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Contraband Mathematics: A Documentary Review of the Resources Available to George Green at the Nottingham Subscription Library 1823–1828

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In 1828, at the age of 35, George Green (1793–1841) published *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*. This was a remarkable essay for three significant reasons. Firstly, it exerts even today a wide-ranging influence in physics and engineering, as well as in areas not known to Green at the time, such as modern mathematical economics, thermodynamics, and meteorology (Gray, 2015; Friedlander and Powell, 1989; Grattan-Guinness, 1995).

Secondly, Green received only minimal formal education, in Sneinton, Nottingham, where his father was a miller (Green's Mill, 2012), and he did not attend university in Cambridge until 1833, at the age of 40. Green's mathematical brilliance demonstrated in his 1828 essay was in spite of this scant schooling, and he was greatly influenced by his neighbor John Toplis (1774/1775–1857), who had access to current European mathematical methods. Thirdly, Green's essay uses the calculus notation of

Gottfried Leibniz (1646–1716) and Joseph-Louis Lagrange (1736–1813) rather than that of Isaac Newton (1643–1727) (Kollerstrom, 2012; Gray, 2015; Cannell, 2001; Grattan-Guinness, 1995; Cannell and Lord, 1993). It is of interest to know how Green managed to achieve such an outstanding publication and to ask why he used European methods. In particular, the mathematical and scientific community in Britain at the time had effectively suppressed European methods, following Newton's lead in asserting his fluxions notation as the preferred—and only—method to be used (Cannell, 2001).

This paper will summarize the Newton–Leibniz controversy and how it relates to Green's 1828 paper, Green's background and the influence of John Toplis, and the methods we used in exploring the Nottingham Subscription Library archives. The findings are divided into possible reading requests during Green's time at the library, the 1828 references available at the library, and some wider cultural contexts, including the Analytical Society and Green's use of European mathematics. The paper ends with suggestions for further research.

The Newton–Leibniz Controversy and Its Relevance for Green

The controversy between Leibniz and Newton over which of them was first to discover calculus is well known. As the mathematical historian Victor Katz writes,

> Leibniz, like Newton, was … interested in … solving differential equations, especially since it turned out that important physical problems could be expressed in terms of such equations. And Leibniz, also like Newton, used power series methods to solve such equations. His technique, however, was different (2009, p. 574).

Similarly, mathematical historian Niccolò Guicciardini explains,

> Newton's method of series and fluxions and Leibniz's differential and integral calculus are … two formalisms that differ in the definitions of their basic concepts as well as in their algorithmic peculiarities (1989, p. 3).

In his *Principia Mathematica Philosophiae Naturalis*, published in 1687, however, Newton included "velocities, accelerations, tangents, and curvatures … presented … in the form of synthetic geometrical demonstrations" (Guicciardini, 1989, p. 246). Newton's methods, which he called the "method of fluxions," thought of calculus in terms of motion, and he used dots over the characteristic letters in his particular mathematical notation. As Katz writes,

> For Newton, the basic ideas of calculus had to do with motion … The **fluxion** $\dot{x}$ of a quantity x dependent on time (called the **fluent**) was the speed with which x increased via its generating motion (2009, p. 552).

In contrast, however, Leibniz developed

a method of procedure for determining sums and differences of infinitesimals. This he appears to have done by about 1676, the time at which Newton composed *De quadratura*. He had, about a year before, adopted his characteristic notation (Boyer, 1959, p. 255).

The distinctive notation of Leibniz included $\int$ for "summa" and d for "differentia," which characterizes his calculus, as will be seen in Green's paper. Katz writes,

> Leibniz discovered his transmutation theorem and the arithmetical quadrature of the circle in 1674. During the next two years, he discovered all of the basic ideas of his calculus of differentials (2009, p. 572).

Thus Leibniz derived functions in terms of sums and differences, and his methods were comparably more efficient than those of Newton.

Newton and Leibniz each claimed to have discovered calculus first, with Newton publicly accusing Leibniz of copying his ideas in 1711, although Leibniz managed to prevent criminal charges being brought against him (Sastry, 2006). Inevitably, the Royal Society at the time decided in Newton's favor. National pride, historical significance, and claims of original mathematical scholarship were of huge importance at the time. Guicciardini (1989) also writes of differences in politics, philosophy, and theology between France and England, providing significant cultural contexts for the mathematics at the time. Fortuitously for Newton, Leibniz died in 1716, leaving a further decade for Newton to assert his methods in Britain over those of his European rival (Wilson, 2017). It is now generally accepted that both Newton and Leibniz discovered calculus independently of each other, hence the differences in methods, concepts, and notation. Their work and publication time scales overlapping, with claims of originality and plagiarism leading to an international row, Newton and Leibniz developed calculus in different ways that resulted in national variances in mathematical progress after their deaths.

