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ERRATUM

Erratum to: Some Smooth Compactly Supported Tight Wavelet Frames with Vanishing Moments

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The line between the displayed formulas (16) and (17) was copied incorrectly from [41, Theorem 1]. It should read as follows: "Suppose that there exist trigonometric polynomials $\widetilde{P}_1(\mathbf{t}), \ldots, \widetilde{P}_M(\mathbf{t})$ such that". In addition, in the proof of Lemma 3 we overlooked to prove that the functions $\widetilde{P}^{(j)}_{n,m}(\mathbf{t})$ are $\mathbb{Z}^n$-periodic. This makes it necessary to reformulate Lemma 3. The statement and proof of Theorem 3 remain the same, but we wish to emphasize that the polynomials $L_0(A^T\mathbf{t})$ and $L_1(A^T\mathbf{t})$ are generated by the algorithm described in Theorem E.

Lemma 3 *Let $\Omega := \{0, 1/2\}^d \setminus \boldsymbol{\Gamma}_{A^T}$, let $u_{n,m}(t)$ and $h_{n,m}(t)$ be trigonometric polynomials that satisfy (19), let $P_{n,m}(\mathbf{t})$ be defined by (11), let $\mathbf{u} \in \mathbb{Z}^d$ be such that $r_1(A) \cdot \mathbf{u} = 1/2$, let $K = 2^d - 2$, and let $\rho : \Omega \to \{d+1, \ldots, K+d\}$ be a bijection. Let*

$$\widetilde{P}^{(j)}_{n,m}(A^T\mathbf{t}) := h_{n,m}(t_j) \prod_{s=j+1}^{d} u_{n,m}(t_s), \; j = 1, \ldots, d-1,$$
$$\widetilde{P}^{(d)}_{n,m}(A^T\mathbf{t}) := h_{n,m}(t_d),$$

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and

$$\widetilde{P}_{n,m}^{(\rho(\mathbf{r}))}(A^T\mathbf{t}) := \frac{1}{2}\big[(P_{n,m}(\mathbf{t}+\mathbf{r}) + P_{n,m}(\mathbf{t}+\mathbf{r}+\mathbf{r}_1(A))) \\ + e^{i2\pi\mathbf{t}\cdot\mathbf{u}}(P_{n,m}(\mathbf{t}+\mathbf{r}) - P_{n,m}(\mathbf{t}+\mathbf{r}+\mathbf{r}_1(A)))\big], \qquad \mathbf{r}\in\Omega,$$

then $\widetilde{P}_{n,m}^{(j)}(\mathbf{t})$*,* $j = 1\ldots, K+d$*, are trigonometric polynomials and*

$$\sum_{\mathbf{r}\in\mathbf{\Gamma}_{A^T}} |P_{n,m}(\mathbf{t}+\mathbf{r})|^2 + \sum_{j=1}^{K+d} |\widetilde{P}_{n,m}^{(j)}(A^T\mathbf{t})|^2 = 1. \tag{20}$$

Proof We start by showing that the $\widetilde{P}_{n,m}^{(j)}(\mathbf{t})$ are $\mathbb{Z}^d$-periodic polynomials. Assume first that $1\le j\le d-1$. Since $g_{n,2m}(t) + g_{n,2m}(t+1/2)$ has period $1/2$ we readily see that also the polynomials $h_{n,m}(t)$ and $u_{n,m}(t)$ have period $1/2$. This in turn implies that $P_{n,m}(A^T\mathbf{t})$ is $(1/2)\mathbb{Z}^d$-periodic. It will therefore suffice to show that if $\mathbf{k}\in\mathbb{R}^d$ and $\mathbf{x} = (A^T)^{-1}\mathbf{k}$, then $\mathbf{x}\in(1/2)\mathbb{Z}^d$. Since the determinant of A^T equals ± 2 and the columns of A^T are in $\mathbb{Z}^d$ this readily follows by an application of Cramer's rule.

From the definition it is also obvious that $\widetilde{P}_{n,m}^{(d)}(\mathbf{t})$ is $\mathbb{Z}^d$-periodic.

We now establish the $\mathbb{Z}^d$-periodicity of the functions $\widetilde{P}_{n,m}^{(\rho(\mathbf{r}))}(\mathbf{t})$. Let $k\in\mathbb{Z}^d$. If $\mathbf{k} = A^T(\mathbf{k}_1)$ for some $\mathbf{k}_1\in\mathbb{Z}^d$, then the $\mathbb{Z}^d$-periodicity of the polynomials $P_{n,m}(\mathbf{t})$ readily imply that $\widetilde{P}_{n,m}^{(j)}(\mathbf{t}+\mathbf{k}) = \widetilde{P}_{n,m}^{(j)}(\mathbf{t})$. On the other hand, if $\mathbf{k} = A^T(\mathbf{r}_1(A) + \mathbf{k}_2)$ for some $\mathbf{k}_2\in\mathbb{Z}^d$, the assertion follows by observing that $2\mathbf{r}_1(A)\in\mathbb{Z}^n$ and $e^{i2\pi(\mathbf{t}+\mathbf{r}_1(A))\cdot\mathbf{u}} = -e^{i2\pi\mathbf{t}\cdot\mathbf{u}}$.

