative order. In other words, traders exchange only in order to balance their inventory. These trades do not convey any information as the inventories' distribution is orthogonal to the liquidation value of the asset **v**. Only in this instance, an informational cascade occurs, bid-ask spread is equal to zero, the public belief does not vary anymore, and prices are steady. Given that the distribution f is constant across time, an informational cascade never ends. Simple computation shows that the thresholds that π_t must reach in order to trigger a cascade depend on traders's risk aversion and inventories. The smaller the lower bound for risk aversion coefficient $\underline{\gamma}$ and the closer traders' inventories are to 1/2 or to $-1/2$, the more extreme public belief (i.e., π_t closer to 1 or to 0) are necessary for a cascade to occur. The conditions on the distribution f given in (ii) and $\underline{\gamma} > 0$ imply that all traders in the economy have inventories bounded away from 1/2 and $-1/2$ and are strictly risk averse. Consequently, a cascade will always happens for π_t sufficiently close to 0 or to 1.

2.2. Risk averse market makers

We now suppose that two risk averse dealers make a market for risk neutral traders. Consider market maker $i \in \{1, 2\}$ at time t, and let B_t^i, A_t^i and x_t^i be his bid and ask reservation prices and his inventory respectively. We make the simplifying assumption that market makers are myopic, i.e., when they fix their quotes for the current trade, they do not take into account that they might be lead to trade also in the following periods. Still we assume that in each period market makers set their quotes taking into account the informational content of buy or sell orders that come indeed from informed traders. Consider a risk averse market maker at time t and let x_t^i be his inventory, then the maximum price that this market maker is willing to pay for one additional unit of the asset is his bid reservation price given that the asset is sold by an informed trader, i.e. $B_t^i = \beta(\Pr(\mathbf{V} = \overline{V}|H_t, sell), x_t^i)$. Similarly, the minimum price that this market maker is willing to accept for selling one unit of the asset is his ask reservation price given that the asset is bought by an informed trader, i.e. $A_t^i = \alpha(\Pr(\mathbf{V} = \overline{V}|H_t, buy), x_t^i)$. Following Ho and Stoll [1983], as there is no asymmetry of information between the two market makers, in period t both of them will post quotes equal to $A_t = \max\{A_t^1, A_t^2\}$ and $B_t = \min\{B_t^1, B_t^2\}$. If time-$t$-trader has signal s, then he will buy if $E[\mathbf{V}|H_t, s] \geq A_t$, sell if $E[\mathbf{V}|H_t, s] < B_t$ and he will not trade elsewhere. We show the following.

Proposition 2. *If traders are risk neutral, market makers are risk averse, then as soon as* π_t *is sufficiently close to* 1 *or to* 0 *and market makers' inventories are bounded away from* $\frac{1}{2}$ *and* $-\frac{1}{2}$*,*

(i) *all traders take the same action and they engage in herding or contrarian behavior depending on their initial attitude to buy or to sell;*
(ii) *an informational cascade occurs.*

As in Proposition 1, proposition 2 describes both individual behavior of agents (part (i)), and long term behavior of prices (part (ii)). We show that if traders are risk neutral and market maker risk averse, then expressions (1) and (2) imply that when the public belief π_t is close to 1 or to 0, all traders will take the same action no matter the signal they received. For example, when π_t is close to 1, all traders will buy (resp. sell) the asset if $A_t < \overline{V}$, i.e., $\min\{x_t^1, x_t^2\} > 1/2$ (resp. $B_t > \overline{V}$, i.e., $\max\{x_t^1, x_t^2\} < -1/2$). Thus, a trader who initially would have sold the asset will engage in buy herding for a public belief π_t sufficiently close to 1. The following table provides the different cases of herding and contrarian behavior when market makers are risk averse:

Market makers' inventories at t	Trader's initial attitude	Positive history (π_t close to 1)	Negative history (π_t close to 0)
$\min\{x_t^1, x_t^2\} > \frac{1}{2}$	seller	buy herding	buy contrarian behavior
$\max\{x_t^1, x_t^2\} < -\frac{1}{2}$	buyer	sell contrarian behavior	sell herding

With regard to the long term behavior of prices, as in Proposition 1, when the inventory of all risk averse agents in the economy is bounded away form $-1/2$ or 1/2, then as soon as the public belief π_t is sufficiently close to 0 or to 1, all traders' orders will not be informative and an informational cascade occurs. The difference with proposition 1 is that, when dealers are risk averse, in the presence of an informational cascade, quotes move for inventory purposes even if trades do not conceal any new information. These prices movement might interrupt an informational cascade. Nevertheless, there is no economic force that leads market maker inventories to be exactly equal either to $-1/2$ or to 1/2, and so in the long run an informational cascade will occur provided that π_t is sufficiently close to 0 or to 1.

3. Concluding remarks

Avery and Zemsky [1998] draw the general conclusion of their paper as follows:

> "The existence of history-dependent behavior (in either its herd or contrarian form) requires (i) that there exists multiple dimensions of uncertainty, and (ii) that traders' asymmetric information about value uncertainty be sufficiently poor relative to their information about one of the other dimensions of uncertainty."