It is thought that the use of Newton's fluxions hindered the development of British mathematics in the eighteenth century, compared to European mathematics, which used Leibnizian methods (Kollerstrom, 2012). Katz comments:

> The unfortunate result of the controversy was that the interchange of ideas between English and Continental mathematicians virtually ceased. As far as the calculus was concerned, the English all adopted Newton's methods and notation, while on the continent, mathematicians used those of Leibniz. It turned out that Leibniz's notation and his calculus of differentials proved easier to work with. Thus, progress was faster on the Continent. To its ultimate detriment, the English mathematical community deprived itself for nearly the entire eighteenth century of the great progress (2009, pp. 576–577).

Newton's legacy can be said to have been damaging for British mathematics and science as far as his calculus

methods of fluxions is concerned. As Cannell writes in her biography of Green regarding the relevance of this controversy for Green's essay,

> Unfortunately for Green, mathematical development in England was at a low ebb in the early decades of the nineteenth century, with Cambridge stagnating in the shadow of Newton, who had produced his mathematics nearly a century and a half earlier. This dead hand of tradition, which stifled much initiative and originality, was in sharp contrast to the situation in France, where a man such as Green might well have been nurtured and reached his full potential (2001, p. xxviii).

Nevertheless, Green overcame the limitations of the conventional British scientific community, as will be seen in his 1828 essay.

It is obvious from details of the mathematical notation in Green's 1828 essay that he made use of European calculus. Green formulates his equations using the Leibnizian notation $\int$ and d,

$$V = \int \frac{\varrho' dx' dy' dz'}{r'}$$

(Green, 1828, p.13),

as well as Lagrange's notation[1]

$$\psi'(a) = -\frac{\varphi(a)}{a} - \varphi'(a)$$

(Green, 1828, p. 36).

Yet having access in Britain to European mathematics using Leibnizian methods was controversial (Wilson, 2017; Westfall, 1980). We find that Green's use of European methods rather than Newton's fluxions can be explained only by undertaking a review of the mathematical works accessible in Nottingham in the early nineteenth century. As historian Jeremy Gray writes of the development of potential functions, "Gauss aside, one name stands out in the 1830s and 1840s, that of the Englishman George Green" (2015, p. 133). However, as discussed below, Green's 1828 essay was no isolated case showing a preference for European methods.

Green's Background

The details of Green's school education in Nottingham are obscure. It is generally understood that he attended an academy school on Upper Parliament Street in central Nottingham for four terms under Robert Goodacre (1777–1835), who was an unusually progressive mathematics educator and later a traveling lecturer in popular science (Cannell, 2001). This short span of schooling was by no means unusual for a local Nottingham lad. As scientific historian Ioan James explains, "Not many of the pupils at the Academy stayed more than a year or two, so it was unremarkable that the boy was withdrawn after four terms" (2004, p. 119). However, Cannell (2001) suggests that Green outstripped his teachers with his mathematical knowledge, and that it was fortunate for the originality of the 1828 essay that he was largely self-taught. Similarly, Gray also suggests, given the continued influence of Newton on British mathematics, that Green "read what he could of the relevant literature … more, by the way, than he could have learned if he had gone to Cambridge" (2015, p. 137). Green would have undoubtedly made contact with John Toplis, one of the top graduates from Cambridge in 1801, and head teacher in Nottingham from 1806 until his return to Cambridge in 1819, when he became the dean of Queen's College, where he had been a student (Craik, 1999; Cannell, 2001).

John Toplis is assumed to have been instrumental in influencing Green in European mathematics:

> George Green and his family lived around the corner from Toplis for some time and, despite direct evidence, it is inconceivable that the two men, with such a particular interest in common and in such a small society, did not have much to share (Green's Mill, 2012).

Similarly, historian Alex Craik writes,

> Though no definite evidence has been found, it has been plausibly conjectured that he [Green] received encouragement from the Rev John Toplis (1999, p. 165).

Toplis published two works that shed some light on his influence on Green. The first was "On the Decline of Mathematical Studies, and the Sciences Dependent upon Them," published in the *Philosophical Magazine* in 1805.

In this article, Toplis makes a passionate case for the current failings of Britain, opening with the following lament:

> It is a subject of wonder and regret to many, that this island, after having astonished Europe by the most glorious display of talents in mathematics and the sciences dependent upon them, should suddenly suffer its ardour to cool, and almost entirely to neglect those studies in which it infinitely excelled all other nations (1805, p. 25).

Toplis criticizes the Royal Society's publications' lack of impact, "The generality of the papers in the *Philosophical Transactions* are no longer of that importance they were formerly" (p. 26), and argues for the benefits of European analysis:

> the wonderful and matchless powers of modern analysis … what is called analysis possesses boundless and almost supernatural powers in its application to science (p. 28).

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[1] The "prime" notation $\psi'(a)$ is shorthand for the derivative of the function $\psi(a)$, which is a notation that Lagrange introduced, and we are attributing the use of the prime notation to Lagrange as well.

To Toplis, those who shun analysis "obstinately attach themselves to geometry" (p. 28), which "denotes a very narrow and prejudiced mind" (p. 29). Toplis's 1805 paper therefore builds a clear case for European mathematics.

