Symmetric collocation BEM/FEM coupling procedure for 2-D dynamic structural–acoustic interaction problems

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Abstract
This paper presents a symmetric collocation BEM (SCBEM)/FEM coupling procedure applicable to 2-D time domain structural–acoustic interaction problems. The use of symmetry for BEM not only saves memory storage but also enables the employment of efficient symmetric equation solvers, especially for BEM/FEM coupling procedure. Compared with symmetric Galerkin BEM (SGBEM) where double boundary integration should be carried out, SCBEM can reduce significantly the computing cost. Two numerical examples are included to illustrate the effectiveness and accuracy of the proposed method.

Keywords
Symmetric collocation BEM, FEM, Coupling procedure, Structural–acoustic interaction, Fluid–structure interaction

1 Introduction
Over the past four decades, significant attention has been devoted to the development of computational methods for time domain fluid–structure interaction problems [1–4]. When finite element method (FEM) is used to model infinite acoustic domains, the artificial boundary reflection should be considered [5]. Mindlin and Bleich [1] presented the early time plane wave approximation (PWA) to simulate the effect of the infinite fluid medium. The PWA method was applied by DiMaggio et al. [6] and Hamdan and Dowling [7] to submerged spherical and spheroidal shells. Fan et al. [8] applied PWA together with the spline shell elements to fluid–structure interaction problems. After Mindlin and Bleich [1], Geers [9] presented an analytical method based on the virtual mass approximation (VMA) of the infinite acoustic medium. Numerical results demonstrated the superior performance of VMA over PWA for late time behaviours and low frequencies. By superimposing PWA and VMA, Ranlet et al. [10] used the doubly asymptotic approximation (DAA) to model the infinite fluid medium, while modal analysis was employed for the structure. DAA has been proved to be accurate for both early and late time behaviours, and has been used by Zilliacus et al. [11] to analyze the response of a submerged fluid-filled cylinder subjected to an incident plane step wave caused by a far field explosion.

The above are some simplified methods to simulate the infinite fluid. When the structure is not in regular form or when the input is not plane wave, the interaction among different points through fluid (structural–acoustic interaction) should be considered. Application of the above methods may cause serious errors in some applications, and the BEM/FEM coupling procedure would be the best alternative.

Boundary element method, which is suitable for both finite and infinite domains, has been applied to many engineering problems during the past two decades [12]. One of the major disadvantages for BEM is the lack of symmetry for its coefficient matrix, which will not only increase the memory storage but also disable the employment of efficient symmetric computation techniques. Thus it makes the computer code less efficient, especially for BEM/FEM coupling procedure where more unknowns often exist in the finite element domain. Symmetric Galerkin BEM (SGBEM) was first proposed by Sirtori [13] for linear elastic analysis, and then used by many researchers in various applications [14–16]. The main difficulty for SGBEM is the existence of hypersingular integrals. Although numerous papers can be found in dealing with the hypersingular integrals [17–19], there are still some spaces that need more research works. The double boundary integration can increase the accuracy for SGBEM, but with a cost of computing time [20].

In this paper, the SCBEM formulation is obtained through matrix multiplication performed only at the first time step. As only one boundary integration has been involved in SCBEM, the computing cost is greatly reduced compared with SGBEM. Symmetry of coefficient matrix not only saves memory storage but also enables the employment of efficient symmetric computation techniques which will subsequently save the computing cost at every time step. As no hypersingularity appears, the numerical implementation for SCBEM is much easier than SGBEM. Therefore, SCBEM can overcome the disadvantages of SGBEM while maintaining its advantages.

SCBEM/FEM procedure is applied in this paper to 2-D time domain fluid–structure interaction problems. The fluid is modelled by BEM, and the structure is modelled by FEM. Two numerical examples are included to illustrate the effectiveness and accuracy of the proposed method. Numerical stability is guaranteed by the linear $\theta$ method for the coupling procedure [21].
2 SCBEM formulation for 2-D acoustic problems

The fluid is governed by the acoustic equation:
\[ \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\gamma(q,t) \]  
(1)

where \( \nabla^2 \) is the Laplacian operator; \( p \) is the transient fluid pressure; \( c \) is the sound speed in the fluid; and \( \bar{p} \) is the second partial time derivative of \( p \); \( \gamma(q,t) \) is the space \( q \) and time \( t \) dependence of source density, which is caused by explosion or any other source.

