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AUTHOR CORRECTION

Correction to: Stochastic Control of Memory Mean-Field Processes

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The original version of this article unfortunately contained a few mistakes in Theorems and notation. The corrected information is given below.

- In Definition 1.1 the space $\mathcal{M}$ is not a Hilbert space, but a *pre-Hilbert* space. Throughout the paper $\mathcal{M}$ denotes the completion of this space.
- The proofs of Theorems 4.5 and 4.6 are not correct as stated.
  However, the proofs can easily be corrected—and even simplified—as follows: For a given Fréchet differentiable operator $G : \mathcal{M}_0 \mapsto \mathbb{L}^1(\mathbb{P})$ let $\nabla_m G$ denote its Fréchet derivative at the measure $m \in \mathcal{M}_0$, and for $X \in \mathbb{L}^2(\mathbb{P})$ let $M = \mathcal{L}(X)$ denote the law of X. Define the dual operator $\nabla_m^* G(\cdot) \in \mathbb{L}^2(\mathbb{P})$ by the property

$$\mathbb{E}[\langle \nabla_m G, M \rangle] = \mathbb{E}[\nabla_m^* G X]; \quad X \in \mathbb{L}^2(\mathbb{P}), \tag{0.1}$$

  and let the dual operator $\nabla_{\bar{m}} H^t$ be defined in the similar way as $\nabla_{\bar{x}} H^t$ in (4.6)–(4.7) in the paper. The existence and the uniqueness of $\nabla_m^* G$ and $\nabla_{\bar{m}} H^t$ follow by the Riesz representation theorem, as explained in the paper.

The original article can be found online at https://doi.org/10.1007/s00245-017-9425-1.

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With the introduction of these dual operators the proofs of Theorems 4.5 and 4.6 can be corrected and simplified. Specifically, there is no need for the adjoint variables p^1, q^1 and r^1, if we modify the backward stochastic differential equation (BSDE) for the adjoint variables p^0, q^0, r^0 accordingly. More precisely, everything concerning the processes p^1, q^1, r^1 can be deleted throughout the paper if we change the BSDE (4.13) to

$$\begin{aligned} dp^0(t) &= -\left\{\frac{\partial H}{\partial x}(t) + \mathbb{E}[\nabla_{\bar{x}} H^t|\mathcal{F}_t] + \nabla_m^* H(t) + \mathbb{E}[\nabla_{\bar{m}} H^t|\mathcal{F}_t]\right\}dt \\ &\quad + q^0(t)dB(t) + \int_{\mathbb{R}_0} r^0(t,\zeta)\tilde{N}(dt,d\zeta); \quad t \in [0,T] \\ p^0(t) &= \frac{\partial h}{\partial x}(X(T),M(T)) + \nabla_m^* h(X(T),M(T)); \quad t \geq T \\ q^0(t) &= 0; \quad t > T \\ r^0(t,\cdot) &= 0; \quad t > T, \end{aligned} \tag{0.2}$$

- The BSDE (4.14) is not needed and should be deleted.
- The expression (4.23) should be corrected to

$$\begin{aligned} &= \mathbb{E}\Bigg[\int_0^T \hat{p}^0(t)\tilde{b}(t) \\ &\quad - \int_0^T \left\{\frac{\partial \hat{H}}{\partial x}(t) + \mathbb{E}[\nabla_{\bar{x}} \hat{H}^t|\mathcal{F}_t] + \nabla_m^* \hat{H}(t) + \mathbb{E}[\nabla_{\bar{m}} \hat{H}^t|\mathcal{F}_t]\right\}\tilde{X}(t)dt \\ &\quad + \int_0^T \hat{q}^0(t)\tilde{\sigma}(t)dt + \int_0^T \int_{\mathbb{R}_0} \bar{r}^0(t,\zeta)\tilde{\gamma}(t,\zeta)\nu(d\zeta)dt\Bigg] \end{aligned} \tag{0.3}$$

- Equation (4.24) can be deleted.
- The first part of the proof of Theorem 4.6 (up to and including (4.29)) can be deleted. All terms in the proof involving p^1 should be deleted. The equation for $\mathbb{E}[p^0(T)Z(T)]$ should be corrected to

$$\begin{aligned} &\mathbb{E}[p^0(T)Z(T)] \\ &\quad = \mathbb{E}\left[\int_0^T p^0(t)dZ(t) + \int_0^T Z(t)dp^0(t) + [p^0, Z]_T\right] \\ &\quad = \mathbb{E}\Bigg[\int_0^T p^0(t)(\nabla b(t))^T\ (Z(t), Z_t, DM(t), DM_t, \pi(t), \pi_t)dt \\ &\quad - \int_0^T \left\{\tfrac{\partial H}{\partial x}(t) + \mathbb{E}[\nabla_{\bar{x}} H^t|\mathcal{F}_t] + \nabla_m^* H(t) + \mathbb{E}[\nabla_{\bar{m}} H^t|\mathcal{F}_t]\right\} Z(t)dt \\ &\quad + \int_0^T q^0(t)(\nabla\sigma\ (t))^T\ (Z(t), Z_t, DM(t), DM_t, \pi\ (t)\ , \pi_t)dt \\ &\quad + \int_0^T \int_{\mathbb{R}_0} r^0(t,\zeta)(\nabla\gamma\ (t,\zeta))^T\ (Z(t), Z_t, DM(t), DM_t, \pi(t), \pi_t)\nu(d\zeta)dt\Bigg]. \end{aligned}$$