The second publication by Toplis was his translation of the first book of Pierre-Simon Laplace's (1749–1827) celestial mechanics, published in Nottingham in 1814 with the title *A treatise upon Analytical Mechanics : Being the First Book of the Mécanique Céleste of M. le Comte Laplace*. And he is also thought to have worked on translations of other continental mathematics that used Leibnizian calculus, including work of Sylvestre François Lacroix (1765–1843) (Grattan-Guinness, 1995). As Craik writes, "Toplis … was one of the first to urge British mathematicians to adopt the new style of French analysis" (1999, p. 165). His obvious enthusiasm for European, rather than conventional British, mathematics would have undoubtedly made an impact on Green (Cannell and Lord, 1993). It is also thought that Toplis almost certainly enabled Green to access European mathematical publications that might not have been available in Nottingham. Records of the Nottingham Free School, now Nottingham High School, do not go back as far as the time of Green or Toplis (Gunther, 2017), but those of the Subscription Library, now Bromley House, in Nottingham, include catalogues of all works accessible by Green during his membership. It is these records that are key to the possible influences on Green's 1828 essay.

A Brief Note on Methods and Access

A systematic and comprehensive documentary analysis was undertaken of the publications available to George Green during his membership in the Nottingham Subscription Library from 1823 to 1832, when he left for Cambridge at the age of 40 (Potts, 2017). We used a social constructionist epistemology (Holstein and Gubrium, 2008) in that following Henn et al. (2006), we were sensitive not only to the documents accessible, but also to how those publications reflected the early-nineteenth-century mathematical community. We regarded the Bromley House archives as private documents in Green's era, but today the books listed are public and can be freely accessed (Bryman, 2015, referring to Scott, 1990).

After requesting access to the records at Bromley House, we were allowed to visit by arrangement as nonmembers undertaking research. We then looked up each possibly significant book and cross-referenced its title with what we were able to find as to the dates of George Green's membership immediately prior to his 1828 essay to consider which books Green might have used, and why.

Findings: George Green's Library and Possible Reading Requests

The Nottingham Subscription Library was founded in 1816 and moved to Bromley House in 1821, two years before Green became a member. The purpose was to "become a focus for the cultural and intellectual life of Nottingham" (Hoare, 1991). The library benefited from the charitable Steadfast Library, a gift of theological works and additional scientific publications from a nearby rector who wished to help establish a lending library (Hoare, 1991).

Membership in the library was defined in the first point of the "General Rules" as follows: "This Institution shall be called, the Nottingham Subscription Library; and shall consist of not more than 200 members, whose interests and rights in the property of the institution, shall be equal" (Catalogue of the Books, 1816, p. 3). Each member was therefore a shareholder, each share costing five guineas, and each member's annual subscription was two guineas, clearly indicating the middle-class status of the shareholders, or members, including Green as a miller and businessman (Hoare, 1991). A new member could be admitted only in the event of a vacancy arising, sometimes in the event of a shareholder's death, as might have been the case for Green, and "transfers" were noted retrospectively once a year at the Annual General Meeting (Annual Report, 1823–1824). The Annual General Meeting minutes of April 7, 1824, chaired by Rev. Robert White Almond, second president of the Nottingham Subscription Library, and rector of St. Peter's Church in Nottingham town center, list several "transfers in 1823," the third listing being that of a Mr. Clifton Tomson to Mr. George Green (Figure 1).

The library opening hours are given as "from Eleven o'clock in the morning till two o'clock, – and from three o'clock till nine in the evening" (Catalogue of the Books, 1816, p. 12; General Rule XXVI "Library"). As well as reading and borrowing, Green would have been able to use this time to request that the library purchase specific titles. The rules (point XXVI) explained:

> A subscriber wishing to propose a book, shall enter the title, with his signature, in a book kept by the librarian for that purpose, seven days previous to the meeting of the open committee" (Catalogue of the Books, 1816, p. 12; General Rule XXVII "Library").

Unfortunately, Committee Minutes recorded the books requested under the section "The following books were ordered" but not who made the request. While Cannell's biography lists a number of books available, which we were also able to see from the catalogues, she also acknowledges this vagueness, commenting, "Thus it is not always possible to establish with any accuracy which books were available to Green" (2001, p. 47). The process of identifying Green's sources as suggested by Bromley House librarian Frances Potts (2017) is therefore to look at Green's essay, list his references, trace these back to the library and specific orders, and infer that he might have requested or read the titles himself, which is exactly what we attempted.

