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CORRECTION

Correction to: The Other Closure and Complete Sublocales

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In the original publication of the article, the formulation of the *c-subfitness* condition (c-sfit) in Subsection 5.2 is inaccurate, with effect in Theorem 5.3. A corrected version of 5.2 and more explanation in the proof of 5.3 is provided in this Corrigendum note.

5.2. A Formal Relaxation

Since an open sublocale of a subfit locale is subfit, we can replace (iii) in 5.1.2 by a formally stronger claim concerning an arbitrary open $U \subseteq L$ instead of L. Thus, we can characterize subfitness by

$$\text{for any open } U \subseteq L \text{ and any sublocale } S \subseteq L, \quad S^\circ = U \;\Rightarrow\; S = U \qquad \text{(sfit'')}$$

(compare with 4.3). Now, we will formally relax this condition to

$$\begin{array}{c}\text{for any open } U \subseteq L \text{ and any } \textit{complete} \text{ sublocale } S \subseteq L,\\ S^\circ = U \;\Rightarrow\; S = U.\end{array} \qquad \text{(c-subfit)}$$

We will present two necessary and sufficient conditions for c-subfitness, one of them technical, another stating that it is precisely the borderline of the coincidence of completeness and openness.

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5.3 Theorem *A frame L is c-subfit iff, for every complete S and open U in $\mathcal{S}(L)$,*

$$\downarrow(S \smallsetminus \{1\}) = \downarrow(U \smallsetminus \{1\}) \quad \Rightarrow \quad S = U.$$

Proof $\Rightarrow$: Since $S^\circ = \mathfrak{o}(a)$ is obviously the same as claiming that $S \subseteq \mathfrak{o}(a)$ together with $\downarrow(S \smallsetminus \{1\})$ being cofinal in $\downarrow(\mathfrak{o}(a) \smallsetminus \{1\})$, it suffices to prove that $S \subseteq \mathfrak{o}(a)$.

Let $\downarrow(S \smallsetminus \{1\}) = \downarrow(\mathfrak{o}(a) \smallsetminus \{1\})$. We shall show that $\mathfrak{c}(a) \cap S = \mathsf{O}$. Indeed, if $1 \neq s \in S$ and $a \leq s$, then $a \in \downarrow(\mathfrak{o}(a) \smallsetminus \{1\})$, hence $a \leq a \to x \neq 1$ for some x. But then $a \leq x$ and therefore $a \to x = 1$, a contradiction.
$\Leftarrow$ is trivial. □