We show in this note that if traders or market makers are risk averse, neither requirements (i) or (ii) are necessary to obtain herd or contrarian behavior. For what regards the relation between market efficiency and herding, Avery and Zemsky [1998] show that after herd behavior started the flow of information to market makers is not stopped. Thus, in their setting informational cascade never occurs and prices eventually converge to the asset's fundamentals. In our setting, an informational cascade can arises provided that market makers and traders differ in risk aversion and the inventories of risk averse agent are bounded away from 1/2 and $-1/2$. If market makers are risk neutral and an informational cascade occurs, buy or sell order do not convey any new information, spread is zero, the public information and prices are constant and therefore market is not efficient. In this instance, traders with different inventory unbalance will take different actions, but no trader will use his private signal to determine the sign of his transaction. By contrast, if market makers are risk averse, then during a cascade all risk neutral traders will take exactly the same action, as their inventory plays no role. Moreover, quotes move even in presence of informational cascade because trades change market makers' portfolio composition. This can break herd behavior, and temporally interrupt the informational cascade.

In a related paper (Décamps and Lovo [2004]) we show that information aggregation failure does not relay on the restriction to CARA utility functions nor on the assumption that agents can just choose the sign of their trade but not its size.

Appendix

Reservation prices

Take a risk averse agent that attaches probability π_t to the event $\{\mathbf{V} = \overline{V}\}$ and that holds an amount m of money and x unit of the risky asset. The buy reservation price $\beta(\pi_t, x)$ is the minimum amount of money that this agent is willing to pay in exchange for one additional risky asset. In other words $\beta(\pi_t, x)$ in expression (1) is the β that solves

$$E[U(\mathbf{v}, x, m) - U(\mathbf{v}, x+1, m-\beta)|H_t] = 0$$

where

$$\begin{aligned} E[U(\mathbf{v}, x, m)|H_t] &= E\big[-\gamma e^{-\gamma(\mathbf{V}x+m)}|H_t\big] \\ &= \pi_t E\big[\big(-\gamma e^{-\gamma((\overline{V}+\varepsilon)x+m)}\big)\big] + (1-\pi_t)E\big[\big(-\gamma e^{-\gamma((\underline{V}+\varepsilon)x+m)}\big)\big]. \end{aligned}$$

Similarly, the sell reservation price $\alpha(\pi_t, x)$ in expression (2) is the minimum price at which our agent is willing to sell one unit of the asset. That is to say the α that solves

$$E[U(\mathbf{v}, x, m) - U(\mathbf{v}, x-1, m+\alpha)|H_t] = 0.$$

Taking into account that ε is normally distributed and that agents have CARA utility function, we have expressions (1) and (2) for the reservation prices.

Proofs of Propositions 1 and 2

Propositions 1 and 2 are direct consequences of the following lemma

Lemma 1. *Take π_t sufficiently close to 1 or to 0, and let $x > 1/2$ and $x' < -1/2$. Then for any triple of signals s, s' and s'' it results*

$$\begin{aligned} \alpha\big(\pi_t^{s}, x\big) &< E[V|H_t, s'] < \alpha\big(\pi_t^{s''}, x'\big), \\ \beta\big(\pi_t^{s}, x\big) &< E[V|H_t, s'] < \beta\big(\pi_t^{s''}, x'\big). \end{aligned}$$

Proof: First, as $\pi_t^l = \frac{\pi_t(1-p)}{\pi_t(1-p)+(1-\pi_t)p}$ and $\pi_t^h = \frac{\pi_t p}{\pi_t p+(1-\pi_t)(1-p)}$, π_t^s is continuous in π_t, for $s = h, l$. Second, from expressions (1) and (2), α and β are continuous in π. Third, $\alpha(1, \frac{1}{2}) = \beta(1, -\frac{1}{2}) = \overline{V}$, $\alpha(0, \frac{1}{2}) = \beta(0, -\frac{1}{2}) = \underline{V}$ and α and β are decreasing in x. Then, the results follows from an easy continuity argument. □

Proofs of Propositions 1 and 2: Without loss of generality we reason with sell orders. Take an informed trader who strictly prefer not to sell at date 0. Assume first that the trader is risk averse with inventory $x' < -\frac{1}{2}$ whereas dealers are risk neutral. From Lemma 1, as π_t is sufficiently close to 0, we have $\alpha(\pi_t^s, x') > B_t$ for $s \in \{l, h\}$. Thus, our trader will engage in sell herding and his trade will not be informative. Second, suppose the trader is risk neutral and dealers are risk averse. If at date t, the public belief π_t is close to 0 and dealers inventories satisfy $\max\{x_t^1, x_t^2\} < -\frac{1}{2}$ then, from Lemma 1, $B_t = \min\{B_t^1, B_t^2\} > E[V|H_t, s]$ for $s \in \{l, h\}$ and the trader will engage in sell herding. Once again trades are not informative. Similarly, π_t close to 1 leads to sell contrarian behavior. Using an analogous argument it can be easily checked that if all risk averse agents' inventories are bounded away from $\frac{1}{2}$ and $-\frac{1}{2}$ then, as soon as π_t is close to 1 or to 0, trades are not informative and therefore an informational cascade occurs.

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