

Holomorphic extension of CR functions from quadratic cones

Debraj Chakrabarti · Rasul Shafikov

Published online: 17 June 2009
© Springer-Verlag 2009

Erratum to: Math. Ann. (2008) 341:543–573
DOI 10.1007/s00208-007-0200-5

We correct in this note some errors in [1]. While these do not affect the main results (Theorem 1.1 and Corollary 1.1), some statements need to be revised. We are grateful to Adam Coffman, who pointed out that our classification of cones in Theorem 1.2 was missing one case.

The mistake occurred in Lemma 4.2 (p. 553). The correct hypothesis should read: *Let $Q \in \text{Sym}(2, \mathbb{R})$, $\det Q \leq 0$, and suppose that the sum of the four entries of Q is different from 0.* Then the proof offered in the paper goes through, since the last displayed formula has a non-zero denominator.

This affects Theorem 1.2: there is a new type $\mathcal{M}_{(1,1)}^4$ of quadratic cone with hermitian signature (1,1) in the table accompanying the statement of Theorem 1.2 on p. 547. The row to be inserted in the table is:

(π, ν)	Type	Parameters	Defining function
(1,1)	$\mathcal{M}_{(1,1)}^4$	$A > 0$	$\text{Re}(z_1^2 + iAz_1z_2) + \text{Im}(z_1\bar{z}_2)$

The proof of Theorem 1.2 has to be modified as follows.

The online version of the original article can be found under doi:[10.1007/s00208-007-0200-5](https://doi.org/10.1007/s00208-007-0200-5).

D. Chakrabarti (✉)
Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556, USA
e-mail: dchakrab@nd.edu

R. Shafikov
Department of Mathematics, The University of Western Ontario, London, Canada N6A 5B7
e-mail: shafikov@uwo.ca

- On p. 555, in Case 2, we need to note in the second sentence that thanks to Equations (16) and (17), the sum of entries of $g^t Q g$ cannot be zero.
- On p. 556, in Case 3, we only consider P such that the sum of the entries of $g^t P g$ is non-zero.
- At the end of p. 556, after Case 3, we deal with the remaining cones:

“Case 4 After reducing the matrix Q to the diagonal form $q \text{diag}(1, -1)$, $q \in \mathbb{R} \setminus \{0\}$, assume that in the new coordinates the entries of P satisfy $p_{11} + p_{22} + 2p_{12} = 0$. It then follows from (17) that

$$P = p \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},$$

where $p \in \mathbb{R}$. Thus the equation of the cone becomes

$$\text{Re} \left((p + iq)z_1^2 - 2pz_1z_2 + (p - iq)z_2^2 \right) + \text{Im}(z_1\bar{z}_2) = 0.$$

If $p = 0$, this reduces to the form considered in Case 3, i.e., $\mathcal{M}_{(1,1)}^1$. If $q < 0$, we make the coordinate change $z^* = iz$. If now $p < 0$, we make the additional change of coordinates $z_1^* = z_2$, $z_2^* = z_1$, so that we have $p > 0$, $q > 0$. Finally, we make the coordinate change $z_1^* = \sqrt{p}(z_1 - z_2)$, $z_2^* = \frac{1}{2\sqrt{p}}(z_1 + z_2)$, which reduces the cone to type $\mathcal{M}_{(1,1)}^4$, with $A = 2q$.”

- Following Lemma 5.1 on p. 557, we prove the uniqueness of cones of type $\mathcal{M}_{(1,1)}^4$ in the same way as for types $\mathcal{M}_{(1,1)}^1$ and $\mathcal{M}_{(1,1)}^2$. In fact, this is an immediate consequence of Lemma 5.1.

A quadratic cone of type $\mathcal{M}_{(1,1)}^4$ is non-minimal as it contains the complex line $\{z_1 = 0\}$. Therefore, the proof of Theorem 1.1 is unchanged. However, on p. 569, $\mathcal{M}_{(1,1)}^4$, should be included to the list of types in the statement of Proposition 7.1, conclusion (i).

Finally, we correct a misleading statement made in the abstract (p. 543). The words “if and only if” in the second line should be replaced by “whenever”. We prove in Corollary 1.1 that absence of two-sided support implies one-sided holomorphic extendability of CR functions. The converse is not discussed in this paper.

Reference

1. Chakrabarti, D., Shafikov, R.: Holomorphic extension of CR functions from quadratic cones. *Math. Ann.* **341**(3), 543–573 (2008)