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# Multiple solutions for quasi-linear elliptic problems in $\mathbb{R}^2$ with exponential growth

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Theorem 1 in our paper [1] does not hold in the form it was stated, since the growth estimate from below of the sequence of min–max values $b_k$ obtained in Sect. 5, Lemma 3, contains a technical mistake. The proof of Lemma 3 fails since it relies upon erroneous application of Theorem 5, which states that for each $1 < p < 2$ and $R > 1$ there is a $\vartheta = \vartheta(R) > 0$ such that

$$\int_\Omega \left(e^{|u|^p} - 1\right) dx \le C_0 \|u\|^{1/\vartheta}$$

for any $u \in H_0^1(\Omega)$ with $\|u\|_{1,2} = R$. We have mistakenly applied Theorem 5 in the proof of Lemma 3 to obtain inequality (39), which is not correct: it has to be substituted with the following inequality

$$\begin{aligned}\int_\Omega \left(e^{|u|^p} - 1\right) dx &\le \|u\|_{\alpha\beta}^{\beta} \left\{\int_\Omega \left(e^{\frac{\alpha}{\alpha-1}|u|^p} - 1\right) dx\right\}^{\frac{\alpha-1}{\alpha}} + c_1 \\ &\le \|u\|_{\alpha\beta}^{\beta} C_{\alpha,\vartheta'} R^{\frac{\alpha-1}{\alpha\vartheta'}} + c_1,\end{aligned}$$

where $\alpha = \alpha(\vartheta)$ has been defined in the lines above (39) as a function of $\vartheta = \vartheta(\|u\|_{1,2})$ and $\vartheta' = \vartheta'((\frac{\alpha}{\alpha-1})^{1/p}\|u\|_{1,2})$ is obtained by applying Theorem 5 to the term $\int_\Omega \left(e^{\frac{\alpha}{\alpha-1}|u|^p} - 1\right)dx$ (in (39), instead, we have used the same $\vartheta$). The value of $\vartheta$ is different from the value of $\vartheta'$, so that the arguments that follow in the proof fail.

Nevertheless, a large part of the paper builds up correctly the machinery to implement the classical Bahri–Beresticki–Rabinowitz perturbation method to work with second order quasi-linear elliptic equations of variational type in the plane with the exponential nonlinearity $|u|^{p-2}ue^{|u|^p} + \varphi$, where $1 < p < 2$. Thus as soon as a suitable growth estimate from below for the $b_k$'s is available in our setting (currently no result in the literature is, to our knowledge), a multiplicity result would follow.

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Changing nonlinearity, a weaker result can be stated. Indeed, for the exponential nonlinearity $ue^{|u|^p}$ with $0 < p < \frac{1}{2}$, Sugimura proved in [2] a logarithmic type estimate from below for the $b_k$'s, in the case of semi-linear elliptic equations. This estimate allows us to obtain a multiplicity result similar to the one claimed in [1], that can be stated as follows.

**Theorem 1.** *Assume that $a_{ij}(x, s)$ and $\varphi(x, s)$ satisfy the same hypotheses of Theorem 1 in [1] and, in addition, that there exist $\gamma > 0$ and $R > 0$ such that a.e. in $\Omega$ and all $(s, \xi)$ in $\mathbb{R} \times \mathbb{R}^2$*

$$|s| \geq R \implies \sum_{i,j=1}^{2} s D_s a_{ij}(x, s)\xi_i \xi_j \leq \gamma \sum_{i,j=1}^{2} a_{ij}(x, s)\xi_i \xi_j \, .$$

*For each $p \in (0, \frac{1}{2})$ the problem*

$$\begin{cases} -\sum\limits_{i,j=1}^{2} D_j(a_{ij}(x, u)D_i u) + \frac{1}{2} \sum\limits_{i,j=1}^{2} D_s a_{ij}(x, u)D_i u D_j u = ue^{|u|^p} + \varphi, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

*has a sequence $(u_h)$ of solutions in $H_0^1(\Omega)$ such that $f_\varphi(u_h) \to +\infty$.*

## References


1. Squassina, M., Tarsi, C.: Multiple solutions for a class of quasilinear elliptic problems in $\mathbb{R}^2$ with exponential growth. Manuscripta Math. **106**, 315–337 (2001)
2. Sugimura, K.: Existence of infinitely many solutions for a perturbed elliptic equation with exponential growth. Nonlinear Anal. **22**, 277–293 (1994)