With a set of discrete points \( Q_{ij} \), \( j = 1, 2, \ldots, J \), on the \( \Gamma \) boundary and a set of discrete time \( t_n, n = 1, 2, \ldots, N \), the traditional asymmetric BEM formulation can be written as [22]:
\[ H_p^n - G_u^n = \sum_{m=1}^{n-1} G_{mn} u_m^n - \sum_{m=1}^{n-1} H_{mn} p^m + S^n \]  
(2)

where \( c(S) \) has been included in \( H \), matrices or vectors are written in black letter in this paper. \( \bar{u}_n = -\frac{L}{\rho \bar{p}} \) is the normal acceleration on the boundary; \( n \) is the coordinate for the unit outward normal \( \bar{n} \) at point \( Q_i \). Subscript \( n \) represents the normal part of a variable at the boundary (normal to the boundary), while superscript \( n \) represents the variable at the time \( t_n \).

There exist only weak and strong singularities in Eq. (2). If linear space interpolation function is used for the pressure, there will be no strong singularity [22] and the numerical integration method can be used.

Multiplying Eq. (2) by \( G^T \) and \( H^T \) respectively leads to:
\[ L_{GH}^n p^n = L_{GG}^n u_n^n + \sum_{m=1}^{n-1} L_{GGmn} u_m^n \]
\[- \sum_{m=1}^{n-1} L_{GHmn} p^m + V^G S \]  
(3)

and
\[ \left( L_{GH}^T \right)^n \bar{u}_n^n = L_{HH}^n p^n + \sum_{m=1}^{n-1} L_{HHmn} p^m \]
\[- \sum_{m=1}^{n-1} L_{HGMn} u_m^n - V^H S \]  
(4)

where
\[ L_{GG} = G^T G \]  
(5)
\[ L_{GH} = G^T H \]  
(6)
\[ L_{HH} = H^T H \]  
(7)
\[ L_{GGmn} = G^T G_{mn} \]  
(8)
\[ L_{GHmn} = G^T H_{mn} \]  
(9)
\[ L_{HGMn} = H^T G_{mn} \]  
(10)
\[ L_{HHmn} = H^T H_{mn} \]  
(11)
\[ V^G S \]  
(12)
\[ V^H S \]  
(13)

\[ L_{GH} \text{ and } L_{HH} \text{ are symmetric matrices.} \]

It is the same with the traditional asymmetric BEM that double nodes cannot be used here if the accelerations for both of these two nodes are unknown. For corner points, two nodes with small distance can be used. However, if the acceleration for at least one node is known, double nodes can be used.

Time domain SCBEM formulation can be established if both Eqs. (3) and (4) are used. Equation (3) is applied on all those sections of the boundary where fluid pressure is prescribed \( (\Gamma_1) \), while Eq. (4) is applied on those sections where normal acceleration is prescribed \( (\Gamma_2) \). The results can be written as:
\[ \begin{bmatrix} L_{GH} & L_{GH}^T \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \end{bmatrix} = \begin{bmatrix} L_{GH}^T \end{bmatrix} \begin{bmatrix} u_{n1} \\ u_{n2} \end{bmatrix} + \sum_{m=1}^{n-1} \begin{bmatrix} L_{GGmn} & L_{GGmn}^T \end{bmatrix} \begin{bmatrix} u_m^n \\ u_m^n \end{bmatrix} \]
\[- \sum_{m=1}^{n-1} \begin{bmatrix} L_{GHmn} & L_{GHmn}^T \end{bmatrix} \begin{bmatrix} p_m^n \\ p_m^n \end{bmatrix} + V^G S \]  
(14)