There were two catalogues that spanned Green's 1823–1832 membership, and the cataloguing was updated every five to ten years (Potts, 2017). Both the 1816 and 1841 catalogues included publications available in the library at the time of Green's membership and listed numerous works. In the 1816 catalogue, 1,225 books are listed across the seven "classes" of subject types. These were Theology (class A), Philosophy (class B), History (class C), General Literature (class D), The Fine Arts (class E), Law, Politics, Naval and Military Tactics (class F), and Periodical
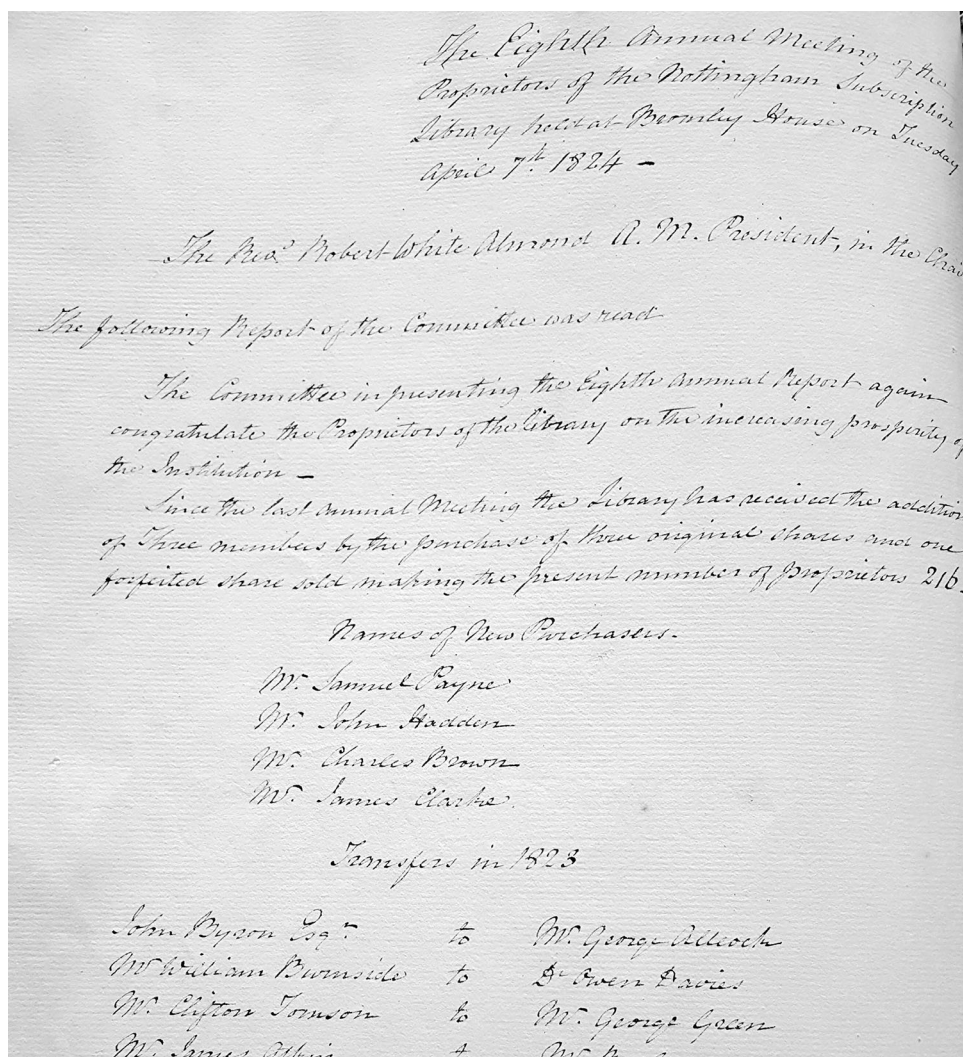
Figure 1. Transfer document of 1823 from a Mr. Clifton Tomson to Mr. George Green (Annual Report, 1823–1824).

Publications (class G) (Coope and Corbett, 1991, p. 134), and those "presented" or donated whose titles were added separately at the end of the catalogue (Catalogue of the Books, 1816). We were particularly interested in the 106 books listed in class B (Philosophy) section b: "Natural and Mathematical." These included Hutton's *Course of Mathematics* (1810), Laplace's *1stBook of Celeste Mechanique translated by Toplis,* and *Philosophical Transactions vol 1–18.* The 1841 catalogue is similarly classified, with a total of 8,797 books listed, and 887 listed under "Natural and Mathematical" (class B Philosophy, section b). From 1816 to 1841, the growth of the "Natural and Mathematical" section in terms of the numbers of books available seems roughly proportionate to the growth of the entire catalogue collection. By the time of the 1841 catalogue, the mathematics and physics books listed included, for example, up to volume 60 of the *Philosophical Transactions,* as well as *Treatise of Mechanics* (1826) 2 vols by Olinthus Gregory (1774–1841), *On Mechanics* and *History of the Inductive Sciences* (1837) by William Whewell (1794–1866), although Gregory's and Whewell's works would have been available

only after Green's 1828 essay (Cannell, 2001). Books about chemistry, biology, geology, medicine, and zoology far outnumbered the mathematics and physics books that were available to Green. Most of the "Natural and Mathematical" books were in English, with the exception of a few early publications from the *Philosophical Transactions* in Latin, and some later publications in French.

Bromley House Library's *Catalogue of the Books* (1841) has the entry "114: Mathematical Investigations, by George Green," a later work by Green, and the entry "115: Analysis of Electricity and Magnetism, by George Green" looks very similar to, and is probably a shorthand for, the title of the 1828 essay (Catalogue of the Books, 1841). No date is given for either of these two entries from Green, nor of any other works listed. We note that *Hutton's Mathematical and Philosophical Dictionary* was ordered according to the committee minutes of Monday October 6, 1823 (Figure 2).

We also note that the *Elementary Illustrations of the Celestial Mechanics of Laplace*, in French, was ordered Monday, January 5, 1824 (Figure 3).

