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*Erratum*

# Erratum to: Construction of KMS States in Perturbative QFT and Renormalized Hamiltonian Dynamics

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The formula for the ground state ($\beta = \infty$) given in Theorem 3 of the paper is wrong. It is based on Theorem 2 and Proposition 3 which, however, are formulated and proved only for $0 < \beta < \infty$. In order to include the case $\beta = \infty$ one may modify Theorem 2 in the following way.

**Theorem 2.** *Let $\omega_\beta$ be a $\beta$-KMS state (for $0 < \beta < \infty$ ) or a ground state (for $\beta = \infty$ ) on $\mathfrak{A}$ with respect to $\alpha_t$. Then the following statements hold in the sense of formal power series in the interaction:*

- *For $A_1, \ldots, A_n \in \mathfrak{A}_h \in \mathfrak{A}_h$ the function*

$$G_{A_1,\ldots,A_n}(r, t_1, \ldots, t_n, s) = \frac{\omega_\beta \left(U_h(r)^{-1}\gamma \left(\alpha^h_{t_1}(A_1)\cdots\alpha^h_{t_n}(A_n)\right) U_h(s)\right)}{\omega_\beta(U_h(s-r))}$$

  *can be extended to a continuous function on the closure of $\mathfrak{T}^\beta_{n+2}$, which is analytic in the interior.*
- *For $0 < \beta < \infty$ the linear functional $A \to G_A(-i\beta/2, 0, i\beta/2) =: \omega^h_\beta(A)$ is a KMS state with respect to $\alpha^h_t$.*
- *If for $\beta = \infty$ the limit $\lim_{\beta'\to\infty} G_{A_1,\ldots,A_n}(-i\beta'/2, \ldots, i\beta'/2)$ exists uniformly on compact sets of $\mathfrak{T}^\infty_n$ for all $A_1, \ldots, A_n \in \mathfrak{A}_h$ then $\omega^h_\infty(A) = \lim G_A(-i\beta'/2, 0, i\beta/2)$ is a ground state with respect to $\alpha^h_t$.*

The proof for finite $\beta$ remains essentially the same. In the case of infinite $\beta$ one needs to have control on the behavior of the functions $G_{A_1,\ldots,A_n}$ as the arguments tend to infinity.

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In Proposition 3 one has to replace the formula by

$$\omega_\beta^h(A) = \sum_n (-1)^n \int_{S_{n,\beta}} d^n u\, \omega_\beta^c \left( \bigotimes_{j:u_j<0} \alpha_{iu_j}(K_h) \otimes \gamma(A) \otimes \bigotimes_{j:u_j\geq 0} \alpha_{iu_j}(K_h) \right) \tag{29}$$

with

$$S_{n,\beta} = \{-\beta/2 \leq u_1 \leq u_2 \ldots \leq \beta/2\}$$

For finite $\beta$ this coincides with the original formula due to the KMS condition. For infinite $\beta$ one has to control the convergence of the integrals

$$\int_{S_{n,\infty}} d^n u\, \omega_\beta^c ( \bigotimes_{j:u_j<s_1} \alpha_{iu_j}(K_h) \otimes \gamma(\alpha^h_{t_1+is_1}(A_1)) \otimes \bigotimes_{j:s_1\leq u_j<s_2} \alpha_{iu_j}(K_h) \otimes \gamma(\alpha^h_{t_2+is_2}(A_2)) \otimes \cdots \bigotimes_{j:u_j\geq s_k} \alpha_{iu_j}(K_h)).$$

If these integrals converge for all $A_1, \ldots, A_k \in \mathfrak{A}_h$ uniformly in $(t_1 + is_1, \ldots, t_k + is_k)$ on compact sets in $\mathfrak{T}_k^\infty$, then $\omega_\infty^h$ is a ground state.

Proposition 4 was formulated and proven for finite $\beta$. To extend it to the case of infinite $\beta$ requires additional work that goes beyond the purpose of this erratum.

In Theorem 3, where the infinite volume limit is treated, one has to use the formula (29) from the revised Proposition 3. The results for the clustering of connected functions of the free massive field also yield the required convergence properties for the revised expression. We expect that the resulting state is Lorentz invariant and coincides with the vacuum state constructed in the seminal paper of Epstein and Glaser [1], but this remains to be shown.

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## Reference

1. Epstein, H., Glaser, V.: The role of locality in perturbation theory. Ann. Inst. H. Poincaré A **19**, 211–295 (1973)

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