and
\[ \begin{bmatrix} \left( L_{GH}^T \right) & \left( L_{GH}^T \right)^T \end{bmatrix} \begin{bmatrix} u_{n1} \\ u_{n2} \end{bmatrix} = \begin{bmatrix} L_{HH} & L_{HH}^T \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \end{bmatrix} + \sum_{m=1}^{n-1} \begin{bmatrix} L_{HHmn} & L_{HHmn}^T \end{bmatrix} \begin{bmatrix} p_m^n \\ p_m^n \end{bmatrix} \]
\[- \sum_{m=1}^{n-1} \begin{bmatrix} L_{HGMn} & L_{HGMn}^T \end{bmatrix} \begin{bmatrix} u_m^n \\ u_m^n \end{bmatrix} - V^H S \]  
(15)

Moving unknown variables to the left and known variables to the right, one can get the following BEM formulation from Eqs. (14) and (15):
\[ \begin{bmatrix} L_{GH} & L_{GH}^T \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \end{bmatrix} = \begin{bmatrix} L_{GH}^T \end{bmatrix} \begin{bmatrix} u_{n1} \\ u_{n2} \end{bmatrix} + \sum_{m=1}^{n-1} \begin{bmatrix} L_{GGmn} & L_{GGmn}^T \end{bmatrix} \begin{bmatrix} u_m^n \\ u_m^n \end{bmatrix} \]
\[- \sum_{m=1}^{n-1} \begin{bmatrix} L_{GHmn} & L_{GHmn}^T \end{bmatrix} \begin{bmatrix} p_m^n \\ p_m^n \end{bmatrix} + V^G S \]  
(16)

or in another form as:
\[ A_0 X^n + B_0 Y^n = \sum_{m=1}^{n-1} \begin{bmatrix} L_{GGmn} & L_{GGmn}^T \end{bmatrix} \begin{bmatrix} u_m^n \\ u_m^n \end{bmatrix} \]
\[- \sum_{m=1}^{n-1} \begin{bmatrix} L_{GHmn} & L_{GHmn}^T \end{bmatrix} \begin{bmatrix} p_m^n \\ p_m^n \end{bmatrix} + V^G S \]  
(17)
As both $\mathbf{L}^{GG}$ and $\mathbf{L}^{HH}$ are symmetric matrices, the time domain collocation BEM formulation given by (16) or (17) is symmetric. Equation (16) can also be derived directly from Eq. (2). Rewrite Eq. (2) according to the boundary conditions for $\Gamma_1$ and $\Gamma_2$ as:

$$
\begin{align*}
\begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \end{bmatrix} &= \begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} u_{n1}^n \\ u_{n2}^n \end{bmatrix} + \sum_{m=1}^{n-1} \mathbf{G}^{mn} u_m^n \\
&- \sum_{m=1}^{n-1} \mathbf{H}^{mn} p_m^n + \mathbf{S}^n
\end{align*}
$$

(18)

Moving all the unknown variables to the left and known variables to the right leads to:

$$
\begin{align*}
\begin{bmatrix} -G_1 & H_2 \end{bmatrix} \begin{bmatrix} u_{n1}^n \\ u_{n2}^n \end{bmatrix} &= \begin{bmatrix} -H_1 & G_2 \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \end{bmatrix} + \sum_{m=1}^{n-1} \mathbf{G}^{mn} u_m^n \\
&- \sum_{m=1}^{n-1} \mathbf{H}^{mn} p_m^n + \mathbf{S}^n
\end{align*}
$$

(19)

Multiplying Eq. (19) by $\begin{bmatrix} -G_1 & H_2 \end{bmatrix}^T$, one can also get Eq. (16) where the coefficient matrix is symmetric and its numerical result is the same with that from the traditional asymmetric BEM formulation.