Figure 2. Committee Minutes of Monday October 6, 1823, and the ordering of Hutton's *Mathematical and Philosophical Dictionary* (Committee Minutes, 1823).

Figure 3. Committee Minutes of Monday January 5, 1824, and the ordering of Elementary Illustrations of the *Celestial Mechanics* of Laplace (Committee Minutes, 1824).

Green refers to Laplace in his 1828 essay, so it is possible that the request was his. It is also possible that Green ordered Hutton's work, as suggested below.

Findings: Green's 1828 Essay References and Nottingham Subscription Library Availability

Green's 1828 essay acknowledges remarkably few references (Cannell, 2001), and his referencing system is confusing. Green names thirteen mathematicians and scientists within the 1828 essay, but cites in full only twelve publications from seven authors at the end. This in itself makes the task of matching references within the 1828 essay with the archives of the Nottingham Subscription Library a difficult process. Green cites five works by Siméon Denis Poisson (1781–1840), two by Jean-Baptiste Fourier (1768–1830), and one each by Laplace, Lacroix, and Jean-Baptiste Biot (1774–1862). In addition are two full references to British scientists, Peter Barlow (1776–1862) and Henry Cavendish (1731–1810). Although Green refers to Charles-Augustin de Coulomb (1736–1806) in the text of the essay, he does not provide a full reference at the end. Furthermore, Green refers in his essay to Laplace's work in shorthand but does not name him on several occasions, referring instead to, "as in the M′ec. C′eleste [sic]" (1828, p.

35). H. Gwynedd Green's *Biography of George Green, Mathematical Physicist of Nottingham and Cambridge* (1945), further identifies sources by Thomas Young (1773–1829) and Louis Arbogast (1759–1803) that were not named but are implied in the 1828 essay. A reader of Green's 1828 essay would therefore need to be familiar with all cited authors and their works, as well as comfortable with Green's preferred referencing style.

Both Barlow's letter "On the Temporary Magnetic Effect Induced in Iron Bodies by Rotation" (1825) and Cavendish's paper "An Attempt to Explain Some of the Principal Phaenomena of Electricity, by Means of an Elastic Fluid" (1771) were published in the *Philosophical Transactions of the Royal Society*, available to Green at the Nottingham Subscription Library. Similarly, the 1814 translation of Laplace by Toplis was also available. The request for Laplace's work in the original French made during the time of Green's shareholding membership at the Nottingham Subscription Library suggests that Green was able to read French. Cannell writes, "Thus with the exception of the *Transactions of the Royal Society,* none of the books used directly by Green was to be found in the library" (2001, p. 49), although Toplis's 1814 translation was in fact listed in the 1816 library catalogue (Catalogue of the Books, 1816)

and therefore presumably available to Green. Instead, Cannell argues, Green would have known of his sources primarily from the *Philosophical Transactions,* from which he would have sought out further works in scientific journals that could be ordered through the Nottingham Subscription Library, but could also be purchased from local booksellers.

Of his few references, Green himself writes in a preface to his 1828 essay,

> had it been practicable, I should have been glad to have given … an historical sketch of its progress; my limited sources of information, however, will by no means permit me to do so; but probably I may here be allowed to make one or two observations on the few works which have fallen in my way (Green, 1828).

Gwynedd Green gives further context, writing, "Working by day as a miller and with the intellectual atmosphere of Bromley House as a background for his scientific interests, George Green published his first and greatest paper in 1828" (1945, p. 558). This also suggests that while Green was quite well read, the 1828 essay remains remarkable in its application of mathematics available to him as a miller in early-nineteenth-century Nottingham.

It is interesting that these works acknowledged by Green use different forms of mathematics. The French mathematicians listed above all use European calculus, and Lagrange, referred to specifically by Green (1828, p. 36), used variational calculus (Gray, 2015), but Cavendish used Newtonian fluxions in his 1771 paper. The inclusion of Cavendish suggests very strongly that Green was able to understand *both* Newtonian fluxions and Leibnizian calculus, as well as the variational calculus of Lagrange. However, his 1828 essay is evidence that he preferred to work *only* with the calculus notation of Leibniz and Lagrange. It is possible, therefore, that the ordering of Hutton's *Dictionary* of 1823, as recorded at the Subscription Library, if indeed by Green, is not necessarily to be perceived as an error on his part because the mathematics in the *Dictionary* was aimed at military cadets as suggested by Cannell (2001), but perhaps so that he could understand British mathematics for the completion of his 1828 essay, citing Cavendish's work of 1771, which uses Newtonian fluxions. Furthermore, Cannell does not comment on Green's use of Lagrange's notation in the 1828 essay. It is therefore worth noting this as yet more evidence of Green's brilliance in being self-taught sufficiently not only to read and understand the fluxions of Cavendish, but also to work proficiently with the calculus notations of both Leibniz and Lagrange.

