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Das Extremum der Formänderungsarbeit (The extremum of the deformation work)

by Georg Prange (ed. Klaus Knothe)

This book, recently edited by K. Knothe, contains the unpublished habilitation thesis (1916) of the mathematician G. Prange. At the library of the University of Hannover there have been preserved a few, slightly varying versions of this thesis, with many handwritten comments by the Author and others. The Editor tried to eliminate all obvious careless statements in the presumed original version, and has added a historical introduction, emphasizing the importance of Prange's work, as well as an Appendix containing interesting documents about the Author's scientific life. Further, he inserted some critical comments as additional footnotes in the original text.

Prange was essentially influenced by his academic teachers Klein, Hilbert, Minkowski and Runge in Göttingen. His habilitation thesis, entitled „The Extremum of the Deformation Work", was submitted to the University of Hannover in 1916. Since it was not published then, it remained more or less unknown. The subject had been motivated by the need for a clarification and extension to continuous system of those procedures in structural mechanics which are nowadays called energy methods, and which had been originated mainly by Menabrea in 1858, Cotterill in 1865 and Castigliano in 1875. Indeed, Prange realized that the term "deformation work", which was used at that time simultaneously for the strain energy and for the complementary potential energy, was an essential source of confusion and, as a consequence, a reason for a bitter controversy between some prominent engineers and scientists. It seems, that even nowadays the lack of clarity has not been eliminated completely. Therefore, and for historical reasons, the publication of Prange's work seems to be useful.

The main part of the book contains two chapters, the first one being devoted to the elastic framework (as an example for a discrete system), and the second one to the continuous elastic body. Both chapters are supplemented by an outline of the historical development. The general procedure is analogous for both cases, and is characterized by the following steps: equilibrium, Hooke's law, principle of virtual displacements, principle of minimum potential energy, Lagrangian multiplier method, Legendre transformation (analogous to the Hamilton-Jacobi theory in analytical mechanics), principle of stationary canonic functional (called nowadays "Hellinger-Reissner principle"), principle of minimum complementary potential energy, Castigliano's theorems and reciprocal relations. In this way, Prange succeeded in a rational derivation of the Menabrea's principle.

In spite of this fundamental work, the transition of the principle of minimum potential energy to the principle of minimum complementary potential energy has been attributed to Friedrichs (1929) due to Hilbert, and is called "Friedrichs transformation". Unfortunately, Prange used the same denotation "deformation work" for the internal potential energy (strain energy) and the internal complementary potential energy (complementary strain energy), which are numerically identical only for a Hookean material in the absence of eigen/thermal stresses. He did not realize that the canonic functional has a saddle property rather than an extremum one. It should be emphasized that the canonic variational principle was established for the nonlinear problem of elastica already by Born in 1906 in his Göttingen dissertation (not mentioned by Prange), and for the elasticity theory by Hellinger in 1913 (without considering boundary conditions). Only in 1950 Reissner, in 1951 de Veubeke and in 1956 Koppe rediscovered Hellinger's result, taking into account additionally the boundary conditions. Nowadays, there exists an extensive mathematical literature about general complementary (or dual) variational principles (Noble 1964, Rall 1966, Sewell 1969, Arthurs 1970, Oden and Reddy 1974, and others). Nevertheless, Prange's contribution is to be considered as an important milestone in the development of the theory.

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