The matrices $\mathbf{A}_0$ and $\mathbf{B}_0$ in Eq. (17) can be obtained at the first time step through matrix multiplication. Since the summation of the last three terms on the right-hand side of Eqs. (18) and (19) (also in Eq. (2)) is a vector, the additional calculation in Eq. (16) or (17) at each following step is only the multiplication of a matrix with this vector. Therefore, the cost increment caused for SCBEM is not big for time domain problems compared with the traditional asymmetric collocation BEM. Usually, such a cost increment may be cancelled out subsequently by the efficient symmetric computation techniques (refer to the following explanations). On the other hand, as only one boundary integration has been involved, SCBEM is much faster than SGBEM.

From [24], in order to solve equations with $N$ unknowns,

1. if the coefficient matrix is not symmetric and Gauss elimination method is used, the total number of multiplications/divisions is $N^3/3 + N^2 - N/3$, and the total number of additions/subtractions is $N^3/3 + N^2/2 - 5N/6$.

2. if the coefficient matrix is symmetric and the $LDL^T$ Cholesky factorization is used, the total number of multiplications/divisions is $N^3/6 + 2N^2 - N/6$, and the total number of additions/subtractions is $N^3/6 + N^2 - 7N/6$.

It can be seen clearly that the cost saving is $N^3/6 - N^2/2 - N/6$ multiplications/divisions and $N^3/6 - N^2/2 + N/3$ additions/subtractions at each time step for SCBEM. While, in order to get the symmetric coefficient matrix, the cost increment caused by matrix multiplications is $N^2(3N + 1)/2$ ($\mathbf{A}_0$ is symmetry in Eq. (17)) multiplications and $N(N - 1)(3N + 1)/2$ additions at the first time step. Therefore, for a big $N$, the cost saving will cancel out the cost increment after nine time steps and SCBEM is faster than the asymmetric traditional BEM. Similar results can be derived for the BEM/FEM coupling procedure while numerical proof can be found next in the first example.

For BEM/FEM coupling procedure, both Eqs. (3) and (4) should be used on the interface section (with subscript “f”):

$$
\begin{align*}
\begin{bmatrix} L_{H1}^f & L_{G1}^f & L_{G1}^f \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \\ p_3^n \end{bmatrix} \\
\begin{bmatrix} L_{G1}^f & L_{G1}^f & \mathbf{G}_{ni} \end{bmatrix} \begin{bmatrix} u_{n1}^n \\ u_{n2}^n \\ u_{ni}^n \end{bmatrix} \\
\begin{bmatrix} L_{G1}^f & L_{G1}^f & \mathbf{G}_{ni} \end{bmatrix} \begin{bmatrix} u_{n1}^n \\ u_{n2}^n \\ u_{ni}^n \end{bmatrix}
\end{align*}
$$

(20)

and

$$
\begin{align*}
\begin{bmatrix} \mathbf{L}_{HH} & \mathbf{L}_{HI} & \mathbf{L}_{II} \end{bmatrix} \begin{bmatrix} p_1^m \\ p_2^m \\ p_3^m \end{bmatrix} \\
\begin{bmatrix} \mathbf{L}_{II} & \mathbf{L}_{IH} & \mathbf{L}_{II} \end{bmatrix} \begin{bmatrix} p_1^m \\ p_2^m \\ p_3^m \end{bmatrix}
\end{align*}
$$

(21)

Rewriting Eqs. (14) and (15) as:

$$
\begin{align*}
\begin{bmatrix} L_{H1}^f & L_{G1}^f & L_{G1}^f \end{bmatrix} \begin{bmatrix} p_1^n \\ p_2^n \\ p_3^n \end{bmatrix} \\
\begin{bmatrix} L_{G1}^f & L_{G1}^f & \mathbf{G}_{ni} \end{bmatrix} \begin{bmatrix} u_{n1}^n \\ u_{n2}^n \\ u_{ni}^n \end{bmatrix} \\
\begin{bmatrix} L_{G1}^f & L_{G1}^f & \mathbf{G}_{ni} \end{bmatrix} \begin{bmatrix} u_{n1}^n \\ u_{n2}^n \\ u_{ni}^n \end{bmatrix}
\end{align*}
$$