Findings: Wider Scientific Cultural Contexts at the Nottingham Subscription Library

1. Newton

The archives of the Nottingham Subscription Library provide insight into the shift of interest in using different calculus methods in Britain at the beginning of the nineteenth century. As already discussed, Newton's fluxions were dominant in Britain throughout the eighteenth century and into the nineteenth. Historian Thomas Sonar writes that British mathematicians, including James Stirling (1692–1770), Colin Maclaurin (1698–1746), Brook Taylor (1685–1731), and Roger Cotes (1682–1716), "could not keep up with the mathematicians on the Continent" (2018, p. 482). Similarly, Boyer states, "In England Newton's lack of clarity and his inconsistency in notation was followed by a confusion of fluxions with moments" (1959, p. 278). The Nottingham Subscription Library catalogue lists the Royal Society's *Philosophical Transactions* from 1665 onward. For Green, access to this publication was hugely significant, because "Green would have been able to follow the course of discoveries in science for the previous 160 years" (James, 2004, p. 121).

In these publications, Green would have found the Royal Society dominated by Newton, Newton's supporters, and the culture of inflexibility for some non-British scientists. For instance, this perspective can be seen in an anonymous letter to the Swiss scientist Castilion, containing a scathing review of his attempts to replicate Newton's methods in Lausanne, Switzerland:

> his process is so illogical and embarrassed, and so far short of a demonstration of Newton's celebrated binomial theorem, that it is undeserving of being reprinted on the present occasion (Anonymous, 1742).

Another publication in the *Philosophical Transactions* was by Davies Gilbert (1826), later president of the Royal Society, whose work was published comparatively late for the fluxions tradition.

2. Mixed Methods

European mathematics began to appear in British mathematical writing shortly after Newton's death in a mixed and sometimes confusing way. For example, fellows of the Royal Society, publishing in the *Philosophical Transactions*, followed Newton's fluxions and developed its methods, but also began referring to European calculus. While Hutton, Thomas Simpson (1710–1761), and Edmond Stone (1700–1768) all published works about fluxions, Blanco writes how the two different calculus methods were beginning to get mixed together by the 1730s:

> It is, therefore, not hard to imagine that the introductory works of the period reflected the contemporary confusion around the nature of fluxions. Even adherents of the same approach could express it in different ways. For instance, Stone adopted again a double standard. In the text, the definition of fluxion matches up with that of dx, according to the Leibnizian approach: "The infinitely small part whereby a variable quantity is continually increased or decreased, is called the fluxion of that quantity" (Stone, 1730, part I, Definition II) (2014, p. 56).

Among the texts that were been available to Green at the Nottingham Subscription Library were examples of these "mixed-methods," including Vince (1786, p. 432) and Woodhouse (1802, p. 88).

However, Green clearly did not follow this convention of mixing notations and traditions. He avoided the shortcomings of seeking to continue the dominance of Newton's fluxions. While the Newtonian and Leibnizian methods are both ways of working with calculus, they do not mix well, as illustrated by Blanco's example above. As already noted, both Gray (2015) and Cannel (2001) view Green's limited education as an advantage for his original approach and thinking in the 1828 essay. One can only assume that had Green attended Cambridge as a young man, he would have been educated in fluxions, limiting his brilliance with the weight of British Newtonian mathematical convention. It is therefore to his advantage that Green was self-taught, influenced by Toplis, and, we can assume, was encouraged to read European mathematics. Therefore, Green chose to *refer* to Cavendish's publication using fluxions, but he *applied* the calculus of Leibniz and Lagrange. Green's ability in all three methods, and ultimately his preference for European mathematics, enabled the content of his 1828 essay.

3. Traditions Overlap: Ivory and Buée
However, despite this emerging interest in European methods, the sentiments favoring Leibnizian calculus were not universal. For example, the Nottingham Subscription Library included an early-nineteenth-century publication in the *Philosophical Transactions* of the Royal Society by James Ivory (1765–1842), in which he critiques Laplace:

> Although the analysis which Laplace has traced out for the attractions of spheroids must be allowed to be very ingenious and masterly, yet still there are some considerations which cannot but lead us to think, that it falls short of that degree of perfection which it is laudable to aim at (1812, p. 33).

As Cannell writes, in many ways Ivory's paper represents how there persisted "prejudice against continental mathematics and the rigidity of the Cambridge establishment" (2001, p. 40). Ivory's work indicates the lasting influence of Newton, even into the early nineteenth century.

There were mixed responses to Newton's fluxions methods in Europe. Gray (2015) cautions against simplifying the history of the development of calculus in Europe, and writes,

> the foundations of the calculus were for at least two centuries the subject of shifting, partial, and largely coherent speculations that form the opening chapters of the history of analysis" (p. 11).

Gray illustrates how Cauchy developed Leibnizian methods, whereas d'Alembert (1717–1783) developed Newtonian methods, with Lagrange rejecting fluxions altogether. Only six years previous to Ivory's critique of Laplace in 1812, Adrien-Quentin Buée (1748–1826) used Leibnizian calculus in his *Mémoire sur les quantités imaginaires*, published in French in the *Philosophical Transactions,* commenting:

> Quelle profondeur, quelle addresse, quelle sagacité n'a t'il pas fallu à Mr. La Place pour poser, avec le seul secours des équations différentielles connues (expliquées à la manière ordinaire) la dernière pierre à l'édifice Newtonien![2] (1806, p. 87).

