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Erratum to: Dependence of Hilbert coefficients

Published online: 15 September 2017

Erratum to: manuscripta math. 149, 235–249 (2016)
DOI 10.1007/s00229-015-0762-6

Unfortunately, there was a gap in the proof of Proposition 2.3, and we have to delete it. Keeping the notation in [2], then the proof of Proposition 2.3 only gives the following result.

Proposition A. *Assume that* $y_1, \dots, y_d \in R_1$ *is an* E*-filter-regular sequence of* R*, that is,* $[0 :_{E/(y_1,\dots,y_{i-1})E} y_i]_n = 0$ *for all* $n \gg 0$*. Put* $B^* = \ell_{R_0}(E/(y_1, \dots, y_d)E)$*. Then,* $|e_i(E)| \le B^*(\mathrm{reg}^1(E) + 1)^i$*, for all* $1 \le i \le d - 1$.

These inequalities could be useful elsewhere. For the local case, we can only prove

Proposition B. *Let* $x_1, \dots, x_d \in I$ *be an* $\mathbb{M}$*-superficial sequence for* I *and* $B = \ell(M/(x_1, \dots, x_d)M)$*. Then,* $|e_i(\mathbb{M})| < B(2\,\mathrm{reg}(G(\mathbb{M})) + 2)^i$ *for all* $1 \le i \le d$.

Proof. We do induction on d. Let $a = \mathrm{reg}(G(\mathbb{M}))$ and $e_i = e_i(\mathbb{M})$. By [2, Lemma 1.5],

$$H_{\mathbb{M}}(a) = P_{\mathbb{M}}(a) = \sum_{i=0}^{d} (-1)^i e_i \binom{a+d-i}{d-i}.$$

By [1, Lemma 1.7],

$$H_{\mathbb{M}}(a) = \ell(M/M_{a+1}) \le \ell(M/I^{a+1}M) \le B\binom{a+d}{d}.$$

Note that $\binom{a+j}{j} \le (a+1)^j$ and $e_0 = e_0(I, M) \le B$.

If $d = 1$, then

$$|e_1| = |H_{\mathbb{M}}(a) - e_0(a+1)| \le \max\{B(a+1), e_0(a+1)\} = B(a+1).$$

The online version of the original article can be found under doi:10.1007/S00229-015-0762-6

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DOI: 10.1007/s00229-017-0975-y

Let $d \geq 2$. First, we prove the statement for $0 < i \leq d-1$. Assume that $\operatorname{depth}(M) > 0$. Then, $\dim(M/x_1M) = d-1$, and by [3, Proposition 1.2], $e_i(\mathbb{M}) = e_i(\mathbb{M}/x_1M)$ for all $i \leq d-1$. By [2, Lemma 1.9], $\operatorname{reg}(\mathbb{M}/x_1M) \leq a$. Hence, by the induction hypothesis applied to $\mathbb{M}/x_1M$ and the sequence $x_2, \ldots, x_d$, we get

$$|e_i(\mathbb{M})| < B(2\operatorname{reg}(G(\mathbb{M}/x_1M)) + 2)^i \leq B(2a+2)^i.$$

We now assume that $\operatorname{depth}(M) = 0$. Let $\overline{M} = M/H^0_{\mathfrak{m}}(M)$ and $\overline{\mathbb{M}} = \mathbb{M}/H^0_{\mathfrak{m}}(M)$. Note that $e_i\mathbb{M}) = e_i(\overline{\mathbb{M}})$ for all $i \leq d-1$ and $\ell(\overline{M}/(x_1, \ldots, x_d)\overline{M}) \leq B$. In the proof of [1, Lemma 1.9], it was shown that there is an exact sequence

$$0 \to K \to G(\mathbb{M}) \to G(\overline{\mathbb{M}}) \to 0,$$

where K has a finite length. Hence, $\operatorname{reg}(G(\overline{\mathbb{M}})) \leq \operatorname{reg} G(M) = a$, and

$$|e_i(\mathbb{M})| = e_i(\overline{\mathbb{M}}) < \ell(\overline{M}/(x_1, \ldots, x_d)\overline{M})(2\operatorname{reg}(G(\overline{\mathbb{M}})) + 2)^i \leq B(2a+2)^i.$$

Finally, we have

$$\begin{aligned}
|e_d| &\leq H_{\mathbb{M}}(a) + \textstyle\sum_{i=0}^{d-1} |e_i| \binom{a+d-i}{d-i} \\
&< B\binom{a+d}{d} + B \textstyle\sum_{i=0}^{d-1} 2^i (a+1)^i \binom{a+d-i}{d-i} \\
&\leq B(a+1)^d + B \textstyle\sum_{i=0}^{d-1} 2^i (a+1)^i (a+1)^{d-i} \\
&= B2^d (a+1)^d.
\end{aligned}$$

□

Using Proposition B instead of Proposition 2.3 in the proof of [2, Theorem 2.4], we can still derive the same bound, because there we used a very rough estimation $d+1 < \omega^{d+1}$, and now instead of it, we only need to use the estimation $2^d \leq \omega^d$. Also note that there were some misprints in establishing the inequality (8) in the proof of [2, Theorem 2.4], but the inequality is correct.

References

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