(22)
Combining the above four equations and move all unknowns to the left, one can get:

\[
\begin{bmatrix}
-L_{G1} & L_{G1} & L_{G1} & L_{G1} \\
L_{G1} & -L_{G1} & L_{G1} & L_{G1} \\
-L_{G1} & L_{G1} & -L_{G1} & L_{G1} \\
L_{G1} & L_{G1} & L_{G1} & -L_{G1} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_n^1 \\
-\mathbf{u}_n^1 \\
\mathbf{u}_n^1 \\
\mathbf{u}_n^1 \\
\end{bmatrix}
=\begin{bmatrix}
-L_{G1} & L_{G1} & L_{G1} & L_{G1} \\
L_{G1} & -L_{G1} & L_{G1} & L_{G1} \\
-L_{G1} & L_{G1} & -L_{G1} & L_{G1} \\
L_{G1} & L_{G1} & L_{G1} & -L_{G1} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{p}_i^m \\
-\mathbf{p}_i^m \\
\mathbf{p}_i^m \\
\mathbf{p}_i^m \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-L_{G1} & L_{G1} & L_{G1} & L_{G1} \\
L_{G1} & -L_{G1} & L_{G1} & L_{G1} \\
-L_{G1} & L_{G1} & -L_{G1} & L_{G1} \\
L_{G1} & L_{G1} & L_{G1} & -L_{G1} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_m^1 \\
-\mathbf{u}_m^1 \\
\mathbf{u}_m^1 \\
\mathbf{u}_m^1 \\
\end{bmatrix}
=\begin{bmatrix}
-L_{G1} & L_{G1} & L_{G1} & L_{G1} \\
L_{G1} & -L_{G1} & L_{G1} & L_{G1} \\
-L_{G1} & L_{G1} & -L_{G1} & L_{G1} \\
L_{G1} & L_{G1} & L_{G1} & -L_{G1} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{p}_l^m \\
-\mathbf{p}_l^m \\
\mathbf{p}_l^m \\
\mathbf{p}_l^m \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-L_{G1} & L_{G1} & L_{G1} & L_{G1} \\
L_{G1} & -L_{G1} & L_{G1} & L_{G1} \\
-L_{G1} & L_{G1} & -L_{G1} & L_{G1} \\
L_{G1} & L_{G1} & L_{G1} & -L_{G1} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_n^1 \\
-\mathbf{u}_n^1 \\
\mathbf{u}_n^1 \\
\mathbf{u}_n^1 \\
\end{bmatrix}
=\begin{bmatrix}
-L_{G1} & L_{G1} & L_{G1} & L_{G1} \\
L_{G1} & -L_{G1} & L_{G1} & L_{G1} \\
-L_{G1} & L_{G1} & -L_{G1} & L_{G1} \\
L_{G1} & L_{G1} & L_{G1} & -L_{G1} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_n^1 \\
-\mathbf{u}_n^1 \\
\mathbf{u}_n^1 \\
\mathbf{u}_n^1 \\
\end{bmatrix}
\]

Obviously, Eq. (24) is symmetric.

As instabilities may show up frequently in time domain BEM algorithm [23], the linear $\theta$ method has been used in this paper to guarantee the stability of the proposed SCBEM scheme [21].

3 SCBEM/FEM formulation

As shown in Fig. 1, the continuous domain $\Omega$ is divided into two sub-domains $\Omega_{BE}$ and $\Omega_{FE}$ ($\Omega = \Omega_{BE} \cup \Omega_{FE}$) with a common interface $\Gamma_i$. The sub-domain $\Omega_{BE}$ is acoustic and modelled by boundary elements. The sub-domain $\Omega_{FE}$ is elastodynamic and modelled by finite elements. The following coupling conditions on the interface should be considered:

(i) equilibrium condition (for fluid–structure interaction problems only):

\[
\mathbf{u}^n_{Bi} - \mathbf{u}^n_{Fe} = \mathbf{u}^n_{Bi} - \mathbf{u}^n_{Fe} \quad \text{for } s \in \Gamma_i
\]