Bueé, whose "views on imaginary numbers conform one of the last episodes in the acceptation of negative and imaginary numbers as valid mathematical entities on metaphysical grounds" (Pacheco Castelao et al, 2006, p. 8), was a French priest who escaped late-eighteenth-century and revolutionary France to settle in England. Pacheco Castelao et al. further comment that for the *Philosophical Transactions* to publish Bueé in French was remarkable, given the convention for articles accepted by the Royal Society to be published in English, and that more were published in Latin than French by the early nineteenth century. Gray (2015) notes that Bueé helped influence European mathematicians, including Augustin-Louis Cauchy (1789–1857), a contemporary of Green.

Therefore, as these examples of Ivory and Bueé in the *Philosophical Transactions* show, it is important to appreciate that the history of the development of calculus in nineteenth-century Britain and Europe was one of evolving concepts and change in methods. Thus, concepts of infinity, continuous function, limit, continuum and notions of time and space, and the infinitesimal, as well as the differential and the derivative, were all explored and reviewed beyond the achievements of either Newton or Leibniz.

4. The Cambridge Analytical Society
Change in British mathematics was inevitable, and by the beginning of the nineteenth century, British mathematicians were using Leibnizian calculus. The "young savages" (Guicciardini, 1989) of the Analytical Society began as an undergraduate stunt, a distribution of pamphlets about European mathematics imitating a debate about how the university was to hand out Bibles to students (Enros, 1983). The society took a stance against conventional British and Newtonian mathematics as taught in Cambridge, soon developing a serious appreciation of European methods, and first publishing their own volume in 1813. Charles Babbage (1791–1871), Sir Edward Bromhead (1789–1855), George Peacock (1791–1858), and John Herschel (1792–1871) all imported Leibnizian calculus after Babbage, traveling in Europe as an undergraduate, bought *Traité du calcul differential et du calcul integral* by Lacroix, although the result was increased frustration in knowing more than their university tutors.

The Analytical Society must be contextualized as an indication of the reform of mathematics that was yet to take place at Cambridge:

> the Analytical Society … actually played no real part in the movement to reform Cambridge mathematics: rather, it was a precursor of that movement. Much of the Society's significance for the history of mathematics lies in the way it illuminates the diversity of

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[2] "It fell to Mr Laplace to put, with such depth, address, and wisdom, and solely with recourse to those known differential equations (explained in the ordinary manner), the last stone in Newton's edifice!" Translation from the French by Dr. Paul Omar.

forces that were at work in the transformation of mathematics in Cambridge and England (Enros, 1983, p. 26).

Nevertheless, the activities of the Society's members were relevant, with Bromhead actively encouraging reading the translation by George Peacock and William Whewell in 1816 of Lacroix's *Calcul Integral et Differentiel* (Durand-Richard, 2010), both established university tutors in mathematics at Cambridge by this time (Cannell and Lord, 1993). However, it is not thought that Bromhead knew of Green's work on the 1828 essay until after its publication (Cannell and Lord, 1993), and so it remains questionable what direct influence Bromhead could have had on Green's essay. The Analytical Society did not last, since various other members graduated and left Cambridge, but through Toplis, and later, following the 1828 essay, Bromhead and his associates, Green could take advantage of the evolving European mathematics environment in Britain.

Historian Thomas Sonar writes that the reform of calculus in nineteenth-century Britain was mainly due to Laplace, with the first four volumes of Laplace's *Mécanique Celeste* published between 1799 and 1805 having "a stimulating effect concerning a reform of British analysis" (2018, p. 485). By the time of Green's essay of 1828, European mathematics was beginning to take hold in Cambridge. As Cannell notes in her biography of Green,

> A few years before Green published his first Essay, a notable revival of learning swept the university: the Fluxional symbolism, which since the time of Newton had isolated Cambridge from continental schools, was abandoned in favour of the differential calculus, and the works of the great French analysts were introduced and eagerly read (Whittaker, 1910, p. 153, in Cannell, 2001, p. 29).

Furthermore, the *Philosophical Transactions* also included publications using the variational calculus of Lagrange, such as Knight (1817, p. 466) and Horner (1819, p. 311). As Sonar writes, different traditions were by now running parallel:

> In the second decade of the nineteenth century, three forms of the new analysis could be found in Britain: Newton's calculus of fluxions was dominant but not further developed for some time, the infinitesimal analysis of Leibniz and Euler, and finally the differential and integral calculus of Lagrange (2018, p. 485).

It was this time of critical mathematical discovery and the development of calculus that Green navigated superbly, citing three different methods and using Leibniz's and Lagrange's methods in his 1828 paper.

