manuscripta math. 133, 545–546 (2010) © Springer-Verlag 2010

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Erratum to: Hyperelliptic curves with prescribed p-torsion

Published online: 1 September 2010

Erratum to: Manuscripta Math. (2005) 117(3):299–317
DOI 10.1007/s00229-005-0559-0

The statement of [4, Prop. 2] is correct, but a case is missing from the proof. Here is the material necessary to complete the proof.

Consider the moduli space $\mathcal{H}_g$ of smooth hyperelliptic curves of genus g and its closure $\overline{\mathcal{H}}_g$ in $\overline{\mathcal{M}}_g$. Let $V_{g,f}$ denote the sublocus of $\overline{\mathcal{M}}_g$ whose points correspond to curves of genus g with p-rank at most f.

Proposition 1. [4, Prop. 2] *The locus $V_{g,f} \cap \overline{\mathcal{H}}_g$ is pure of codimension $g - f$ in $\overline{\mathcal{H}}_g$.*

The proof uses the intersection of $V_{g,f}$ with $\Delta_0 := \Delta_0[\overline{\mathcal{H}}_g]$, the sublocus of $\overline{\mathcal{H}}_g$ whose points correspond to stable hyperelliptic curves which are not of compact type, i.e., whose Picard variety is not represented by an abelian scheme. The text of page 3 contains the incorrect statement: "The boundary $\overline{\mathcal{H}}_g - \mathcal{H}_g$ consists of components Δ_0 and Δ_i for integers $1 \leq i \leq g/2$." In fact, Δ_0 is not irreducible for $g \geq 3$. By [3,6], it is the union of components $\Xi_i := \Xi_i[\overline{\mathcal{H}}_g]$ for $0 \leq i \leq g-2$, where each Ξ_i is an irreducible divisor in $\overline{\mathcal{H}}_g$ and where Ξ_i and Ξ_{g-i-1} denote the same substack of $\overline{\mathcal{H}}_g$.

The following description of Ξ_0 and Ξ_i for $i \geq 1$ can be found in [1, Sect. 2.3]. If η is the generic point of Ξ_0, then the curve Y_η is an irreducible hyperelliptic curve self-intersecting in an ordinary double point P. The normalization of Y_η is a smooth hyperelliptic curve Y_1 of genus $g-1$ and the inverse image of P in the normalization consists of an orbit under the hyperelliptic involution. For $1 \leq i \leq g-2$, if η is the generic point of Ξ_i, then the curve Y_η has two components Y_1 and Y_2, which are smooth irreducible hyperelliptic curves, of genera $g_1 = i$ and $g_2 = g - 1 - i$, intersecting in two ordinary double points P and Q. The hyperelliptic involution ι stabilizes each of Y_1 and Y_2. The points P and Q form an orbit of the restriction of ι to each of Y_1 and Y_2.

The online version of the original article can be found under doi:10.1007/s00229-005-0559-0.
Rachel Pries is partially supported by NSF grant DMS 07-01303.

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DOI: 10.1007/s00229-010-0392-y

Let C_0 be a component of $V_{g,f} \cap \overline{\mathcal{H}}_g$. The first paragraph of [4, proof of Prop. 2] shows that $\dim(C_0) \geq g-1+f$ and that C_0 intersects Δ_0. The second and third paragraphs of [4, proof of Prop. 2] prove the result in the case that C_0 intersects Ξ_0.

Proof. (Material needed to complete the proof of [4, Prop. 2].) To complete the proof, it suffices to consider the case that C_0 intersects Ξ_i for some $i \geq 1$. A point of $C_0 \cap \Xi_i$ is the moduli point of a curve Y with two components Y_1 and Y_2 as described above. Let f_1 (resp. f_2) be the p-rank of Y_1 (resp. Y_2). Since the toric part of $\mathrm{Jac}(Y)[p]$ contains a copy of the group scheme μ_p, [2, Ex. 9.2.8] implies that $f_1+f_2 \leq f-1$. For $\ell = 1, 2$, the curve Y_ℓ corresponds to a point of $V_{g_\ell, f_\ell} \cap \overline{\mathcal{H}}_{g_\ell}$. The hyperelliptic orbit $\{P, Q\}$ is determined by the choice of hyperelliptic orbits $\{P_1, Q_1\}$ on Y_1 and $\{P_2, Q_2\}$ on Y_2.

The clutching morphism

$$\lambda_{g_1,g_2} : \overline{\mathcal{H}}_{g_1;1} \times \overline{\mathcal{H}}_{g_2;1} \to \overline{\mathcal{H}}_{g_1+g_2+1}.$$

is a finite, unramified morphism between moduli spaces of labeled curves [5, Cor. 3.9], see also [1, Sect. 2.3]. Now $C_0 \cap \Xi_i$ is in the image of the restriction of λ_{g_1,g_2} to

$$(V_{g_1,f_1} \cap \overline{\mathcal{H}}_{g_1;1}) \times (V_{g_2,f_2} \cap \overline{\mathcal{H}}_{g_2;1}).$$

Since $g_1, g_2 < g$, one can apply an inductive approach. Applying the inductive hypothesis gives $\dim(V_{g_1,f_1} \cap \overline{\mathcal{H}}_{g_1;1}) = g_i + f_i$; (note that the marked orbit increases the dimension by 1). Thus $\dim(C_0 \cap \Xi_i) \leq (g_1+f_1)+(g_2+f_2) = g+f-2$. Since Ξ_i is an irreducible divisor in $\overline{\mathcal{H}}_g$, this yields $\dim(C_0) \leq g+f-1$, and thus $\mathrm{codim}(C_0, \overline{\mathcal{H}}_g) = g-f$.

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