(ii) compatibility condition:

\[
\mathbf{u}^n_{Fe}(s, t) = \mathbf{u}^n_{Bi}(s, t) \quad \text{for } s \in \Gamma_i
\]

Combining Eq. (25), one can get the relationship between the nodal load vector, $\mathbf{R}^n_{Fe}$ and traction vector, $\mathbf{p}_i^n$, as [25]:

\[
\mathbf{R}^n_{Fe} = -\mathbf{Fp}^n_{Bi}
\]

The time domain FEM equation for undamped systems reads [25]:

\[
\mathbf{K}_{eff}\mathbf{u}^n = \mathbf{R}^n_{eff}
\]

where

\[
\mathbf{K}_{eff} = \mathbf{M} + \frac{\Delta t^2}{4} \mathbf{K}
\]

\[
\mathbf{R}^n_{eff} = \mathbf{R}^n - \mathbf{K}\left(\mathbf{u}^{n-1} + \Delta t \mathbf{u}^{n-1} + \frac{\Delta t^2}{4} \mathbf{u}^{n-1}\right)
\]

Considering Eq. (27), Eq. (28) can be written as:

\[
\begin{bmatrix}
\mathbf{K}_{eff} & \mathbf{F} & \mathbf{K}_{reff} \\
\mathbf{T} & \mathbf{K}_{reff} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_{Bi}^n \\
\mathbf{p}_{Bi}^n \\
\mathbf{u}_{Fe}^n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{R}_{Bi}^n \\
\mathbf{R}_{Fe}^n
\end{bmatrix}
\]
\[ \mathbf{R}^{n+1} = -\mathbf{K} \left( \mathbf{u}^{n+1} + \Delta t \mathbf{u}^n + \frac{\Delta t^2}{4} \mathbf{u}^{n-1} \right) \]  

Multiplying Eq. (31) with \( [\mathbf{K}]^T = \begin{bmatrix} \mathbf{K}_{\text{eff}}^T & \mathbf{F}^T & \mathbf{K}_{\text{ref}}^T \end{bmatrix} \) leads to:
\[
\begin{bmatrix}
\mathbf{K}_{\text{eff}}^T & \mathbf{K}_{\text{ref}}^T
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_{\text{eff}}^n \\
\mathbf{u}_{\text{ref}}^n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{R}_{\text{eff}}^n \\
\mathbf{R}_{\text{ref}}^n
\end{bmatrix}
\]

or in abbreviated form:
\[
\begin{bmatrix}
\mathbf{K}_{\text{eff}}^T & \mathbf{K}_{\text{ref}}^T
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_{\text{eff}}^n \\
\mathbf{u}_{\text{ref}}^n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{F}_1^n \\
\mathbf{F}_2^n
\end{bmatrix}
\]

When the compatibility condition in Eq. (26) is considered, the symmetric collocation BEM/FEM coupling formulation can be obtained from Eqs. (34) and (35) for 2-D dynamic structural-acoustic interaction problems as:

\[
\begin{bmatrix}
-\mathbf{L}_{11}^G & \mathbf{L}_{12}^G & -\mathbf{L}_{11}^G & \mathbf{L}_{12}^G \\
\mathbf{L}_{12}^G & -\mathbf{L}_{22}^G & \mathbf{L}_{12}^G & -\mathbf{L}_{22}^G \\
-\mathbf{L}_{11}^G & \mathbf{L}_{12}^G & -\mathbf{L}_{11}^G & \mathbf{L}_{12}^G \\
\mathbf{L}_{12}^G & -\mathbf{L}_{22}^G & \mathbf{L}_{12}^G & -\mathbf{L}_{22}^G
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_{\text{eff}}^n \\
\mathbf{p}_{\text{eff}}^n \\
\mathbf{u}_{\text{ref}}^n \\
\mathbf{p}_{\text{ref}}^n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{B}_1^n \\
\mathbf{B}_2^n \\
\mathbf{B}_3^n \\
\mathbf{B}_4^n
\end{bmatrix}
\]

Symmetry of the coefficient matrix in Eq. (36) can save the memory storage and enable the employment of efficient symmetric computation techniques.