5. Green's Access to European Mathematics
By the early nineteenth century, therefore, it was possible for Green to gain access to European mathematics, not only through the Nottingham Subscription Library, but also from ordering or purchasing scientific publications, as already

discussed. It is therefore not possible to know which books were available at the time of Green's membership, since the catalogue was updated again only in 1841, after his membership had terminated in 1832. In particular, it is not possible to know which of the books listed in the 1841 catalogue were available and could have been used by him for the 1828 essay. However, Laplace's work *Céleste Mécanique* (Laplace, 1814) also gave Green access to the works of Adrienne-Marie Legendre (1752–1833) and Sylvestre François Lacroix (1765–1843), among others, although James cautions that these were not "the particular books to which Green refers or makes use of in the [1828] *Essay*" (2004, p. 121). Cannell and Lord write, "Toplis was a keen protagonist for Leibnizian mathematics, to the point of publishing his translation at his own expense" (1993, p. 28). While we might assume Green's continued contact with Toplis until 1819, the translation of *Celeste Mécanique,* and Toplis's references to other works by Laplace, Lacroix, Lagrange, and Legendre in his preface provide some documentary evidence of how Green's interest in Leibnizian calculus would have been possible (Cannell, 2001).

Cannell and Lord suggest that Green's choice of electricity and magnetism as the topic of his 1828 essay would have required European rather than conventional British mathematics:

> The "Mathematical Analysis" in the *Essay* title gives pause for thought, since it was the term applied to the mathematics used on the Continent based on the calculus as formulated by Leibniz, in contradiction to the fluxions of Newton: in Babbage's memorable phrase "*d*-ism as opposed to *dot*-age" (1993, pp. 27–28).

In addition, Gray writes,

> The very word "analysis" in the title reminds us that Green was self-taught, for it refers to the calculus as it had become in France, not to the sterile exercises in Newtonian methods preferred in England (2015, p. 137).

It was therefore noteworthy for a British mathematician to choose a European topic for his essay. If Green's access to the Nottingham Subscription Library resources was significant, it was because of his self-teaching, allowing him the freedom to focus on the subject of his 1828 essay without being encumbered by Newtonian traditions at a British university. As Gray further comments,

> Green had become interested in the hugely popular topic of the emerging new physics of electricity and magnetism … The new physics was to call for new mathematics, and was based on intuitions of a non-Newtonian kind (2015, p. 137).

Thus, though by no means isolated in his choice of European calculus given the developments of 1820s Cambridge, Green's 1828 essay indicates the beginning of the end of British mathematics being dominated by Newtonian fluxions.

Green after 1828 and Further Research

Green's essay was presented to the Nottingham Subscription Library on its publication (Hoare, 1991; Mastoris, 1991), having been advertised in December 1827 in the local Nottingham press. Although many local subscribers and townspeople bought copies, "in provincial Nottingham, in 1828, it was more probably received with polite bewilderment and incomprehension" (Cannell, 2001). After taking over the family milling business on the death of his father in 1829, Green did, however, send a copy to Bromhead as one of a few subscribers to Bromley House who lived outside of Nottingham town. On receiving the paper, Bromhead actively encouraged Green, and recommended him to Gonville and Caius College, Cambridge. Green went to Cambridge in 1833 at the age of 40, published more papers, and is widely known today for his far-reaching contributions in both mathematics and physics.

Green's essay of 1828 was an astonishing achievement:

> The combination of ideas, their elegance, and their clear presentation even when rigour lay out of reach, impressed mathematicians, and rapidly ensured that Green's name was henceforth securely attached to his discoveries, even when some had by then been discovered by others (Gray, 2015, p. 140).

At Cambridge, Green was an outstanding student, and was awarded the Perse Fellowship at Caius. He returned to Nottingham in 1840, dying in 1841. He is buried in St. Stephen's churchyard in Sneinton, Nottingham, across the road from his mill (Green's Mill, 2012; Cannell and Lord, 1993). A memorial plaque was installed in Westminster Abbey, dedicated in 1993 for the bicentenary of Green's birth (Fauvel, 2000), and Bromley House continues to celebrate Green as a former member of the Nottingham Subscription Library.

In following Green's mathematical education, it would be interesting to examine the increasing use of Leibnizian calculus in particular, and European mathematics in general, at the University of Cambridge following Newton's death. It would be valuable to understand how Toplis, for example, was so familiar with Leibnizian calculus and European mathematics that he was presumably able to share his interests with Green while they were living as neighbors in Nottingham. To look through these archives, and publications of the *Analytical Society* at Cambridge in the 1820s, would be fascinating further research.

The story of Green's use of European calculus illustrates a cultural change in British mathematics. The controversy between Newton and Leibniz shows the fierce cultural and national differences under which mathematics developed in the eighteenth and nineteenth centuries. By undertaking a systematic documentary analytical review of the publications accessible to Green as a member of the Nottingham Subscription Library, we can see that he had access to many publications, some of which used Newtonian fluxion methods, particularly from the Royal Society's *Philosophical Transactions,* and others that were by European and British mathematicians using Leibnizian calculus and the calculus of Lagrange. These publications at Bromley House library available to Green during his 1823–1832 membership suggest significant variations in the social construct of the scientific communities in Britain and Europe. Green's use of European calculus in his 1828 essay signaled a bigger change in British mathematics away from Newton's fluxion methods, embracing European calculus to the ultimate benefit of the British mathematical and scientific communities. Green's personal story might remain shrouded in obscurity, but it reflects one of the invaluable roles that a local library can play in shaping the development of mathematics in Britain.

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