Let $\mathbf{\Gamma} = \mathbf{\Gamma}_{A^T}$. We claim that for $\mathbf{r}\in\Omega$ there exists an unique $\widetilde{\mathbf{r}}\in\Omega$, $\widetilde{\mathbf{r}}\ne\mathbf{r}$, such that $\mathbf{r}+\mathbf{r}_1(A)+\mathbf{k}_3 = \widetilde{\mathbf{r}}$ for some $\mathbf{k}_3\in\mathbb{Z}^d$. Let us verify this assertion. Since $\mathbf{r}+\mathbf{r}_1(A)\in\{0,\frac{1}{2},1\}^d$, there exists an unique $\mathbf{k}_3\in\mathbb{Z}^d$ such that $\mathbf{r}+\mathbf{r}_1(A)+\mathbf{k}_3\in\{0,\frac{1}{2}\}^d$. Let $\widetilde{\mathbf{r}} := \mathbf{r}+\mathbf{r}_1(A)+\mathbf{k}_3$. We need to show that $\widetilde{\mathbf{r}}$ is neither $(0,\ldots,0)$ nor $\mathbf{r}_1(A)$ nor r. If $\widetilde{\mathbf{r}} = (0,\ldots,0)$ then $\mathbf{r}+\mathbf{r}_1(A)\in\{0,1\}^d$. This implies that $\mathbf{r} = \mathbf{r}_1(A)$, which contradicts the hypothesis that $\mathbf{r}\in\Omega$. In similar fashion we can see that $\widetilde{\mathbf{r}}$ is neither $\mathbf{r}_1(A)$ nor $\mathbf{r}$.

Conversely, there exists an unique $\mathbf{k}_4\in\mathbb{Z}^d$ such that $\widetilde{\mathbf{r}}+\mathbf{r}_1(A)+\mathbf{k}_4 = \mathbf{r}$. Indeed, repeating the preceding argument we conclude that there exists an unique $\mathbf{k}_5\in\mathbb{Z}^d$ such that $\widetilde{\mathbf{r}}+\mathbf{r}_1(A)+\mathbf{k}_5\in\{0,\frac{1}{2}\}^d$. Let $\mathbf{k}_4 := \mathbf{k}_5$. Since $\widetilde{\mathbf{r}} = \mathbf{r}+\mathbf{r}_1(A)+\mathbf{k}_3$, it follows that $\mathbf{r}+2\mathbf{r}_1(A)+\mathbf{k}_3+\mathbf{k}_4\in\{0,\frac{1}{2}\}^d$. Bearing in mind that $2\mathbf{r}_1(A)\in\mathbb{Z}^d$ and $\mathbf{r}\in\Omega$, we have $2\mathbf{r}_1(A)+\mathbf{k}_3+\mathbf{k}_4 = \mathbf{0}$. Thus

$$\widetilde{\mathbf{r}}+\mathbf{r}_1(A)+\mathbf{k}_4 = \mathbf{r}+2\mathbf{r}_1(A)+\mathbf{k}_3+\mathbf{k}_4 = \mathbf{r}.$$

We have therefore shown that there exist two disjoint sets $\Omega_1,\Omega_2\subset\Omega$, such that $\Omega = \Omega_1\cup\Omega_2$ and for any $\mathbf{r}\in\Omega_1$ there exists an unique $\widetilde{\mathbf{r}}\in\Omega_2$ such that $\widetilde{\mathbf{r}} = \mathbf{r}+\mathbf{r}_1(A)+\mathbf{k}$ and $\mathbf{r} = \widetilde{\mathbf{r}}+\mathbf{r}_1(A)+\mathbf{m}$ for some $\mathbf{k},\mathbf{m}\in\mathbb{Z}^d$. Since, moreover, $\widetilde{P}_{n,m}^{(\rho(\mathbf{r}))}(A^T\mathbf{t})$ and $\widetilde{P}_{n,m}^{(\rho(\widetilde{\mathbf{r}}))}(A^T\mathbf{t})$ are complex conjugates of each other, we readily see that

$$|\widetilde{P}_{n,m}^{(\rho(\mathbf{r}))}(A^T\mathbf{t})|^2 + |\widetilde{P}_{n,m}^{(\rho(\widetilde{\mathbf{r}}))}(A^T\mathbf{t})|^2 = |P_{n,m}(\mathbf{t}+\mathbf{r})|^2 + |P_{n,m}(\mathbf{t}+\mathbf{r}+\mathbf{r}_1(A))|^2.$$

Therefore

$$\begin{aligned}
&\sum_{j=d+1}^{K+d} |\widetilde{P}_{n,m}^{(j)}(A^T\mathbf{t})|^2 \\
&\quad = \sum_{\mathbf{r}\in\Omega} |\widetilde{P}_{n,m}^{(\rho(\mathbf{r}))}(A^T\mathbf{t})|^2 = \sum_{\mathbf{r}\in\Omega_1} |\widetilde{P}_{n,m}^{(\rho(\mathbf{r}))}(A^T\mathbf{t})|^2 + \sum_{\widetilde{\mathbf{r}}\in\Omega_2} |\widetilde{P}_{n,m}^{(\rho(\widetilde{\mathbf{r}}))}(A^T\mathbf{t})|^2 \\
&\quad = \sum_{\mathbf{r}\in\Omega} |P_{n,m}(\mathbf{t}+\mathbf{r})|^2.
\end{aligned}$$

The remainder of the proof is a repetition of the argument used in the original version of this lemma. □