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ERRATUM

Erratum to: The algebraic structure of non-commutative analytic Toeplitz algebras

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The algebraic structure of the non-commutative analytic Toeplitz algebra $\mathfrak{L}_n$ is developed in the original article. Some of the results fail for the case $n = \infty$, and this implies that certain other results are not established in this case. In Theorem 3.2 of the original article, we showed there is continuous surjection $\pi_{n,k}$ from $\mathrm{Rep}_k(\mathfrak{L}_n)$, the space of completely contractive representations of $\mathfrak{L}_n$ into the $k \times k$ matrices $\mathfrak{M}_k$, onto the closed unit ball $\overline{\mathbb{B}_{n,k}}$ of $\mathcal{R}_n(\mathfrak{M}_k)$ by evaluation at the generators. It is further claimed that if $T = [T_1, \dots, T_n] \in \mathcal{R}_n(\mathfrak{M}_k)$ with $\|T\| < 1$, then there is a unique representation in $\pi_{n,k}^{-1}(T)$. Further information is obtained for $k = 1$ in Theorem 3.3 of the original article. Our proof of these results is valid for $n < \infty$, however, for $n = \infty$ the uniqueness claim is incorrect. An example due to Michael Hartz (see [2, Example 2.4]) shows that $\pi_{\infty,1}^{-1}(0)$ is very large—it contains a copy of the $\beta\mathbb{N}\backslash\mathbb{N}$.

The difficulty in the proof of Theorems 3.2 and 3.3 of the original article stems from the use of the factorization $A = WX$ used in Lemma 3.1 of the original article. In the case $n = \infty$, this factorization comes from Corollary 2.9. The problem is that the infinite sum in Corollary 2.9 converges in the strong topology, not the norm topology, so that when the representation Φ is not strongly continuous (or equivalently,

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WOT-continuous), the calculation of the norm in the last lines of the proof of Lemma 3.1 is invalid.

Theorem 3.3 is used throughout Section 4 of the original article. The results of Section 4 are valid when $n < \infty$. However, we are no longer certain of the validity of the following results when $n = \infty$: Theorem 4.1, Proposition 4.3, Theorem 4.6, Theorem 4.7, and Corollary 4.12. Proposition 4.3 is used in Theorem 4.6 to establish that all automorphisms are WOT-continuous. When attention is restricted to the class of WOT-continuous automorphisms, our proofs of Theorem 4.1 and Theorem 4.7 remain valid when $n = \infty$. We do not know whether WOT-continuity is automatic.

It is worth noting that the results of Section 4 are valid for the class of *isometric* isomorphisms even when $n = \infty$. This is because any isometric automorphism is WOT-continuous. Indeed, by [3, Theorem 3.3], $\mathfrak{L}_n$ has a unique predual, $\mathfrak{L}_{n*}$, which sits naturally inside the dual space $\mathfrak{L}_n^*$. The uniqueness of the predual implies that Θ^* fixes the image of $\mathfrak{L}_{n*}$, so there is a surjective isometry $\Theta_* : \mathfrak{L}_{n*} \to \mathfrak{L}_{n*}$ such that $\Theta = (\Theta_*)^*$. Hence Θ is weak-$*$ continuous. By [1, Corollary 2.12], the weak-$*$ and WOT topologies on $\mathfrak{L}_n$ coincide, so Θ is WOT-continuous.

References

1. Davidson, K.R., Pitts, D.R.: Invariant subspaces and hyper-reflexivity for free semigroup algebras. Proc. Lond. Math. Soc. **78**(2), 401–430 (1999)
2. Davidson, K.R., Hartz, M., Shalit, O.M.: Multipliers of embedded disks. Complex Anal. Oper. Theory (2014). arXiv:1307.3204v3; doi:10.1007/s11785-014-0360-8
3. Davidson, K.R., Wright, A.: Operator algebras with unique preduals. Canad. Math. Bull. **54**(3), 411–421 (2011)