Preface

This special volume contains the proceedings of the Workshop and International Conference on Spherical Varieties, which was held at the Tsinghua-Sanya International Mathematical Forum (TSIMF) from October 31st to November 11th 2016. It consists of eleven contributions, all invited from the speakers.

Spherical varieties form a remarkable class of algebraic varieties equipped with an action of an algebraic group having an open dense orbit. This class contains those of toric varieties, flag varieties, and symmetric spaces, and is stable under natural operations such as equivariant modifications and degenerations.

Spherical varieties can be defined in an algebro-geometric way, as the normal varieties on which a connected reductive group G acts, and a Borel subgroup B has an open orbit. They also admit a representation-theoretic characterization: a normal projective G-variety X is spherical if and only if the space of global sections of any G-linearized line bundle is a multiplicity-free G-module. Accordingly, spherical varieties play an important role in algebraic geometry and in representation theory.

In equivariant algebraic geometry, the classification of spherical varieties by combinatorial invariants has been a major open problem. It was recently solved via earlier contributions of Luna and Vust, Knop and others, and recent work of Luna, Bravi and Pezzini, Cupit-Foutou and others. Yet the geometry of spherical varieties presents many open questions; for example, the study of B-orbit closures (there are finitely many of them), with contributions by Achinger, Gandini, Perrin and Pezzini; the description of moduli spaces of spherical varieties, with work of Avdeev and Cupit-Foutou, Bravi and van Steirteghem.

Some of these questions are motivated by recent developments related to harmonic analysis. Work of Gaitsgory and Nadler introduced the rich methods of the geometric Langlands program in the study of spherical varieties; stringy motivic invariants of spherical varieties were subsequently studied by Batyrev and Moreau, while Sakellaridis and Venkatesh developed harmonic analysis on spherical varieties over non-archimedean local fields. The structure and classification of real spherical spaces and harmonic analysis on these spaces are investigated by Knop, Krötz, Sayag, Schlichtkrull and others.

In this volume, four contributions give a comprehensive introduction to these diverse aspects of spherical varieties:

- Embeddings of spherical homogeneous spaces, by J. Gandini.
- Harmonic analysis for real spherical spaces, by B. Krötz and H. Schlichtkrull.

- Sanya lectures: geometry of spherical varieties, by N. Perrin.
- Lectures on wonderful varieties, by G. Pezzini.

These expository texts are extended notes of the courses given during the workshop. In addition, seven contributions, originating in talks at the conference, present some of the latest developments on spherical varieties:

- Orbits of real semisimple Lie groups on real loci of complex symmetric spaces, by S. Cupit-Foutou and D. Timashev.
- Spherical tropical geometry: a survey of recent developments, by K. Kaveh and C. Manon.
- Standard embeddings of smooth Schubert varieties in rational homogeneous manifolds of Picard number 1, by S.-Y. Kim and K.-D. Park.
- Geometric counting on wavefront real spherical spaces, by B. Krötz, E. Sayag, and H. Schlichtkrull.
- Two-dimensional irreducible algebraic semigroups, by D. Li.
- The log minimal model program for horospherical varieties via moment polytopes, by B. Pasquier.
- On some families of smooth affine spherical varieties of full rank, by K. Paulus, G. Pezzini, and B. Van Steirteghem.

We hope that these articles will contribute to popularize the multifaceted domain of spherical varieties, and stimulate further developments. We sincerely thank the TSIMF for the wonderful support of the conference, and the staff of TSIMF for their professional services. We would like to thank all the authors and referees for their cooperation. We are very grateful to Acta Mathematica Sinica for allowing us to publish this special volume.

The organizers,
Michel Brion (CNRS, Grenoble)
Baohua Fu (Chinese Academy of Sciences)