4 Numerical examples

4.1 Submerged cylinder subjected to a plane incident

The first example is to analyze the dynamic response of an underwater cylinder subjected to a series of far field explosions, as shown in Fig. 2. The far field explosions are chosen so that it will cause a nearly constant plane wave pressure in the free field (without cylinder) at the position where the cylinder is located. The purpose is to compare the numerical results from the SCBEM/FEM coupling procedure proposed in this paper with the analytical solution given by Huang in 1978 and reported by Zilliacus [11]. Altogether 48 quadrilateral finite elements and 48 boundary elements are used to model the cylinder and the infinite fluid respectively. The non-dimensional time step is chosen to be \( \Delta T = c \Delta t/r = 0.1 \), and the pressure is normalized by \( P_0 = \rho c^2 \).
Figure 3 shows the non-dimensional radial velocity of the cylinder \( (v_r) \) at different points on the outer surface, obtained from the SCBEM/FEM coupling procedure and from the analytical method presented by Zilliacus [11]. The velocity is normalized with respect to the sound speed in the fluid medium, \( c \), and the time is normalized with respect to \( r/c \) where \( r \) is the radius of the cylinder. Reasonable agreement can be observed, although the slight difference between the incident wave used here (see Fig. 4) and that used by Zilliacus [11] may produce some errors.

The computing time on P-III 500 for this example is 24 s by the traditional asymmetric BEM/FEM procedure, while by SCBEM/FEM procedure it is 23 s. Therefore, one can conclude that the cost increment, caused by the matrix multiplication at the first time step and the multiplication of matrix with vector at each step, has been cancelled out by the efficient symmetric computation techniques. However, this may not always happen, especially when the number of time steps (it is 200 for this example) is smaller than nine as stated before.

4.2 Submerged double-layer cylinder subjected to a plane incident

The second example is to analyze the dynamic response of a double-layer cylinder subjected to a series of far field explosions, as shown in Fig. 5. Altogether 96 quadrilateral finite elements are used to model the cylinder and 144 boundary elements are used to model the infinite fluid and the fluid between the two layers. The non-dimensional time step is chosen to be \( \Delta T = c\Delta t/r = 0.1 \), and the pressure is normalized by \( P_0 = \rho c^2 \).

---

Fig. 3. Radial velocity of the cylinder from BEM/FEM compared with Huang’s result, \( \Delta T = 0.1 \)

Fig. 4. Fluid pressure around the cylinder, \( \Delta T = 0.1 \)
Figure 6a shows the non-dimensional radial velocity \(v_n\) at point A, obtained from the SCBEM/FEM coupling procedure \((\theta = 1.4)\) and the analytical method presented by Zilliacus [11], where one can see a reasonable agreement. The velocity is normalized with respect to the sound speed in the fluid medium, \(c\), and the time is normalized with respect to \(r/c\) where \(r\) is the radius of the cylinder.

In order to show the necessity of the linear \(\theta\) method, the result for \(\theta = 1.0\) is shown in Fig. 6b. As one can see, instability appears quickly for this example. So, the linear \(\theta\) method should and can be used in the SCBEM/FEM coupling procedure.

5 Conclusions
A SCBEM/FEM coupling procedure is given and applied to dynamic fluid–structure interaction problems. As only one boundary integration has been involved, SCBEM is much faster than SGBEM. Symmetry of coefficient matrix can save the memory storage and enable the employment of efficient symmetric computation techniques which will subsequently save the computing time, especially for the coupling procedure. The matrix multiplication takes some computing time for SCBEM, but it is not important compared with the merits mentioned above. There is no restriction for the domain shape when using the SCBEM/FEM coupling method. The incident wave is not necessary to be step plane wave; non-plane wave caused by more dangerous near field explosions can be simulated. The SCBEM/FEM coupling method can be easily extended to 3-D problems.

References